Econ 353 Final Project: Computational Forms of Economic Development Models

Xin Scott Chen

Econ 353

Dr. Alan Mehlenbacher

Introduction

The purpose of this paper is to provide an introduction into the field of development economics, both its history and its many innovations. The first section of this paper will provide a brief history into development economics models, from the earliest work by Thomas Malthus, to the revolutionary model by Arthur Lewis and its various extensions. The second part of this paper will formalize the Lewis, Harris-Todaro, and Solow-Swan models mathematically. These three models are then programmed into Wolfram's Mathematica 7.0 to carry out a variety of experiments which will provide greater insight into these development models.

The field of development economics first gained recognition with Malthus' 1798 paper where he made a dire prediction that population growth will only lead to persistent poverty as growth in food production will always be slower than population growth (Malthus, 1826). Malthus, of course, wrote this before the industrial revolution, which increased productivity by leaps and bounds. The study of development economics took a long pause as major conflicts plagued the world in the late 19th and early 20th century. After the second world war, the world was left in tatters, with nearly every nation devastated economically. The victors, the Allies, did not remain allies for long, the United States and the Soviet Union immediately became opponents. The United States and its western allies occupied most of Western Europe; while the Soviet Union occupied most of Eastern Europe. Every part of Europe has been devastated by the war, massive amount of resources were required for reconstruction. The two opposing superpowers both proposed plans to support European nations under their control. The US and its western allies had the Marshall plan, and the Soviet Union had the Molotov plan. Both nations competed to court European nations to follow under Western ideals of Democracy and Capitalism or the Soviet Union's Stalinist Communist policies. The period following WWII is a period of relative peace; with the massive allocation of reconstruction funds, many governments to ask the question of what is the best way to use these funds to grow an economy. This led to the resurrection of the field of development economics.

Before the 1950's, nearly all economic growth models focused on savings and investments as an important driving force of economic development. The Solow-Swan model was written later but a good illustration of an advanced version such a model. There are several major failings with savings driven growth models. It is true that these models will fit the Western nations such as United States (Northern)[1] and the British Empire when they were entering the various stages of the industrial revolution. However, the situation

[1] The North part of the US (Union Regions), were heavily industrialized; whereas the South (Confederate Regions) were more agricultural based with very little industry. One of the major conditions of the Reconstruction Act was for the North to assist in industrializing the South.

in the Less Developed Countries (LDC's)[2] were drastically from the US and Britain during the industrial revolution. Many of these developing nations are not endowed with substantial natural resources, population, or infrastructure for innovation. Infrastructures, in many developing nations, were non-existent or devastated in the war. Some policymakers and economists saw the need for new growth models which would fit the conditions in developing nations.

Model Descriptions

Lewis Model (Surplus Labour Model)

The Lewis (1954) Model was the first model to explicitly focus on dualist economic development. The original Lewis model was simple yet genius with the clarity he expressed his ideas, nearly every development model is some way related to the roots of the Lewis Model. The Lewis model understood that in most LDC's, most workers are in the rural/agricultural sector. Agricultural sector in these nations are not endowed with capital with high productivity like agricultural sectors in the developed nations. Agricultural sector in the LDC's are mostly family farms characterized by low productivity and uncertain output. This assumption made by Arthur Lewis is valid in most LDC's; every development model after the Lewis model has used some form of the assumptions made by Arthur Lewis.

The Lewis model abandoned the traditional assumptions of labour and capital market equilibria. The Lewis model assumes that the rural (agricultural) sector is very primitive; it is labour intensive, with little to no capital endowment resulting in subsistence agriculture. The urban (modern/industrial) sector is capital driven with much higher standards of living than the rural sector. This is what defines Lewis' dualist economy, with the rural sector characterized by subsistence agriculture, and urban sector characterized by capital industries. The most prominent feature of the Lewis model is the introduction of the "Surplus Labour" concept. Lewis defined surplus labour as workers in the agriculture sector with zero marginal productivity. What this implies is that these workers can be taken out of the rural sector without reduction in agricultural output. There are of course strong assumptions behind this. First is that all non-surplus labourers were working at full capacity, so that when withdrawing workers with productivity others cannot work harder to compensate for the lost output from these workers. Another assumption is that there are no concept of landlords and capitalist farmers in rural sector. There are no capitalist farmers paying rent to landlords; no capitalist farmers hiring

[2] UN's definition for LDC's is that gross national income per capita below $750, weak Human Assets Index which includes health, education and other indicators, and vulnerable according to the Economic Vulnerability Index which includes instability of agricultural production, instability in provision of goods and services. See http://www.un.org/special-rep/ohrlls/ldc/ldc%20criteria.htm

workers and paying them wage based on marginal productivity. Instead, the agricultural sector is simply subsistence agriculture with each worker making wage equal to average product. This is an empirically true assumption as in most LDC's, family members share incomes, and in some cases, villages share incomes in order to insure against local disasters which would devastate output.

The Lewis model proposes that withdrawing surplus labourers from the rural sector and sending them to urban sector will not result in decrease in agricultural output hence no change in relative scarcity of agricultural and urban goods. Since wage is determined on the average and not the margin (average wage is simply output divided by number of workers), as surplus labourers are moved to the urban sector, there will be less workers remaining in the rural sector, increasing the rural wage. This increase in wage will reduce the incentive for further workers to migrate. If workers with marginal productivity are withdrawn from the rural sector, in the Lewis model, there would be a reduction in agricultural output. A decrease in agricultural goods implies that workers will have to change their previous agricultural/industrial goods consumption bundle, and this will make them worse off. Lewis' original paper did not explicitly define the relative prices and terms of trade between rural and urban sectors. He believed that the consistent driver of growth is technological advancement, not manipulation of relative prices. Technological advancement in either sector, but especially the rural will allow for development without the problems of food shortages.

