

Modeling Growth, Environmental Agreements, and General Equilibria: Practical Applications of Mathematica, MATLAB, and GAMS

**Economics 353
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Part I: Solow Growth Model and the Human Capital Dimension

Introduction

Developed by Robert Solow in 1956, the Solow growth model explains how economies achieve growth through capital accumulation. The Textbook Solow Model considers only physical capital, while a key publication by Mankiw, Romer, and Weil brings the accumulation of human capital into the picture (1992). While the Textbook Solow does an excellent job of modeling growth in the developed nations over the past five centuries or so, the MRW interpretation of the Solow model is pivotal in explaining the massive income disparities across countries.

In this project, I use Mathematica first to model the Textbook Solow Model, and second to model MRW's interpretation of the Solow Model. That is, in my second notebook I add human capital as a factor of production and as a new dimension in the classic investment vs. capital Solow Model diagram. This is where use of a computational method becomes extremely valuable, if not a complete necessity. It is easy to draw the Textbook Solow Model diagram on paper, but very difficult to visualize and draw MRW's interpretation. Such a diagram is not presented in MRW's critical 1992 paper, perhaps because the computational methods were not readily available at that time.

Economic Model

Model 1: Textbook Solow Model

The Textbook Solow Model is essentially the Basic Solow Model with the introduction of technical progress. I use the Cobb-Douglas form of the production function, although the model may be generalized to any production function that has constant returns to scale and relies only on physical capital and technology-augmented labour (i.e. “effective labour”).

$$F(K, AL) = K^\alpha (AL)^{(1-\alpha)} \quad (1.1)$$

Equation 1.1 is the aggregate production function. Our diagram will examine the labour-intensive production function, so we need things in per effective labour units form:

$$\frac{1}{AL} F(K, AL) = F\left(\frac{K}{AL}, 1\right) = f(k) \quad (1.2)$$

Where 1.2 will always be true so long as the production function exhibits constant returns to scale (i.e. is homogeneous of degree one). Now, we must consider the capital accumulation function, the backbone of the Solow model.

$$\dot{K} = \gamma Y(t) - \delta K(t) \quad (1.3)$$

Where γ is the savings rate and δ is the depreciation rate on physical capital. One of the assumptions of our model is that both of these parameter values are constant through time. Another critical assumption is that everything that isn't spent on consumption in the model is saved and converted to domestic investment. That is, the economy is closed. From here, we must again express our aggregate equation in the form of a state variable (in this case, labour intensive units of capital). Using derivatives and the fact that k is just K/AL , we have:

$$\begin{aligned}\dot{K}/A(t)L(t) &= \gamma f(k) - \delta k(t) \\ \dot{k} &= \gamma f(k) - (\delta + n + g)k(t)\end{aligned}\tag{1.4}$$

According to Solow, the economy will tend towards a balanced growth path where $\dot{k} = 0$ (mathematically, a “steady state”) which it will always achieve if given initial capital levels greater than zero and enough time (the exact time can be found using dynamics, but is outside the scope of this paper). While on the balanced growth path, breakeven investment is equal to actual investment, as follows:

$$(\delta + n + g)k(t) = \gamma f(k)\tag{1.5}$$

That is, investment in capital equals the rate that capital is being “used up” by the economy. The above relationship gives rise to the Solow Model diagram that commonly appears in macroeconomic textbooks (and is likewise presented in this

report's results section). Solving for k yields the balanced growth path level, denoted k^* :

$$k^* = \left[\frac{\gamma}{(\delta+n+g)} \right]^{1/(1-\alpha)} \quad (1.5)$$

Also of importance, k^* that maximizes consumption is:

$$k_{GR}^* = \left[\frac{\alpha}{(\delta+n+g)} \right]^{1/(1-\alpha)} \quad (1.6)$$

Where GR stands for Golden Rule. Since, in this story of the Solow Model, output depends on only one endogenous variable (physical capital), the balanced growth path diagram is two-dimensional. As we will see, this is not the case in Mankiw, Romer, and Weil's Solow growth story.

