Mean-Variance Portfolio Model in Mathematica

Economics 353: Final Project

Prepared for Professor A. Mehlenbacher, University of Victoria

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Economic Model

For my final project, I decided to build upon the Risk vs Return portfolio model from Chapter 7. The chapter presents a Markowitz mean-variance model written in MATLAB, with an alternative model presented in the appendix of the text written in GAMS.

The models use measures of mean return and variance to either maximize return subject to a variance penalty, or minimize variance subject to a desired return. The model therefore requires input data of return, variance, and co-variance for each stock to be used in the portfolio.

While doing my research for this project, and in my reading of the Mathematica documentation, I came across a set of Financial Data functions for Mathematica that would allow me to use actual data for real US stocks instead of hypothetical values, and do so dynamically 'on the fly'. Information on these functions can be found at

http://reference.wolfram.com/mathematica/ref/FinancialData.html

I became excited by this, and challenged myself to write a Markowitz mean-variance model in Mathematica from scratch that would select an optimal portfolio given a set of stocks of interest, and a historical time horizon from which to gather data. Doing so required a great deal of becoming accustomed to Mathematica's programming structure and methods. Thankfully, the documentation provided online was very helpful. It can be found at:

http://reference.wolfram.com/

For my model, I elected to write it such that given a desired return, the model would find the optimal bundle of stocks that would provide for the minimum amount of risk. An alternative would have been to select a comfortable amount of risk, and maximize return. I chose to not to write the model this way as I think selecting a given return is more realistic for my own purposes.

Given a set budget, the model will then quickly tell you how many shares of each stock should be purchased given their current price, and how much your total expenditure would be on each. It will also graph all the possible bundles, to provide a quick visual representation of what other bundles of risk an return are possible.

It should be noted that there is a major weakness with the methodology I am using here, in that we are using data from the past to conveniently paint a picture of the future. The statistical relationship between these two time-periods may simply not exist as we have assumed. As well, our measure of risk (in the case of my model, standard deviation) also does not distinguish between downward and upward price movements. A more in-depth analysis of input data problems can be found at:

http://www.effisols.com/basics/MVO.htm#SinglePeriodMVO

Computational Model

When writing my model, I attempted to provide a narrative of what is happening using the text notes feature of Mathematica's notebook files. The model, including all comments, can be found in the appendix, but I will go into a bit more in-depth analysis of a few components here.

Input Data Selections

I tried to make this model as simple as possible for the user. Therefore, there are only 6 input variable necessary to run the model, and Mathematica either computes or pulls the rest of the necessary data from the Internet in the subsequent sections. These six variables are stock1, stock2, stock3, timehorizon, money, and desannualreturn. The use of an annual return as compared to a horizon specific return is necessary to compare returns when varying the time periods, and ensures the user is comparing "apples with apples".

```
In[799]:= stock1 = "AAPL";
          stock2 = "GOOG";
          stock3 = "YHOO";
          timehorizon = 180;
          money = 150000;
          desannualreturn = 0.5;
```

Measuring Return

To measure mean return, I use the simple formula:

(Current Price – Original Price)/(Original Price)

The resulting return is then converted to an annualized return by dividing it by:

(timehorizon/365)

```
μstock1 =
 Round[
  Part[(FinancialData[stock1] - FinancialData[stock1, {then, then}, "Value"]) /
     FinancialData[stock1, {then, then}, "Value"], 1] / (timehorizon / 365), 0.01]
μstock2 =
 Round[
  Part[(FinancialData[stock2] - FinancialData[stock2, {then, then}, "Value"]) /
     FinancialData[stock2, {then, then}, "Value"], 1] / (timehorizon / 365), 0.01]
μstock3 =
 Round[
  Part[(FinancialData[stock3] - FinancialData[stock3, {then, then}, "Value"]) /
     FinancialData[stock3, {then, then}, "Value"], 1] / (timehorizon / 365), 0.01]
```

Measuring Volatility

My chosen measure of volatility in this model is Standard Deviation. To measure the standard deviation, I simple ask Mathematica to find the cumulative return for our stocks over our given time horizon. I then ask it to find the standard deviation of the list that is returned, and round that to two decimal points.

```
σstock1 =
 Round[StandardDeviation[FinancialData[stock1, "CumulativeReturn", then,
    "Value"]], 0.01]
σstock2 =
 Round[StandardDeviation[FinancialData[stock2, "CumulativeReturn", then,
    "Value"]], 0.01]
σstock3 =
 Round[StandardDeviation[FinancialData[stock3, "CumulativeReturn", then,
    "Value"]], 0.01]
```