Harris-Todaro Model (Two Sector with Unemployment)

The Harris-Todaro (1970) model's key contribution to the field of development economics is by making the migration process a rational choice based on expected earnings. The Harris-Todaro (H-T) model takes most of Lewis models' assumptions as given, such as the rural sector being characterized by subsistence agriculture, and the urban sector being characterized by modernized industries. The Harris-Todaro model takes a standard two sector model and imposes a higher wage in the urban sector which is higher than equilibrium clearing, while wage in agriculture is flexible. Equilibrium clearing is simply when wage across both sectors equalize, minus movement costs or natural advantages (such as better living environment) in 1 or the other sector. By imposing this higher wage in the urban sector, we no longer have market clearing wage which gives the workers in the rural sector an incentive to migrate to the urban sector. These migrant workers are not guaranteed to find a job in the urban sector. There is a probability that they will end being unemployed or in the informal sector.

For modeling simplicity, it is usually assumed that only 1 of these two sectors are in the model. It fits the situation in LDC's better to assume that an informal sector exists in the urban sector than unemployment. LDC's are unlikely to have good social safety nets such as welfare benefits, unemployment benefits, and old

age security. Without these benefits, workers in urban sector must do some work to keep themselves alive. If they were unable to find a job in the urban formal sector, which is the modern industrial sector, they would be forced to work in the informal sector to keep themselves alive. The informal sector is very primitive; work in this sector is labour intensive with little or no capital endowment.

The equilibrium condition of the Harris-Todaro model can be described as the wage in agriculture must be equal to the expected wage in the urban sector. The model in its most basic form ignores disutility from not being at home farm, or cost of mobility, but these omissions do not change the essence of the model, the only implication of this is a downward shift of the urban sector's expected returns. This equilibrium can be defined as,

$$w_a = \left(\frac{L_f}{L_f + L_i}\right) w_f + \left(\frac{L_i}{L_f + L_i}\right) w_i$$

Where

w_a denote the wage in rural (agricultural) sector

w_f denote the wage in urban formal (industry) sector

w_i denote the wage in urban informal sector

L_f denote the number of workers in the urban formal sector

L_i denote the number of workers in the urban informal sector

The left hand side of the equation is simply the agricultural wage. The right hand side, $L_f + L_i$ which is formal sector labour force plus informal sector labour force; combining these results in the entire labour force in the urban sector. $\left(\frac{L_f}{L_f + L_i}\right)$ then is simply the ratio of urban workers in the formal sector, in the Harris-Todaro model, this is what the potential migrant sees as the probability of finding a job in the formal sector. Similarly, $\left(\frac{L_i}{L_f + L_i}\right)$ is what the potential migrant sees as the probability of ending up in the informal sector. The probabilities of each sector is then multiplied by that sector's respective wage, adding the results together yields the right hand side of the Harris-Todaro equilibrium, which is the expected wage from moving to the urban sector.

The Harris-Todaro in essence is an extension of the Lewis model. It simply endogenizes migration decision along with the introduction of a second urban sector. It does not change from the Lewis model in that the fundamental driving force of growth is still technological growth.

Solow-Swan Model (Exogenous Growth Model)

The Solow-Swan model is perhaps the most well known growth model in economics. Similar to the Harrod-Domar model, growth in this model occurs exogenously. But unlike the Harrod-Domar model, where growth is driven purely by savings, the driving force of growth in the Solow-Swan model is technological growth and savings. A caveat of the Solow-Swan model is that it maximizes lifetime consumption rather than utility. The Solow-Swan model does not account discounting, which simply states that people value consumption today more than tomorrow. The Ramsey-Koopmans-Cass model modified the Solow-Swan model from consumption maximization to utility maximization, it also endogenized savings decision. The result was a lower level of Golden Rule level of savings. Despite the popularity of the Solow-Swan model and its extensions, most of its assumptions are inapplicable to LDC's. Even after the more modern endogenous growth models coming into prominence in the recent decades, the Solow-Swan model is still a benchmark model in economics that deserves to be properly understood.

Computational Models

Lewis Model

The Lewis model is a relatively simple model to express mathematically. The agricultural sector will have a simple Cobb Douglas production function, for simplicity, output in rural is assumed to only depend on labour, the production function is,

$$Y = AL^{\alpha}$$

Where Y is agricultural output, A is the technological parameter (A>0), L denoting labour force in agriculture, α (0<α<1) is the parameter.

This production function will yield a concave curve with diminishing marginal productivity. To depict the surplus labour in the Lewis model, a section of the production function must be flat in order to capture the concept of zero marginal productivity (first order derivative being 0). In order to accomplish this in Mathematica, a piecewise defined function is required.

The function is defined in Mathematica as, $\text{Piecewise}[\{\{AL\text{^}\alpha, L < \text{SL}\}, \{A\text{SL}\text{^}\alpha, L \geq \text{SL}\}\}], \{L, 0,1\}$ Basically it states that at Labour allocation level below Surplus Labour (SL), the productive function will be concave, $AL\text{^}\alpha$.