Model 2: MRW's Version of the Solow Model with Human Capital

The MRW version of the Solow Model is essentially the Textbook Model with human capital added as a factor of production. The key to MRW's addition is that they chose to make human capital endogenous. That is, it has its own accumulation function which depends on output. The reader should notice that human capital resembles physical capital in many respects (more on this later).

With human capital, our production function becomes:

$$F(K, AL) = K^\alpha H^\beta (AL)^{(1-\alpha-\beta)} \quad (2.1)$$

Where H is human capital. Similar, to physical capital, human capital per effective unit of labour is denoted h and has an accumulation function:

$$\dot{h} = \phi f(k, h) - (\delta + n + g)h(t) \quad (2.1)$$

Like physical capital, $\dot{h} = 0$ while the economy is on a balanced growth path. This gives rise to k^* and h^* as follows:

$$k^* = \left[\frac{\gamma^{1-\beta} \phi^\beta}{(\delta+n+g)} \right]^{1/(1-\alpha-\beta)} \quad (2.2)$$

$$h^* = \left[\frac{\gamma^\alpha \phi^{1-\alpha}}{(\delta+n+g)} \right]^{1/(1-\alpha-\beta)} \quad (2.3)$$

Mankiw, Romer, and Weil used real world data to prove that adding human capital helps to explain the huge income disparities between countries with similar sized labour forces and levels of physical capital.

Now that we have a basic theoretical understanding of both models, we can move on to this report's main objective: computational representation.

Experiments

The goal of this report is to explore a new visualization of an old model, not to develop and illustrate a new model. That is to say, the goal of experimentation is to develop the reader's understanding of an important and well understood macroeconomic model, not to prove the robustness of a new model.

This being the case, no explicit experiments were performed. Instead, let me invite the reader to explore the *Mathematica* files provided by dragging the manipulator back and forth, observing the effects on values such as k^* and h^* . Also of interest are how the break even lines change in the MRW model as g , n , and δ are adjusted. I would also encourage the reader to explore how output and actual investment change as a result of alteration to the factor shares devoted to physical and human capital.

Results

Model 1: Textbook Solow Model

The Textbook Solow Model model is presented below, adapted from a basic version (which can be found on *Mathematica's* demonstration website) to include technical change (with the option of adjustment), as well as the option of adjusting capital's factor share, α (rather than fixing it at $1/3$). *Mathematica* code can be found in the appendix, entry A1.