Measuring Correlation

To measure the correlation of movements in the stock prices for each of our stocks, I ask Mathematica to again find the list of Cumulative Returns for each stock over our given time horizon. I then ask it to find the correlation between these lists for each set of stocks 1 & 2, 2& 3, and 1 & 3. I then ask Mathematica to round the output to 2 decimal points.

```
r12 = Round[Correlation[FinancialData[stock1, "CumulativeReturn", then, "Value"],
   FinancialData[stock2, "CumulativeReturn", then, "Value"]], 0.01]
r13 = Round[Correlation[FinancialData[stock1, "CumulativeReturn", then, "Value"],
   FinancialData[stock3, "CumulativeReturn", then, "Value"]], 0.01]
r23 = Round[Correlation[FinancialData[stock2, "CumulativeReturn", then, "Value"],
   FinancialData[stock3, "CumulativeReturn", then, "Value"]], 0.01]
```

Finding Possible and Optimal Proportions

Now that we have found all our input data, the model begins to do its real work by finding all the different possible portfolios given varying weights (proportions). This is done in preparation to graph these points, and the variable xmin finds the point to the left of the absolute minimum variance given all possible portfolios. This is then used when graphing the points to establish the boundaries for which we draw the graph. The code I used here to draw the graph was adapted from a Three-Asset Efficient Frontier demonstration by Fiona Maclachlan, which can be found on Wolfram's website at:

http://demonstrations.wolfram.com/ThreeAssetEfficientFrontier/

Once the graph is drawn, we must solve for the optimal weights (proportions). To do so, we must first find the closest return value for which we have established an optimal bundle. The variable return does just that, finding the next closest return value as defined by the points that make up the efficient frontier. The variable volatility then finds the value for the minimum volatility at this return.

Portfolio is then defined to be the list of weights at this given amount of return, where pstock 1 through 3 are the individual weights for each stock as extracted from portfolio. The following variables find the amount to be spent on each stock, and how many shares that equates at current prices.

```
weights =
  DeleteCases[Flatten[Table[{a, b, 1 - a - b}, {a, 0, 1, .02}, {b, 0, 1, .02}], 1],
   {_, _, x_ /; x < 0}];
points =
  Transpose[
   {Sqrt /@
     Table[With[{w1 = weights[[i, 1]], w2 = weights[[i, 2]], w3 = weights[[i, 3]]},
       w1^2 σstock1^2 + w2^2 σstock2^2 + w3^2 σstock3^2 +
        2 w1 w2 σstock1 σstock2 r12 + 2 w1 w3 σstock1 σstock3 r13 +
        2 w2 w3 σstock2 σstock3 r23], {i, Length[weights]}],
    Table[With[{w1 = weights[[i, 1]], w2 = weights[[i, 2]], w3 = weights[[i, 3]]},
      w1 μstock1 + w2 μstock2 + w3 μstock3], {i, Length[weights]}]}];

xmin =
  Ceiling[
   (Extract[
      Extract[
       FixedPoint[
        Delete[#, Position[Prepend[Differences[Transpose[#][[2]]], 0],
            d_ /; d < 0]] &, Sort[points]], 1], 1] - .02), 0.01];

graph = ListPlot[points, PlotStyle → PointSize[.01],
   PlotRange → {{xmin, All}, All},
   AxesLabel → {Style["Portfolio Volatility", "Label"],
     Style[ "Portfolio Expected Return", "Label"]}, AspectRatio → 1.5];
line =
  ListLinePlot[
   Tooltip[
    FixedPoint[
     Delete[#, Position[Prepend[Differences[Transpose[#][[2]]], 0],
         d_ /; d < 0]] &, Sort[points]], "Optimal Portfolio Curve"],
   PlotStyle → {Thick, Green}];
Show[graph, line]
```