At levels above SL, the function will be flat. $A\mathrm{SL}\hat{}\alpha$ is a simple constant because SL is defined between 0 and 1 as a proportion of the agricultural labour force being surplus labourers. SL=0.8 means that 20% of the workers are surplus labourers. A, SL, and α are all parametric constants; hence, $A\mathrm{SL}\hat{}\alpha$ is a linear function. Plotting the function above yields Figure 1 below,

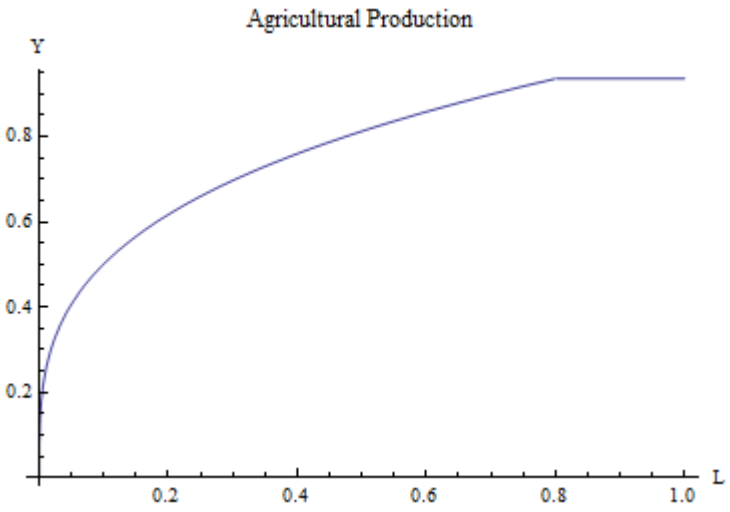

Figure 1

The flat part of the production function depicts the surplus labour, characterized by zero marginal product. The experiments in the following section on the Lewis model are carried out with the Mathematica's dynamic functions.

Harris Todaro Model

The Harris-Todaro model is far more difficult to program than the Lewis model as it is a 2 sector model. For the sake of simplicity in modeling, let us assume that instead of an informal sector, there is unemployment, and that these workers receive some minimum compensation to survive. This simplification does not change the core of the H-T model. Even with an informal sector in the H-T model, the wage in the informal sector is significantly lower than formal wage, and potentially lower than the agricultural wage. By imposing an unemployment sector, it simply makes not being in the urban formal sector receive a wage of zero. This simplifies the Harris-Todaro equilibrium condition to the following,

$$w_a = \left(\frac{L_f}{N_u}\right) w_f$$

Where Nu denote urban population (Lf+U).

We will assume Cobb Douglas production function in both rural and urban sectors. Wage determination in both sector is assumed to be on the margin, instead of average as was the case in the Lewis model.

The following computational model is adapted from a paper on the Harris-Todaro model by Espindola et al. (2005).

Basic Model Setup

Rural sector production function:

$$Y_a = A\mathrm{La}^{\alpha}$$

Where Y_a denote agricultural sector output, A is the technological parameter in the rural sector (A>0), La denote the agricultural labour force, and α is production parameter (0<α<1).

Urban sector production function:

$$Y_f = B\mathrm{Lf}^{\beta}$$

Where Y_f denote urban sector output, B is the technological parameter in the urban sector (B>0), Lf denote the urban labour force, and α is production parameter (0<β<1).

Wage in agricultural sector is flexible and determined at the margin, the marginal productivity in agriculture is,

$$\frac{\partial Y_a}{\partial La} = \alpha A\mathrm{La}^{-1+\alpha}$$

The agriculture wage is the marginal productivity multiplied by the price of agricultural good, let this be denoted by p. The agricultural wage then is,

$$w_a = \alpha A\mathrm{La}^{-1+\alpha}\mathrm{p}$$

The urban wage is determined at the margin; however, as per the H-T model assumption, the wage in urban sector is imposed at a level above market clearing. The marginal productivity in urban sector is,

$$\frac{\partial Y_f}{\partial Lf} = \beta B\mathrm{Lf}^{-1+\beta}$$

The wage in the H-T model is then,

$$w_f = \beta B\mathrm{Lf}^{-1+\beta} \; such\; that\; Lf \leq Nu$$

Where Nu denote the total urban population, if Lf<Nu, then there is unemployment in the urban sector, if Lf=Nu, then there is full employment in the urban sector. The imposed higher than equilibrium wage is denoted by wfbar.

The relative price between the urban and rural sector is based on the terms of trade, or relative scarcity of urban and rural output.

$$p = \rho(\frac{Y_f}{Y_a})^{\gamma}$$

Where p is price of agricultural good, γ and ρ are parameters (γ > 0 and ρ > 0).

Now, a few definitions of labour force and population must be defined. Let Na be population in rural sector, and recall La is the labour force in rural sector, in this model, it is assumed that Na=La. In the urban sector, let Nu be the population in the urban sector, and recall that Lf is the labour force in the urban sector. As defined previously, if Lf<Nu, then there is unemployment in the urban sector, if Lf=Nu, then there is full employment in the urban sector. Let Ntot be total population in the entire economy, both urban and rural sectors. The following identity can then be defined as,

$$Na + Nu = Ntot$$

Short Run Equilibrium

Setting the marginal productivity curves of the two sectors equal will solve for the short run competitive equilibrium in these two sectors, there will be no migration in this equilibrium.