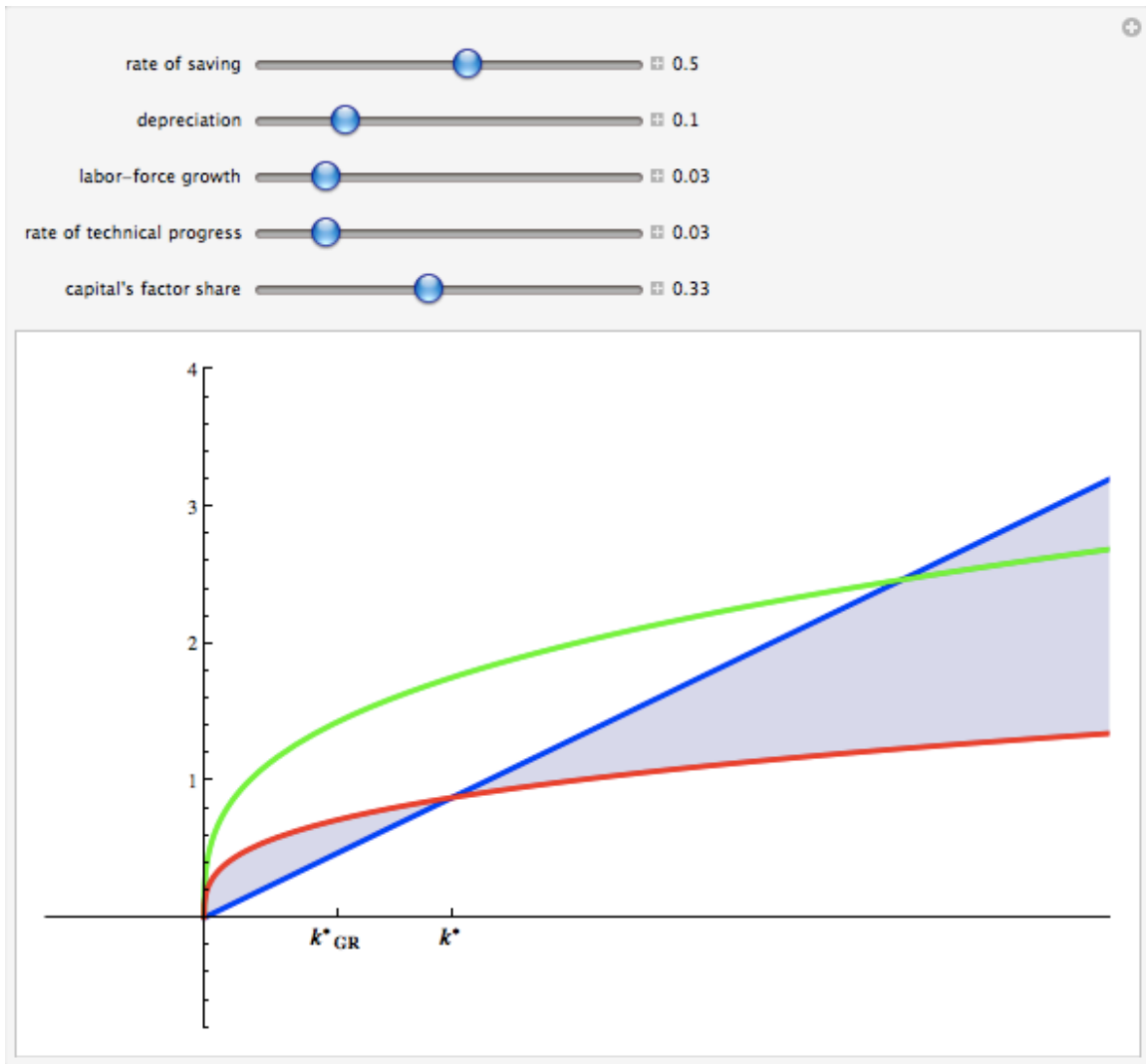


Figure 1.1 Textbook Solow Model

This is the familiar diagram that makes an appearance in nearly every introductory macroeconomic textbook. The code for generating figure 1.1 was relatively straightforward, and writing it was a useful steppingstone to figuring out the code that generated figure 2.1.

Model 2: MRW's Version of the Solow Model with Human Capital

This model has much more computational value than the previous diagram. That is, figure 2.1 is relatively hard to draw or visualize without the aid of a tool like *Mathematica*. It uses a 3D plot to represent how output is affected endogenously by both physical and human capital.