Reporting our Optimal Portfolio

The next block of code creates a report of what the model has found. I relied heavily on the Print function, but revisions may include the use of Export function to output data directly to an Excel file, or create a better looking PDF, perhaps providing a comparative analysis of multiple portfolios automatically. In addition to reporting its findings, the summary presents the input details and pulls the actual name of each stock for the ease of the reader.

```
Print["================ SUMMARY REPORT ==============="]
Print["STOCK 1: The first stock selected in your portfolio is ",
 FinancialData[stock1, "Name"], " (", stock1, ") and in the past ",
 timehorizon, " days it has provided an annualized return of ",
 μstock1 * 100, "% with an approximate standard deviation of ", σstock1, "."]
Print["STOCK 2: The second stock selected in your portfolio is ",
 FinancialData[stock2, "Name"], " (", stock2, ") and in the past ",
 timehorizon, " days it has provided an annualized return of ",
 μstock2 * 100, "% with an approximate standard deviation of ", σstock2, "."]
Print["STOCK 3: The third stock selected in your portfolio is ",
 FinancialData[stock3, "Name"], " (", stock3, ") and in the past ",
 timehorizon, " days it has provided an annualized return of ",
 μstock3 * 100, "% with an approximate standard deviation of ", σstock3, "."]
Print["= = = = = = = = = = = = = = = = = = = = = = = = = = = = = "]
Print["Time Horizon of Input Data: ", timehorizon, " days"]
Print["Desired Annual Return: ", desannualreturn * 100, "%"]
Print["Budget: $", money]
Print["============= OPTIMAL PORTFOLIO ============="]
Show[graph, line]
Print["STOCK 1: Buy ", shares1, " shares of ", FinancialData[stock1, "Name"],
 " (", stock1, ") to account for ", pstock1 * 100,
 "% of your portfolio. This will cost you $", spend1, " at $",
 FinancialData[stock1], " per share." ]
Print["STOCK 2: Buy ", shares2, " shares of ", FinancialData[stock2, "Name"],
 " (", stock2, ") to account for ", pstock2 * 100,
 "% of your portfolio. This will cost you $", spend2, " at $",
 FinancialData[stock2], " per share." ]
Print["STOCK 3: Buy ", shares3, " shares of ", FinancialData[stock3, "Name"],
 " (", stock3, ") to account for ", pstock3 * 100,
 "% of your portfolio. This will cost you $", spend3, " at $",
 FinancialData[stock3], " per share." ]
Print["PORTFOLIO VOLATILITY: ", volatility]
Print["============================================="]
Print["Note: You will have $", money - (spend1 + spend2 + spend3),
 " of extra room in your budget.
   It is recommended you spend this on 5-cent candies,
   and tip the clerk well to increase karma, further reducing risk."]
```

Experiments

Each day the model is run, results will be different as more data becomes available. That said, there are only a few possible experiments: changing the stocks of interest, changing the time horizon for input data, changing the required return, and changing the budget.

Changing the budget is not a good experiment, as proportional composition of our portfolio is what we are really after, and computations for a given budget are really just provided to make life simple for the user. The relative underlying trade-offs would not change.

That said, below I will examine the effects of changing the stocks of interest, changing the time horizon of input data, and changing the required rate of return.

1: Changing the stocks of interest

When changing the stocks of interest, we introduce to the model a varied set of returns and volatilities. The composition of our optimal portfolio will change depending on the relative return and volatility of the stocks we introduce.

I will assess the following 3 sets of stocks over a time horizon of 180 days at a desired return of 30%, except for the last portfolio, in which I specify a desired return of 40% as 30% is not one of the graphed optimal bundles:

(AAPL, GOOG, YHOO), (AAPL, MSFT, DELL), (AAPL, MSFT, YUM)

2: Changing the time horizon

The time horizon of input data is important, as every week day contained in the time horizon is another data point. The longer our time horizon, the more input data we have to use to base our predictions upon. But there is a trade off, as the further into the past we go, the less related the data is to current market circumstances. Therefore, there is a trade-off between relative long-term and short-term trends. Depending on your investment strategy (short term vs medium term vs long term) you may want to change the time horizon used when running the model.

Using the portfolio (AAPL, GOOG, YHOO) I will assess the effects of varying the time horizon from 60, to 180, and to 360 days. For these experiments I will compute an optimal bundle based on a desired return of 50%.