Assuming both sectors are competitive, the short run equilibrium is where the marginal productivity curve from both sectors intersects. The following command is used in Mathematica to solve the short run equilibrium, $\mathrm{Lftemp} = \mathrm{Solve}[\mathrm{MPLa} - \mathrm{MPLf} == 0, \mathrm{Lf}$, note MPLa is marginal productivity of labour in agriculture and MPLf is marginal productivity of labour in urban, this is solved to be

$$\mathrm{Lftemp} = (\frac{A\mathrm{La}^{-1+\alpha}\alpha}{B\beta})^{\frac{1}{-1+\beta}}$$

Imposing full employment in rural sector would imply that Na=La, so population equals labour force. This also implies that,

$$La = Ntot - Nu$$

With Lf, La solved in short as exogenous variables, Outputs Yf and Ya can be solved for the short run.

$$Y_f = B((\frac{ALa^{-1+\alpha}\alpha}{B\beta})^{\frac{1}{-1+\beta}})^{\beta}$$

$$Y_a = A(Ntot - Nu)^{\alpha}$$

Recall that rural wage is marginal productivity multiplied by the price factor, the price can be solved as,

$$p = (\frac{B(Ntot - Nu)^{-\alpha}((\frac{A(Ntot - Nu)^{-1+\alpha}\alpha}{B\beta})^{\frac{1}{-1+\beta}})^{\beta}}{A})^{\gamma}\rho$$

Which yields the agricultural wage to be,

$$w_a = A(Ntot - Nu)^{-1+\alpha}\alpha(\frac{B(Ntot - Nu)^{-\alpha}((\frac{A(Ntot - Nu)^{-1+\alpha}\alpha}{B\beta})^{\frac{1}{-1+\beta}})^{\beta}}{A})^{\gamma}\rho$$

wa is perfectly flexible whereas wfbar is rigidly imposed at a level above clearing. The agricultural wage equation above adjusts to reach a short run equilibrium.

Long Run Equilibrium

The short run equilibrium does not hold in the Harris-Todaro framework as there is expected wage differential, rural workers will want to migrate to the urban sector which pays a higher wage. Recall the Harris Todaro equilibrium condition is,

$$w_a = \left(\frac{L_f}{N_u}\right)w_f$$

The right hand side is simply the expected wage from the urban sector. When the right hand side is greater than the left, there will be migration. Let M denote the wage differential between expected urban wage and agricultural wage,

$$M = \left(\frac{L_f}{N_u}\right)w_f - w_a$$

What is solved with this equation is wage differential; but as part of the Harris-Todaro assumption that difference in expected wage is what drives migration implies that migration will be a simple function (assuming linear for modeling simplicity) of the wage differential. It is reasonable to say that there is a perfect relationship between wage differential and migration in the Harris-Todaro model. This simply means that the higher the wage differential (larger M), the greater the number of migrants. The parametric result of the Long run equilibrium is in the Experiments and Discussion section.

Solow-Swan Model

The Solow-Swan model used for illustration here is in a very simplistic form. Constant returns to scale is assumed. The relevant factors which affect growth in this particular specification of the Solow-Swan model are depreciation, denoted by δ, population growth, denoted by n, and technological growth, denoted by g.

Everything will be expressed in per unit terms, which yields the production function per worker,

$$y = f(k) = k^{\alpha}$$

Where k denotes capital and 0<α<1.

Consumption is defined as

$$c = (1 - s)\, y$$

Where s is rate of savings (marginal propensity to save thus 0<s<1).

Investment (equals savings in this simple autarky model)

$$i = s\, y$$

The equilibrium of the Solow-Swan model is discussed in the following Experiments and Discussions section.

Experiments and Discussions

1. **Lewis Model**

The Lewis model is a simple yet fascinating model and it allows us to analyze how much of the labour force would be considered “surplus labourers”. The output in Figure 2 is simply the benchmark model, the production and technological parameters are both at reasonable levels, along with surplus labour definition set at 0.8. The surplus labour definition here implies that in this particular rural sector, 80% of workers have positive marginal productivity; while 20% of workers are surplus labourers with zero marginal productivity. Surplus labour would be depicted as the flat part on the agricultural production function, as the first order derivative would yield 0.

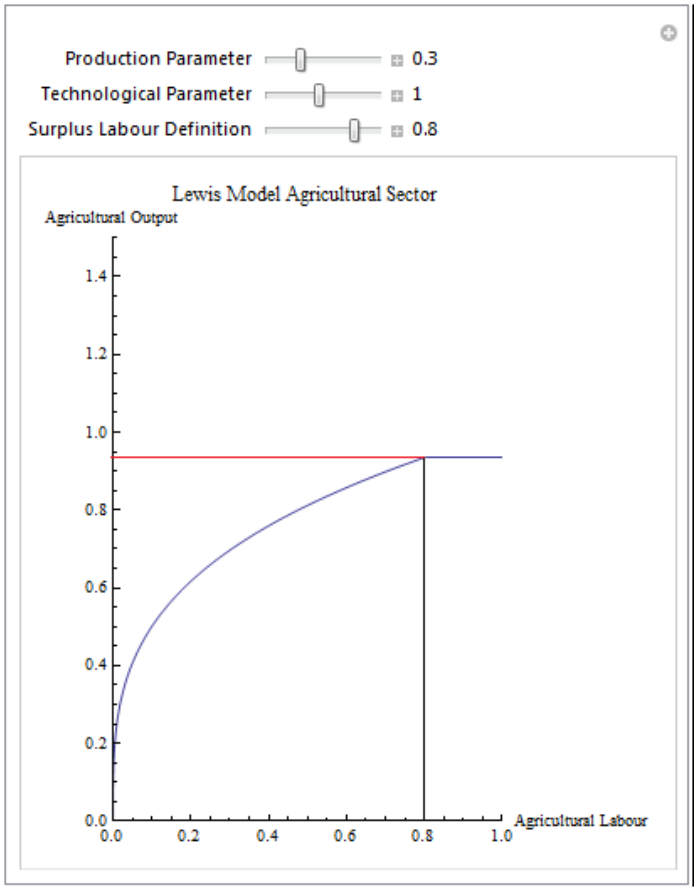

Figure 2

Several experiments can be performed in this Mathematica depiction of the Lewis model. First, a simple intuitive experiment would be to decrease/increase the technological parameter to see its effects. Second, the surplus labour definition can be modified to fit economies of different LDC's. In Sub-Saharan Africa nations, this term could be extremely low, as low as 0.1, which would imply that 90% of the workers are surplus labourers.