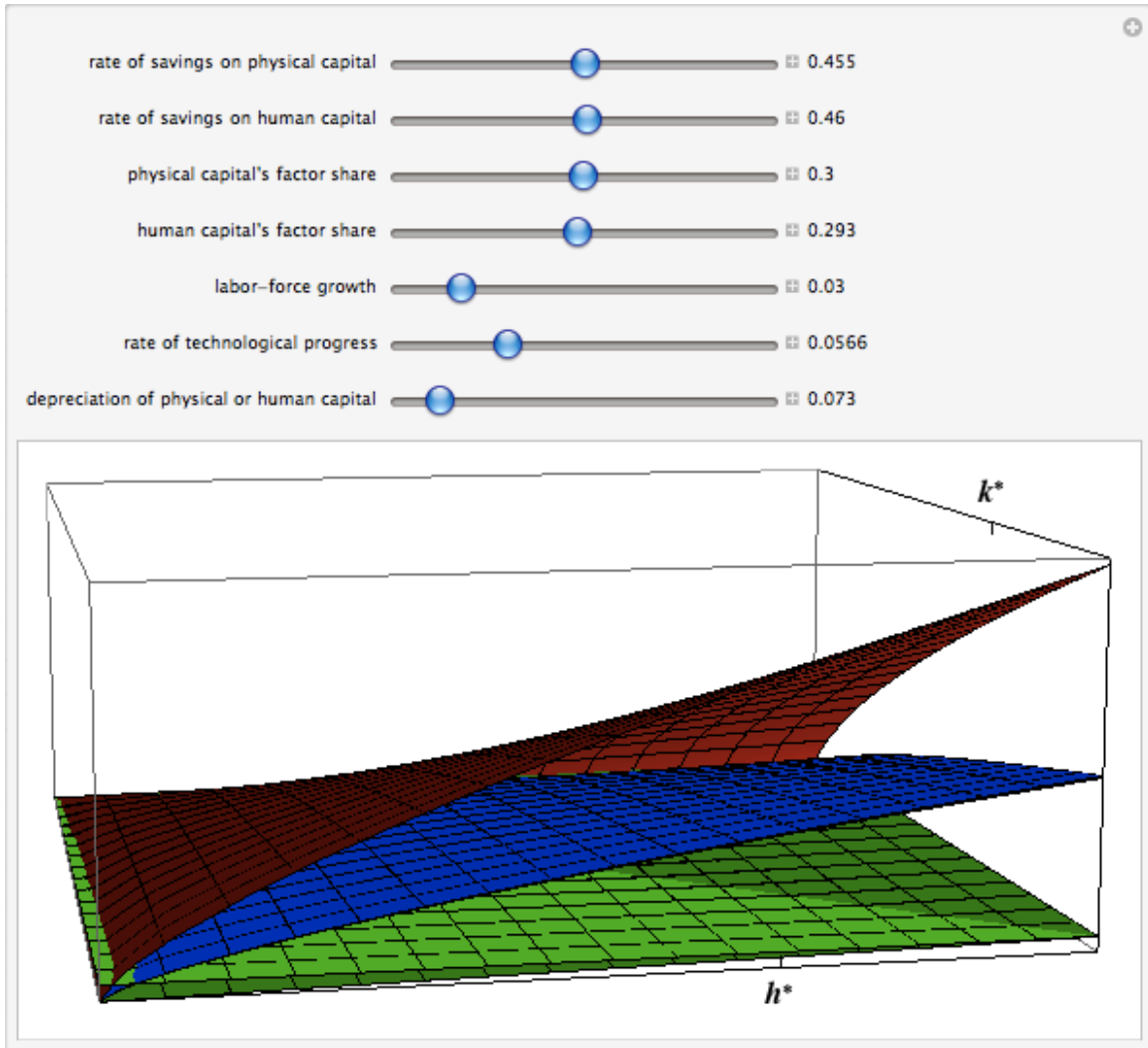


Figure 2.1 MRW Solow Including Human Capital

This figure looks a rather complicated at first glance. But I would encourage the reader to experiment with the *Mathematica* file in order to understand how

changing parameter values like n , g , and δ changes the balanced growth path values of k^* and h^* . Changing the rate of savings on either physical or human capital also has significant effects on the values of k^* and h^* (respectively).

Figure 2.1 is the first of its kind that I have seen, and provides valuable insight into how balanced growth paths are affected by changes in the parameters specified in Mankiw, Romer, and Weil's critical 1992 paper. Although it is an eye-ful on paper, figure 2.1 makes perfect sense in *Mathematica* thanks to the manipulate function and *Mathematica's* mouse-over labels. The code for figure 2.1 is fairly complex, and is presented in the appendix under A2.

Discussion

Mathematica proved to be a powerful tool for creating visualizations of relatively complex economic stories. The Textbook Model diagram, although nothing new, is useful because it allows for manipulation of parameter values. For instance, the user can change γ , the savings rate on physical capital, until $k^* = k_{GR}^*$.

The MRW model visualization, potentially something new, is a useful aid to understanding how production can be affected endogenously by two different variables. Being able to represent both k^* and h^* on the same diagram allows the learner to see how alterations in parameter values affect the balanced growth path.

One limitation to the representation of MRW in figure 2.1 is that things get very busy very quickly. Since k^* depends on h^* and vice-versa (see equations 2.2 and 2.3), their effects on production cannot be separated into two different diagrams. As a result, a busy diagram like figure 2.1 is our only option. The diagram takes some getting used to, and is very hard to grasp without manipulating the values of g , n , δ , etc. and watching what happens to the values of k^* and h^* . For this reason, I don't think there would be much value to including figure 2.1 (or a similar figure) in a growth or macroeconomic textbook.