3. Changing the required rate of return

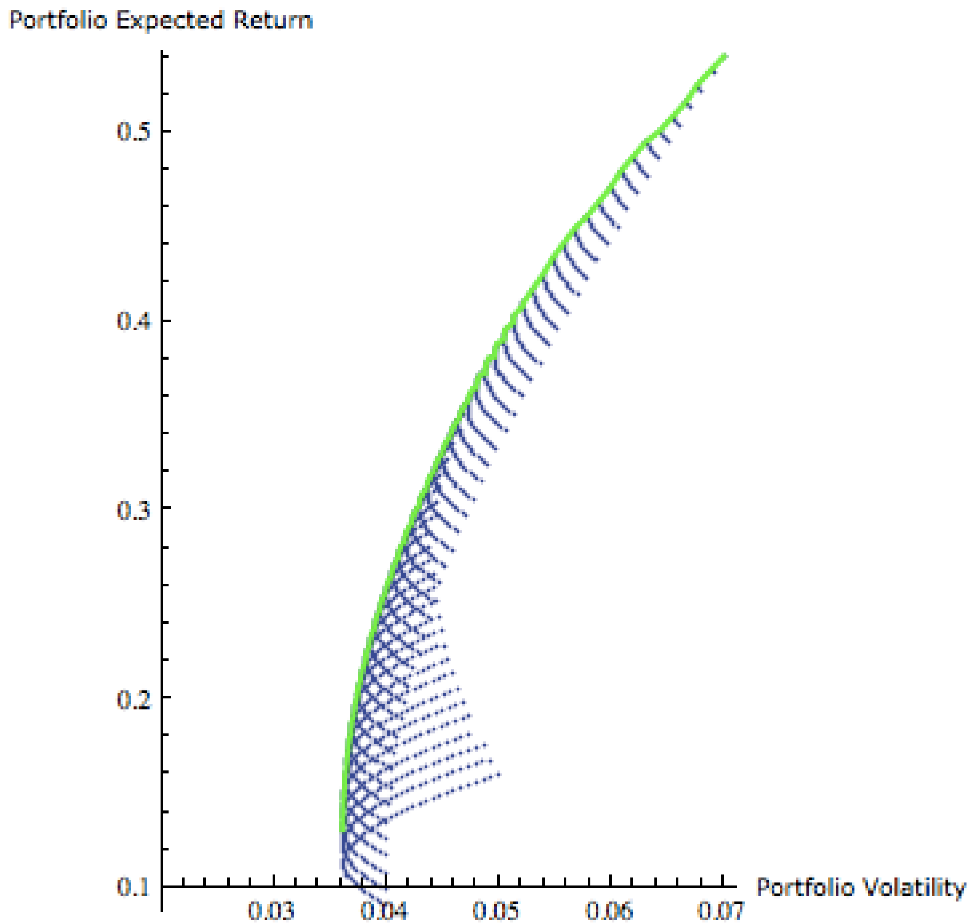
When we are given a different rate of return, the composition of our optimal portfolio will change to reflect the “least risky” weights (proportions) required to obtain that rate of return given our input data. Using the portfolio (AAPL, GOOG, YHOO) I will assess the effects of varying the required rate of return from 30% to 40% and 50%.

Results & Discussion

1: Changing the stocks of interest

<u>*Bundle A:*   AAPL, GOOG, YHOO</u>

The relative volatilities of the portfolio were: 0.07, 0.05, and 0.04, meaning AAPL was the most volatile, followed by GOOG, then YHOO. The mean returns were: 54%, 16%, and 9%.The possible portfolios are graphed below:



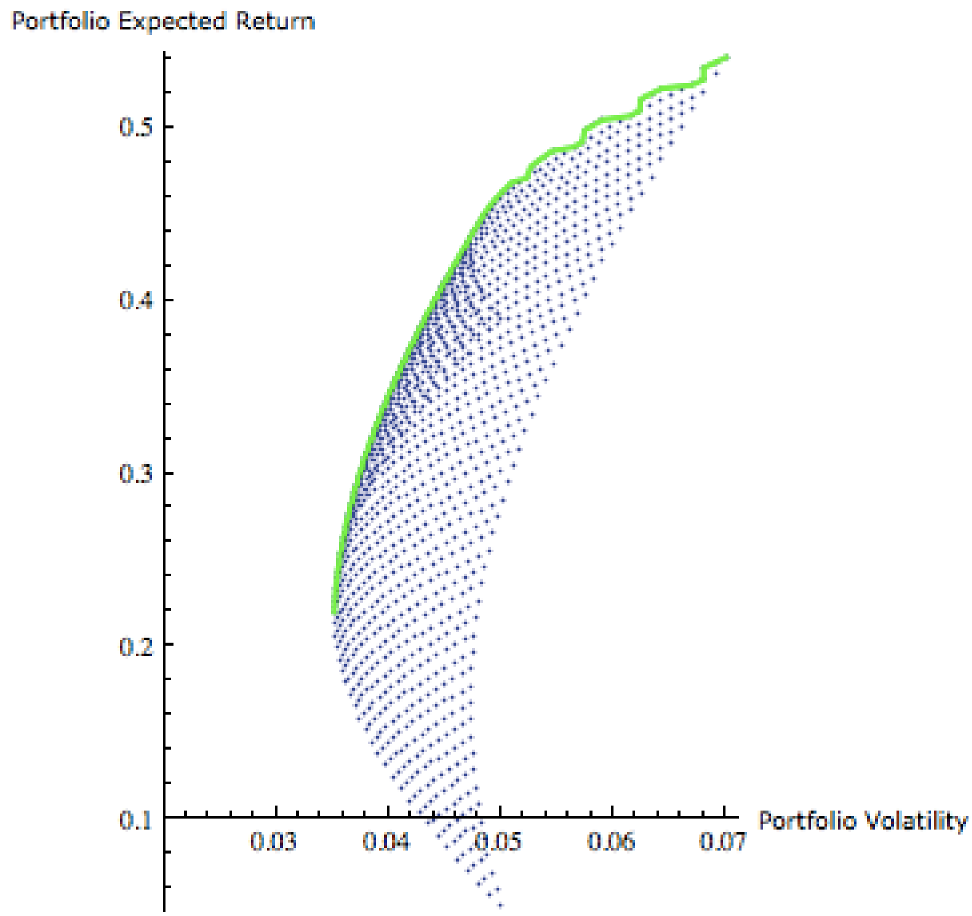
Given our time horizon of 180 days, and required return of 30%, the model makes the following recommendations:

```
STOCK 1: Buy 198 shares of Apple Inc (AAPL) to account for 32.
 % of your portfolio. This will cost you $47 874.4 at $241.79 per share.
STOCK 2: Buy 42 shares of Google Inc (GOOG) to account for 16.
 % of your portfolio. This will cost you $23 781.2 at $566.22 per share.
STOCK 3: Buy 4452 shares of Yahoo Inc (YHOO) to account for 52.
 % of your portfolio. This will cost you $77 999. at $17.52 per share.
```