Experiment 1.1: Higher technological parameter

This experiment increases technological parameter from 1 to 1.5. The intuitions behind this experiment are very simple, with a higher technological parameter, it should result in an upward shift of the production curve. This should lead to an increase in output and if no modifications are made to the surplus labour definition, still only 20% of the labour force would be surplus labourers. The result for this experiment is depicted in Figure 3 below.

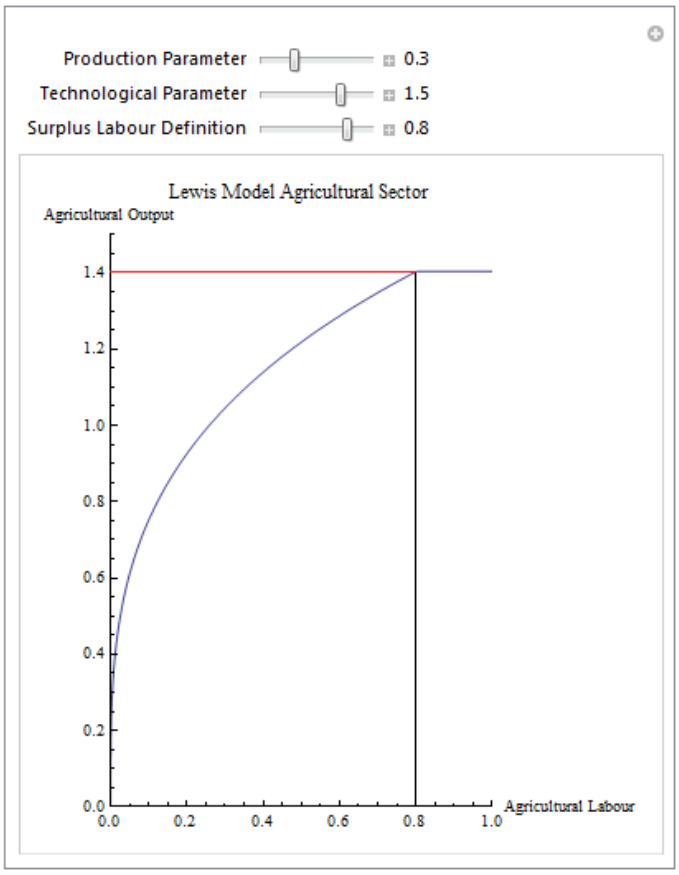

Figure 3

Discussion 1.1

The results in this experiment are exactly as predicted without any surprises. This is a very simple change which shifted up the technological parameter. This results in higher agricultural output which implies higher productivity from workers. As there is no capital in the production function, this change in technological parameter results is a labour-augmenting technological improvement, which results in higher marginal productivity. This of course occurs up to the point of surplus labour definition, where despite the increased productivity, the remaining workers are still considered surplus labourers as they have zero marginal productivity.

The implications of this increased agricultural technological parameter on rural-urban migration are far more interesting. The higher technological parameter in rural sector results in higher output and higher average product, thus wage, for agricultural workers. This higher agricultural wage will reduce the wage differential between the rural and urban sector, which in turn reduces the incentive for rural workers to migrate to the urban sector. The government still has an incentive to try to move the surplus labourers from the rural to the urban sector, as this does not reduce agricultural output, at the same time, providing urban

industries with more workers. However, if in the extreme case, agricultural is so abundant such that average wage in rural sector is greater than wage in urban sector, then there will be no rural-urban migration.

The converse of experiment 1.1 is unnecessary; it is easy to predict that reduced technological parameter will lead to lower agricultural output and marginal productivity. The proportion of surplus labour will not change as the surplus labour definition is not modified. The graph of setting technological parameter at 0.5 instead of 1 is illustrated below in Figure 4.