Part II: The Effect of a Trade Sanction on an International Environmental Agreement Game

Introduction

Barrett argues that international environmental agreements (IEA) must be self-enforcing in order to work (1994). In his pivotal paper, "Self Enforcing Environmental Agreements," he points out that "no country can be forced to sign an IEA, and signatories to an IEA can always withdraw from the agreement" (p. 878). But this view is at odds with the one that Sir Nicholas Stern expresses in the Stern Report.

When asked during COP16 at Cancun whether Canada should slow development in the oil sands, Stern replied "if Canada stays dirty, it will be forced to co-operate at a later date by international trade sanctions" (presentation by Elizabeth May at UVic,

03/11/2011). Barrett seems to feel that trade sanctions are too harsh a penalty for failing to meet international environmental standards, but I think may have reconsidered since his paradigm paper was published in 1994. That is, the looped game that he presents in the latter part of his analysis could probably use some amendment when applied to an issue as serious as climate change.

In this paper I try to adapt Barrett's analysis so it accurately reflects Stern's opinions about the seriousness of climate change. In order to do this, I build functional forms for the costs and benefits of carbon abatement into a prisoner's dilemma program developed by Nourri Najjar (2010). I then loop the program in order to find the efficient sanction level that will induce player 1 (Canada in our case) to accede to the IEA on carbon emissions.

Economic Model

Base Model: Prisoner's Dilemma

One of the paradigm models in game theory, the Prisoner's Dilemma (PD), is used to explain why co-operative outcomes do not necessarily prevail over competitive outcomes, even when the cooperative outcome is more beneficial for both players.

The PD is a two-player, two strategy static game. It has a pure Nash equilibrium where both players have a dominant strategy. That is, both players' best strategy is to defect, regardless of what decision the other player makes. This has the

counterintuitive result that, although the co-operative outcome is better for both players, the competitive outcome still results.

In Economics 381, Alan Mehlenbacher uses this game to explain why IEAs are helpful in trying to reduce pollutants across country boundaries. Without the IEA, both countries can do better by free riding off the abatement made by the other country (i.e. both countries' dominant strategy is to defect). But with the IEA, both countries can be induced to co-operate for fear of spoiling future relations. This co-operative outcome results in higher pay-offs (i.e. better net benefits) for both players.

Developed Model: Sanctions and International Environmental Agreements

As stated earlier, Nicholas Stern seems to reject Barrett's idea that IEA's cannot be enforced. In order to model this sentiment, I have included a trade sanction in player one's pay-off matrix that ratchets up with each loop of the game.

It is important that player 1 is only punished with a trade sanction when it defects (i.e. when it does not conform to the abatement levels agreed upon for developed nations). In order to code this, player one's pay-off matrix must contain a constant term, S , that is subtracted from its net benefits whenever it chooses to defect. This is explained in more detail within the code.

The cost and benefit functions presented in the model are taken from Alan Mehlenbacher's Economics 381 course notes. The important point to take from the equations below is that a country benefits not only from the abatement it does itself, but also from the abatement done by other countries; a country's costs, on the other hand, are determined solely by its own level of abatement. This creates a classic free-rider problem.

$$benefits_i = B(X_a + X_b) \quad i = a, b \quad (1)$$

$$cost_i = C_i X_i \quad i = a, b \quad (2)$$

Where X_a is the abatement done by player 1, and X_b is the abatement done by player 2. In our example, this means that X_a is the carbon reduction made by Canada, while X_b is the reduction made by the rest of the world. B is a scalar that multiplies up the reduction amounts to find a benefit, and C is likewise a scalar that multiplies up the reduction amounts to find a cost. Notice that C differs across players. This will be important to our analysis later on.

Next I make an addition to the benefit and cost functions provided in Economics 381. I add a constant term, S , which represent a sanction levied against player 1 if it does not accede to the agreement. Player 1's cost function and benefit functions, given it has chosen to defect, are now given by:

$$benefits_a = B(X_a + X_b) - S \quad (3)$$

$$cost_a = C_a X_a - S \quad (4)$$

This should provide a disincentive against defecting (a stick rather than a carrot, as Stern predicts), and the hypothesis is that at some S^* player 1 will switch from choosing to defect to choosing to co-operate.