The volatility of this optimal portfolio is: 0.0512789

Bundle B: AAPL, MSFT, DELL

The relative volatilities of the portfolio were: 0.07, 0.05, and 0.05, meaning AAPL was the most volatile, followed by equally risky MSFT and DELL. The mean returns were: 54%, 39%, and 5%. The possible portfolios are graphed



below:

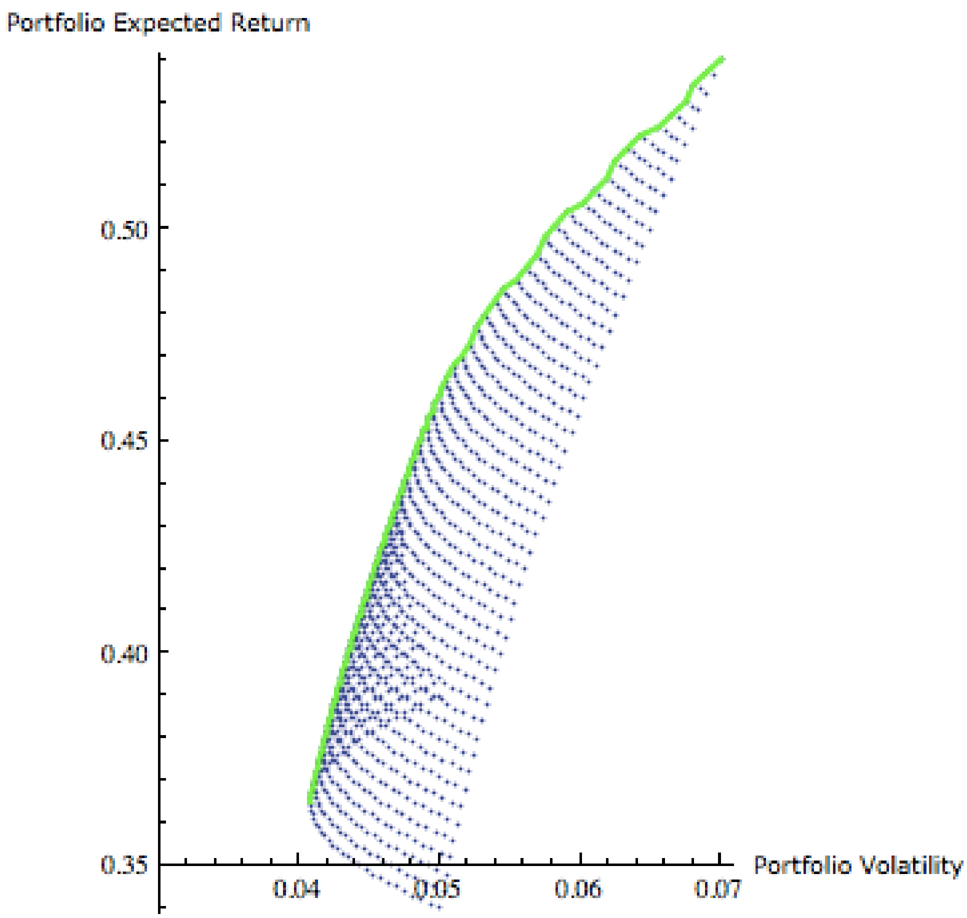
Given our time horizon of 180 days, and required return of 30%, the model makes the following recommendations:

```
STOCK 1: Buy 24 shares of Apple Inc (AAPL) to account for 4.
 % of your portfolio. This will cost you $5802.96 at $241.79 per share.
STOCK 2: Buy 3460 shares of Microsoft Corporation (MSFT) to account for 70.
 % of your portfolio. This will cost you $104 976. at $30.34 per share.
STOCK 3: Buy 2463 shares of Dell Inc (DELL) to account for 26.
 % of your portfolio. This will cost you $38 989.3 at $15.83 per share.
```