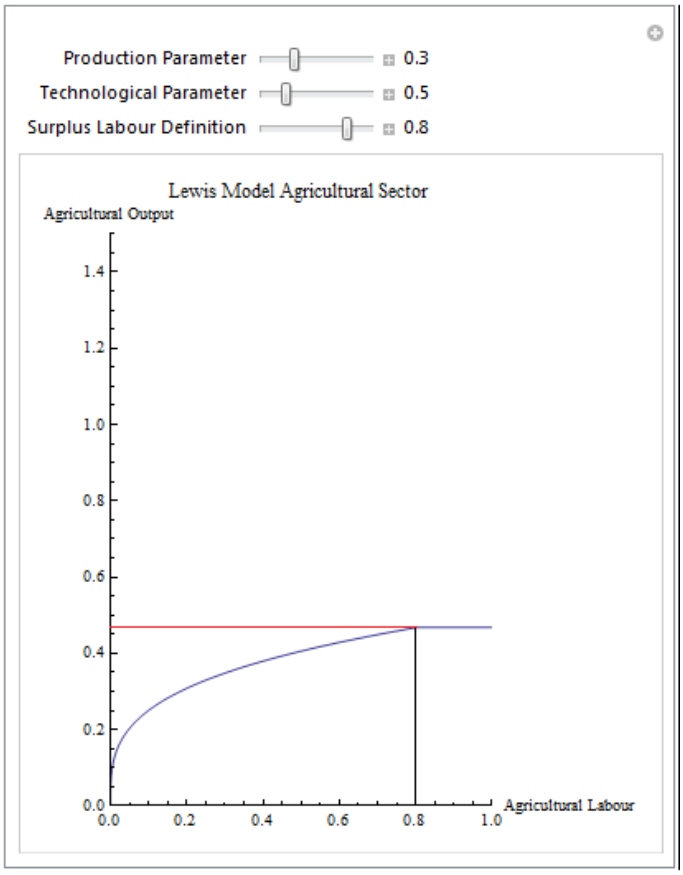

Figure 4

Experiment 1.2: Lower Surplus Labour Definition

A reduction in the surplus labour definition implies that there are proportionally less workers with marginal productivity and more with zero (surplus labourers). The intuition of this is that with less workers actually producing output (positive marginal product), there would be less total output, along with of course a greater proportion of workers being surplus labourers. In this experiment, the surplus labour definition is modified to 0.2, the result is below in Figure 5.

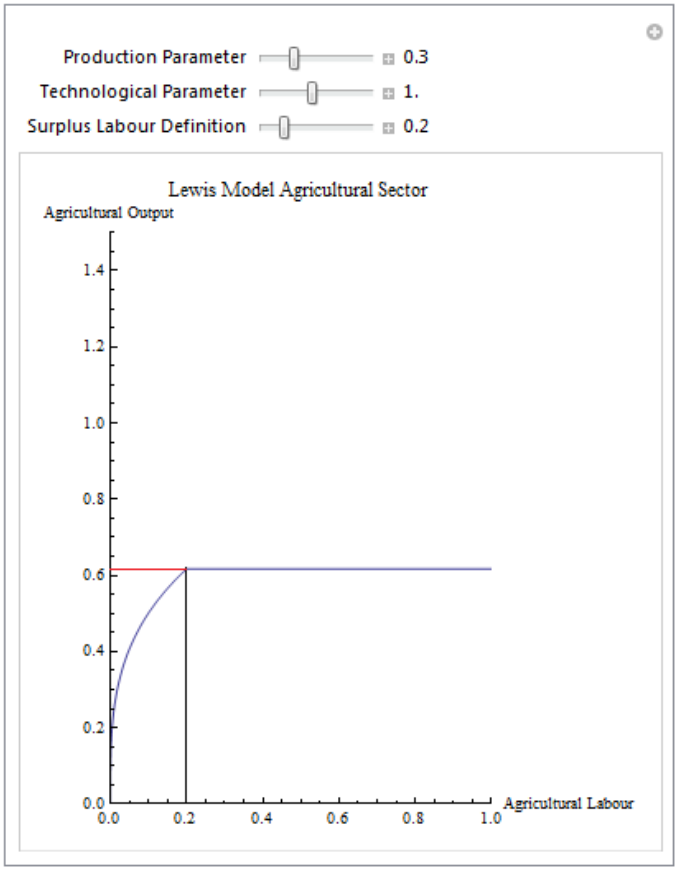

Figure 5

Discussion 1.2

The result matches the intuitions yet again, there is much lower output in agriculture, and surplus labour now occupy 80% of the agricultural labour force. In many parts of Southeast Asia and Sub-Saharan Africa, cultivable lands are incredibly scarce, a few workers is sufficient to complete all tasks required in the agricultural sector; this results in a great proportion of the agricultural population being surplus labourers. With the Lewis model's assumption of agricultural wage being determined based on average product, it results in everybody with lower (near subsistence) wage.

This great proportion of agricultural labour with zero marginal productivity (surplus labourers), results in a lower agricultural wage and at the same time, allows for more worker to be withdrawn from the agricultural sector without reducing output. Agricultural workers have a greater incentive to migrate as wage in urban sector would be relatively much higher; the government, at the same time, could greatly accelerate urban industry growth by incentivizing these low-wage rural surplus labourers to migrate to urban sector.

2. Harris Todaro Model

The Harris-Todaro model is the most complex here to program. The computational model section has described the process of programming the math in this model in depth. Recall from the computational model section, the number of migrants is assumed to have a perfect relationship with the wage differential. Wage differential is defined as $M = w_{urban}^{expected} - w_a$. Solving this condition yields a very messy parametric result,

$$M = \frac{wfbar(\frac{A(Ntot-Nu)^{-1+\alpha}\alpha}{B\beta})^{\frac{1}{-1+\beta}}}{Nu} - A(Ntot-Nu)^{-1+\alpha}\alpha(\frac{B(Ntot-Nu)^{-\alpha}((\frac{A(Ntot-Nu)^{-1+\alpha}\alpha}{B\beta})^{\frac{1}{-1+\beta}})^{\beta}}{A})^{\gamma}\rho$$

Despite the complexity and messy looking equation, every single variable on the right hand side is exogenously determined. After specifying for all the parameters, a numerical solution can be solved for M. Plotting this solution would be meaningless as it would yield a simple linear curve. The solution solved here by itself is of no interest either, it is only a relative term as to how much wage differential is affected by the modification of the exogenous parameters. Table 1 below depicts the parametric constants that needs to be defined,