The volatility of this optimal portfolio is: 0.0441701

Bundle C: AAPL, MSFT, YUM

The relative volatilities of the portfolio were again: 0.07, 0.05, and 0.05, meaning AAPL was the most volatile, followed by equally risky MSFT and YUM. The mean returns were: 54%, 39%, and 34%. The possible portfolios are graphed below:



Given our time horizon of 180 days, and required return of 40%, the model makes the following recommendations:

```
STOCK 1: Buy 37 shares of Apple Inc (AAPL) to account for 6.
 % of your portfolio. This will cost you $8946.23 at $241.79 per share.
STOCK 2: Buy 2373 shares of Microsoft Corporation (MSFT) to account for 48.
 % of your portfolio. This will cost you $71 996.8 at $30.34 per share.
STOCK 3: Buy 1695 shares of Yum Brands Inc (YUM) to account for 46.
 % of your portfolio. This will cost you $68 986.5 at $40.7 per share.
```

The volatility of this optimal portfolio is: 0.0435597

Discussion

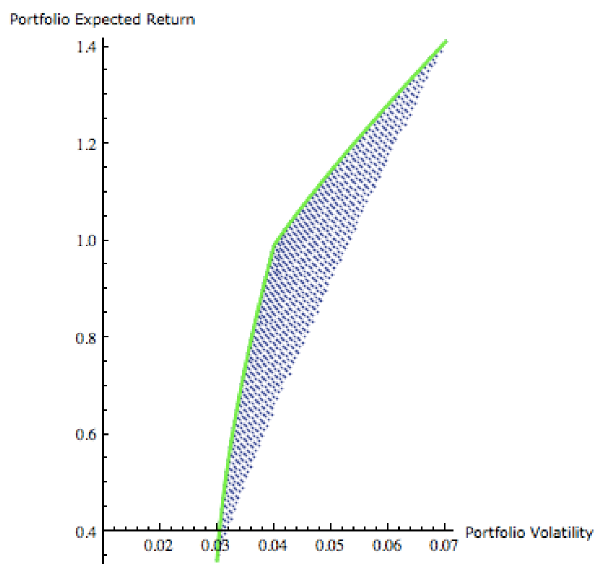
By changing the stocks in each of our bundles, we can vary our portfolio to result in even lower volatility. For example, by switching from bundle A (AAPL, GOOG, YHOO) to bundle B (AAPL, MSFT, DELL) we were able to lower our risk (portfolio volatility) from 0.0512789 to 0.0441701.

In both bundle A and bundle B, the assets YHOO and DELL provided for mean returns that were less than 10%. But in bundle C, no asset had a mean annualized return over the last 180 days that was less than 30%. Infact, our desired return of 30% was no longer present on the efficient frontier, and so we optimized for a desired return of 40% instead.

Interestingly, in bundle C, we were able to optimize for a higher annualized return (40% instead of 30%) but still got the lowest portfolio volatility of all the experiments, at 0.0435597.

2: Changing the time horizon

============== OPTIMAL PORTFOLIO ==============



STOCK 1: Buy 0 shares of Apple Inc (AAPL) to account for 0.
% of your portfolio. This will cost you $0 at $241.79 per share.

STOCK 2: Buy 137 shares of Google Inc (GOOG) to account for 52.
% of your portfolio. This will cost you $77 572.1 at $566.22 per share.

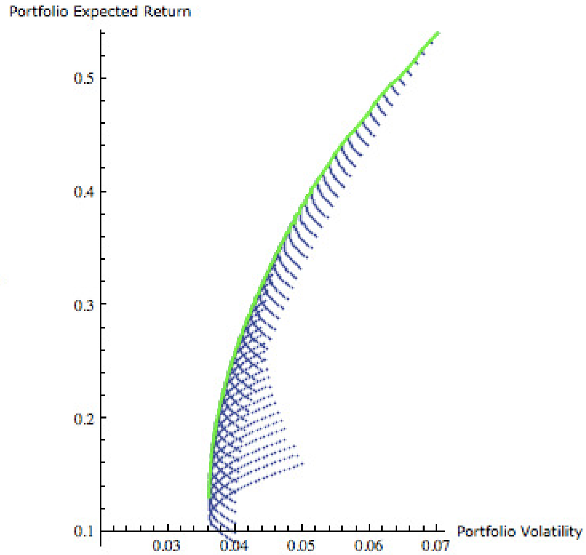
STOCK 3: Buy 4109 shares of Yahoo Inc (YHOO) to account for 48.
% of your portfolio. This will cost you $71 989.7 at $17.52 per share.