Parametric Term	Description	Restrictions	Reasonable Base Value
$\boldsymbol{\alpha}$	agricultural production parameter	$0<\alpha<1$	0.3
$\boldsymbol{\beta}$	urban production parameter	$0<\beta<1$	0.7
A	agricultural technological parameter	A>0	1
B	urban technological parameter	B>0	1.5
Ntot	total population, let Ntot=1, so total population is 1, this simply expresses the population/labour force in urban and rural sector in a proportion term.	Ntot=1	1
Nu	urban population expressed as a proportion of total popn.	0≤Nu≤1	0.3
$\boldsymbol{\rho}$	terms of trade constant	ρ>0	1
$\boldsymbol{\gamma}$	elasticity of relative prices	γ>0	1
wfbar	urban higher than equilibrium imposed wage	wfbar > wf	4

Table 1

The starting value of wfbar requires more explanation. As part of modeling assumptions, wfbar>wf, it is necessary to find a value for wf, imposing Lf=Nu, which implies no unemployment, this simply yields,

$$B\mathrm{Nu}^{-1+\beta}\beta$$

Using the specified values in the table above, this yields a value of 1.50679. wfbar defined at 4 satisfies the restriction and is therefore a reasonable starting value.

Experiments 2: Harris Todaro Model Experiments

With the parametric model defined above, there are countless possible experiments to perform. It is important to ensure all the restriction conditions hold as parameters are modified, the one to be very careful of is the wfbar value; since this value chances as other parameters are modified. The following experiments will be performed using this form of the Harris Todaro model.

a. Impose greater differential in technology between rural and urban sector. A=1, B=2.5
b. Higher proportions of workers start off being in the urban sector. Nu=0.7
c. Terms of trade against agriculture. ρ = 0.5
d. Terms of trade against urban. ρ = 1.5
e. Imposing capitalist agriculture and urban sector, basically assuming industrial organization of both sectors are identical. Imposing A=B, and α=β.
f. As a final experiment, the parameters will be modified to more closely resemble the Chinese economy. Despite the giant leaps forward in the Chinese agricultural sector output since the 1978 economic reforms, urban and rural still are structured differently. The production parameters are modified slightly to reduce the difference in rural and urban structures. The technological differences between the two sectors are substantial, so it is modified again to ensure greater technological differential. The base model's assigned distribution of population is fitting to China. As the Chinese government no longer control prices of agricultural goods after the 1978 economic reforms, there's no manipulation of the terms of trade factor.

The results from all 5 of the above experiments are displayed in Table 2 below.

Using the starting values in Table 1 yielded a result of **370.948**, again this result in absolute terms is meaningless, it's an expression which depicts the wage differential between expected urban wage and rural wage. This value will be used to compare to the result value from the experiments.

	experiment a	experiment b	experiment c	experiment d	experiment e	experiment f
α	0.3	0.3	0.3	0.3	0.7	0.33
β	0.7	0.7	0.7	0.7	0.7	0.66
A	1	1	1	1	1.5	1
B	2.5	1.5	1.5	1.5	1.5	2.5
Ntot	1	1	1	1	1	1
Nu	0.3	0.7	0.3	0.3	0.3	0.3
ρ	1	1	0.5	1.5	1	1
γ	1	1	1	1	1	1
wfbar	4	4	4	4	4	4
Results (M)	**2036.15**	**18.507**	**374.287**	**367.609**	**8.164**	**733.87**

Table 2

Looking at the results from the table above, it is sometimes very difficult to intuitively predict these results, especially the magnitude. One of the more interesting result is that imposing greater technological differential between the two sectors causes massive increase in wage differential between the two sectors as shown in experiment a. Experiment b simply states that with a great proportion of workers in urban sector, rural sector will have labour shortages, which drives up agricultural wage. This reduces wage differential greatly resulting in much less migration. Experiments c and d tests the manipulation of the rural-urban terms of trade and its impacts on migration. The result is that terms of trade have very little impact on wage differential and migration. Experiment e very simply demonstrates that if the two sectors were to be organized in similar industrial structures, the only wage differential comes from labour excess or shortages. As the model defines excess workers in the rural sector, there would be some wage differential and some, albeit very little migration. Experiment f is a simple emulation of the Chinese economy, which is experiencing the greatest rural-urban migration in history. The results here is somewhat fitting of the real life situation in China. The experiment yielded significant wage differential, mostly coming from the difference in technology.

Discussion 2: Harris Todaro Model

There are several interesting results from the Harris Todaro experiments. First, it is very clear that modification to the technological differential between the two sectors has the greatest impact on wage differential. In experiments c and d, it is demonstrated that modification of the term of trade factor, either

against the rural or urban sector has little impact on wage differential, and thus little impact on migration. These two results are very much consistent with the Lewis model's predictions. This does not come as a great surprise since the Harris-Todaro is essentially an extension of the Lewis model. However, it is very interesting that using this computational model proved both of Arthur Lewis' key findings.

The other interesting experiment is f, which attempts to emulate the Chinese economy. With this relatively simple framework, it does capture a few key essence of the Chinese economy. First, most workers are in the rural sector. Second, there is significant technological differential between the two sectors; however, this difference is diminishing since the 1978 economic reforms. There is also no manipulation of agricultural good prices since the reforms. With these specifications, the results depict a situation which resembles the migration situation in China, where 150 million people in the past two decades have migrated from the rural to the urban sector.

3. Solow-Swan Model

The Solow-Swan growth model is not as frequently mentioned in development economics as the Lewis or Harris-Todaro models. Its assumptions simply do fit LDC's; therefore, only a few experiments will be carried out to illustrate several mechanisms of the model. First, a basic model with reasonable parameters assigned (Figure 6).