PORTFOLIO VOLATILITY: 0.0315198

Horizon A: 60 days

==

============== OPTIMAL PORTFOLIO ==============



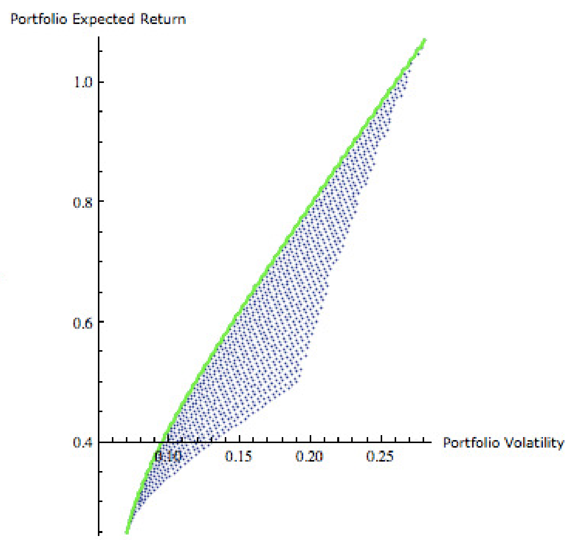
STOCK 1: Buy 558 shares of Apple Inc (AAPL) to account for 90.
% of your portfolio. This will cost you $134 919. at $241.79 per share.

STOCK 2: Buy 0 shares of Google Inc (GOOG) to account for 0.
% of your portfolio. This will cost you $0 at $566.22 per share.

STOCK 3: Buy 856 shares of Yahoo Inc (YHOO) to account for 10.
% of your portfolio. This will cost you $14 997.1 at $17.52 per share.

Horizon B: 180 days PORTFOLIO VOLATILITY: 0.0643048

============== OPTIMAL PORTFOLIO ==============



STOCK 1: Buy 49 shares of Apple Inc (AAPL) to account for 8.
% of your portfolio. This will cost you $11 847.7 at $241.79 per share.

STOCK 2: Buy 174 shares of Google Inc (GOOG) to account for 66.
% of your portfolio. This will cost you $98 522.3 at $566.22 per share.

STOCK 3: Buy 2226 shares of Yahoo Inc (YHOO) to account for 26.
% of your portfolio. This will cost you $38 999.5 at $17.52 per share.

Horizon C: 360 days PORTFOLIO VOLATILITY: 0.119088

Discussion

As can be seen in the graphs above, changing the time horizon of input data creates for drastic changes in the predictions of the graph. In the first instance, with a time horizon of 60 days, the model tells us to allocate 0% of our shares to AAPL. But, with a time horizon of 180 days, the model tells us to allocate 90% of our shares to AAPL, and with 360 days of data, only 8%. These drastic changes are due to the variations of relative returns and volatility over the different time horizons.

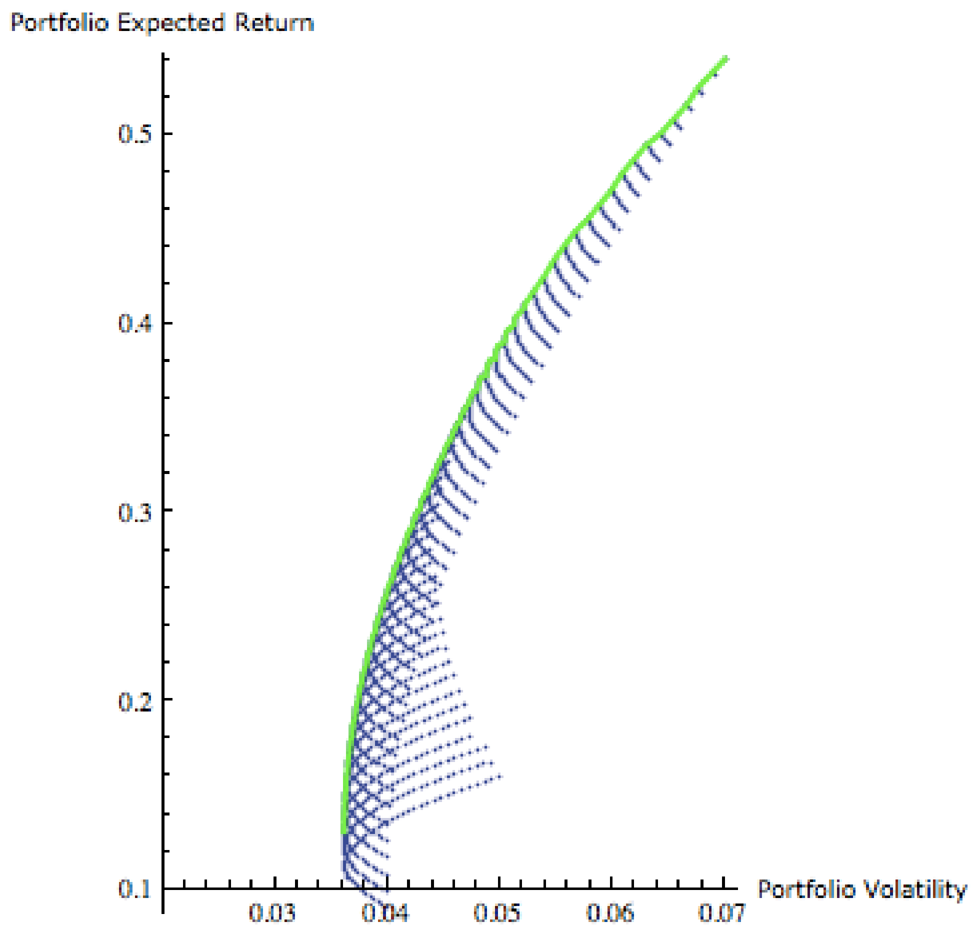
There is a lesson to be learnt here, as I foreshadowed in my discussion of the Economic model above. The model is only as good as its input data, and our predictions are made under the assumption that this data is a good predictor for the future. But, the past few years have shown us just how swiftly the stock market can change, and as the above experiment shows, these changes in our input data will create equally drastic variations in our model's output.