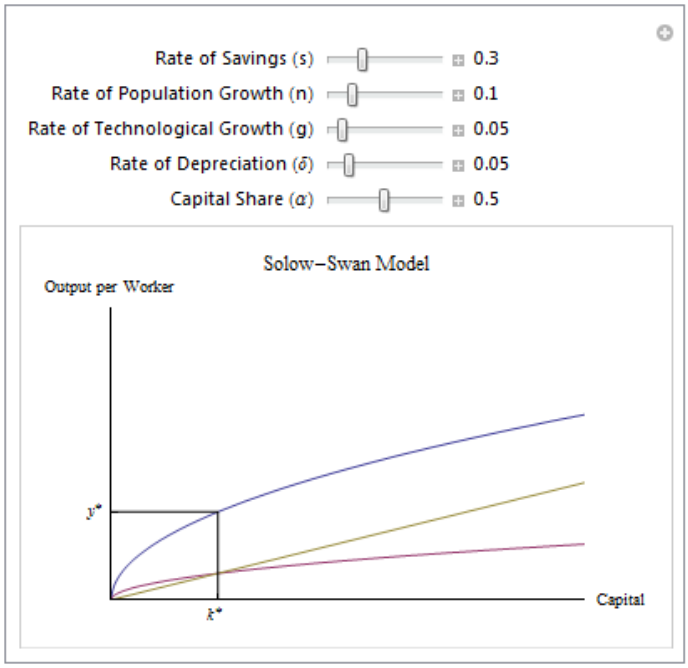

Figure 6

Experiment 3.1: Higher Marginal Propensity to Save

In the Solow-Swan model, an increase in rate savings (less consumption) will lead to higher investments, which in turn increases the golden rule level of capital. Without any changes to production function, this will also lead to a higher level of output associated with this higher capital level. The output for this experiment is in Figure 7 below.

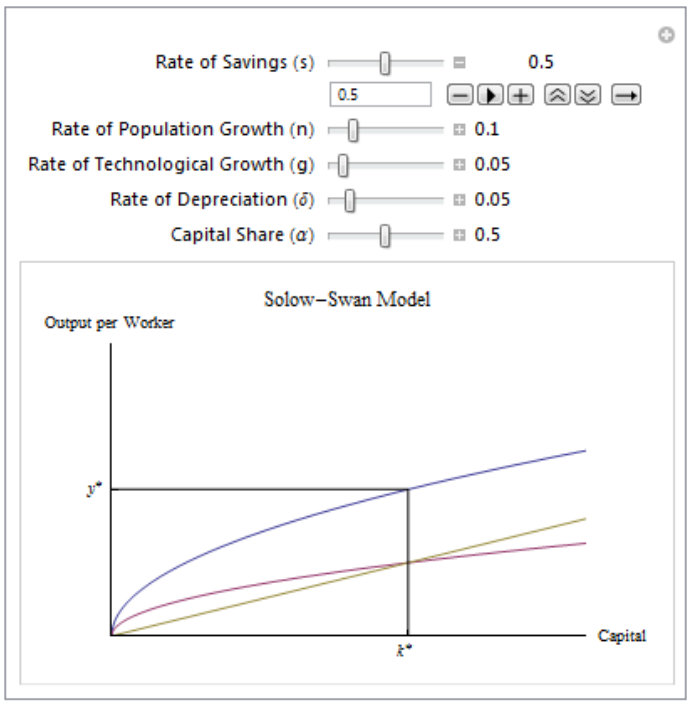

Figure 7

Discussion 3.1

Lacking of any drama, the results are as expected. This experiment simply illustrates the classical belief of savings rate being the strongest driver of growth. This was the advice many international aid organizations gave to developing nations, sometimes as conditions for loans. Increases in savings may lead to income per capita over the long run; however, it does not address the more prevalent issue of distribution of the income. Increased savings and investments result in increased capital; however, capital rent does not go to everyone, it goes to the already wealthy owners of capital. This only worsens the distribution of income in a nation. Income per capita cannot be used as the sole indicator for growth, life expectancy, education, and other measures must be taken account of. This leads to the discussion of negative externalities of inequalities, but that is a discussion for another day.

Concluding Remarks

In this paper, several models frequently used in economic development are programmed into Mathematica 7.0. The three models discussed in this paper are the Lewis model, Harris-Todaro model, and the Solow-Swan growth model. The manipulate feature was used in the Lewis and Solow-Swan models which makes modifications and experiments very easy to perform. The Harris-Todaro model was a much more difficult model to program. The model programmed here solves the wage differential between rural and urban sectors in the Harris Todaro model. As an assumption in the Harris-Todaro model, migration decision is derived from difference between agricultural wage and urban expected wage. The most interesting result is without a doubt derived from the Harris Todaro experiment where the inter-sectoral terms of trade is explored. Manipulations of terms of trade do not affect migration much at all, which is identical to the findings of the Lewis model and grounded in empirical evidences. It is very encouraging to the programmer of the model to see that some of its results match those of empirical studies.

References

Espindola, A. L. et al. (2005). “A Harris-Todaro Agent Based Model to Rural-Urban Migration” *Brazilian Journal of Physics* 36 3A, 603-609.

Harris, J., and Todaro, M. (1970). “Migration, Unemployment, and Development: A Two-Sector Analysis,” *American Economic Review* 40, 126-142.

Lewis, W. A. (1954). “Economic Development with Unlimited Supplies of Labor,” *The Manchester School of Economics and Social Studies* 22, 139-191.

Malthus, T. R. (1826), “An Essay on the Principle of Population: A View of its Past and Present Effects on Human Happiness; with an Inquiry into Our Prospects Respecting the Future Removal or Mitigation of the Evils which It Occasions” (Sixth ed.), London: John Murray.