One more example of this is the range of mean returns possible on the y-axis of the graphs above. In the first horizon returns vary from approximately 0.4 to 1.4, in the second from 0.1 to 0.55, and in the third from 0.2 to 1.1.

Given our desired return of 50%, the volatility of the optimal portfolio has gone up as the time horizon has increased. This is especially true given the wild fluctuations in the market of late. But, that said, this will likely always be true, as a greater time horizon allows for the possibility of greater swings in price than does a shorter time horizon. Therefore, these volatilities should not be compared to each other as to be measures of the same thing. Volatility over one time period is not the same as volatility over another. Volatility should only be compared across different portfolios or desired returns given a consistent time horizon.

3. Changing the required rate of return

Using a 180 day time horizon and stock bundle of AAPL, GOOG, and YHOO.



Return (AAPL,GOOG,YHOO)

Out[704]= 0.54

Out[705]= 0.16

Out[706]= 0.09

Volatility (AAPL,GOOG,YHOO)

Out[707]= 0.07

Out[708]= 0.05

Out[709]= 0.04

Return: 30%

```
STOCK 1: Buy 198 shares of Apple Inc (AAPL) to account for 32.
 % of your portfolio. This will cost you $47 874.4 at $241.79 per share.
STOCK 2: Buy 42 shares of Google Inc (GOOG) to account for 16.
 % of your portfolio. This will cost you $23 781.2 at $566.22 per share.
STOCK 3: Buy 4452 shares of Yahoo Inc (YHOO) to account for 52.
 % of your portfolio. This will cost you $77 999. at $17.52 per share.
PORTFOLIO VOLATILITY: 0.0427458
```

Return: 40%

```
STOCK 1: Buy 384 shares of Apple Inc (AAPL) to account for 62.
 % of your portfolio. This will cost you $92 847.4 at $241.79 per share.
STOCK 2: Buy 58 shares of Google Inc (GOOG) to account for 22.
 % of your portfolio. This will cost you $32 840.8 at $566.22 per share.
STOCK 3: Buy 1369 shares of Yahoo Inc (YHOO) to account for 16.
 % of your portfolio. This will cost you $23 984.9 at $17.52 per share.
PORTFOLIO VOLATILITY: 0.0512789
```

Return: 50%

```
STOCK 1: Buy 558 shares of Apple Inc (AAPL) to account for 90.
 % of your portfolio. This will cost you $134 919. at $241.79 per share.
STOCK 2: Buy 0 shares of Google Inc (GOOG) to account for 0.
 % of your portfolio. This will cost you $0 at $566.22 per share.
STOCK 3: Buy 856 shares of Yahoo Inc (YHOO) to account for 10.
 % of your portfolio. This will cost you $14 997.1 at $17.52 per share.
PORTFOLIO VOLATILITY: 0.0643048
```

Discussion

Changing the required rate of return is a simple experiment, but should confirm to us that a higher rate of return will result in a greater amount of risk. This is confirmed by the example above, as risk (portfolio volatility) increases from 0.0427 to 0.0643 as the desired return increases from 30% to 50%.

This is also confirmed individually by each stock's return VS volatility as shown on the page above. AAPL, with a return of 0.54 has the highest return, but also the highest volatility, at 0.07. The converse is the same for YHOO, with GOOG placing in the middle.

In conclusion, the portfolios determined above for our varying desired returns, in order for the portfolios to obtain a higher return, they had to incorporate higher risk, and therefore increase the risk of the portfolio as a whole.

Appendix