Computational Method

Our first step is to create a game matrix that uses equations 1 to 4. From this matrix we can deduce the payoff matrix for each player. In our case, these are not symmetric since player 1 can suffer from trade sanctions if it chooses to defect.

Next, we need an isolated payoff vector (IPV) to help us determine which decision each player will make (i.e. co-operate or defect) in response to the other player's decision. We call these IPVs C1 and D1 for player 1 where C1 is the IPV for when player 2 chooses to co-operate, and D1 is the IPV for when player 2 chooses to defect. Similarly, we also have C2 and D2, to make a total of four IPVs.

Now that we have IPVs, we can form a best response function for each player. In other words, we can pick the highest scalar in C1 and D1 for player 1, and in C2 and D2 for player 2. This allows us to find the Nash equilibrium, which requires an organized stream of if and elseif statements; I invite the reader to investigate the *Matlab* code provided.

Data

The critical information contained in the model and taken from real world data is the value of C_i . The fact that Canada has an incentive to defect from the IEA in the first place suggests that its abatement costs are relatively high. Indeed, this is what Stern was referring to indirectly when he called Canada "dirty." Canada's abatement costs are high because of activity in the Alberta oil sands, shale gas fields in B.C., and other unconventional oil and gas extraction efforts. This is in contrast to EU nations, who generally express a high willingness to abate carbon emissions because their abatement costs are relatively low.

In order to incorporate this relationship (between Canada and the rest of the world) into my model, I use carbon intensity data from the World Bank. Since the European Union has historically been the biggest proponent of GHG reduction agreements and the most likely body to impose trade sanctions on Canada, I used its GDP/emissions data to represent player 2 (i.e. the rest of the world). I then performed a simple calculation where:

$$C_b = \left[\frac{\text{GHG intensity of } b}{\text{GHG intensity of } a} \right] C_a \quad (5)$$

Please look under references for a link to the International Energy Associations public document on CO₂ emissions, titled "CO₂ Emissions from Fuel Combustion - Highlights" (2008).

Experiments

Role of the Sanction in Inducing Player 1 to Co-operate

The central point of this model is to show that when a sanction is levied against a non-cooperative player in an IEA game, that player will eventually switch their strategy in order to avoid the sanction. Determining the correct level at which this sanction should be set is crucial in order to avoid efficiency losses from actually having to impose the sanction. This is where *Matlab* comes in. By looping the game with a larger value of S in each successive loop, we can observe from the output at which point player 1 switches its strategy from defect to co-operate. Let's call the first value of S that induces player 1 to switch strategies S^* .

In our specific example, S^* is easy to calculate by hand. But this is not necessarily the case for all functional forms. If more complicated cost-benefit equations were used than the ones provided in Economics 381, then S^* could become very difficult to calculate by hand. But with the aid of *Matlab* and the looped algorithm built into our model, S^* is quick and easy to determine.

This paper only examines the value of S^* with the simple cost-benefit expressions provided in Economics 381, but I would invite the reader to try inputting more complex expressions, such as the ones provided by Barrett (1994).

Experiment 1: Set $C_a = \frac{\$3712 \text{ GDP/tonne}}{\$2348 \text{ GDP/tonne}} C_b = 1.581 C_b = 3$

As outlined in the data section, set C_b , the cost of abatement to the rest of the world, according to data from the International Energy Association. For other values, arbitrarily set:

$$\begin{aligned} X_a &= 2 & B &= 2 \\ X_b &= 2 & C_b &= 1.898 \end{aligned}$$

Experiment 2: Set $C_a = 2C_b = 4$

Under this experiment, we assume that the carbon intensity of Canada's economy relative to the world's will increase dramatically in the future. This doesn't seem completely unreasonable when one considers the pace of oil sands development in relation to the steps other countries are taking (for instance, China has included a carbon tax in its latest five year plan). The values for X_a , X_b , and B do not change between experiments.

Experiment 3: Set $C_b = C_a = 2$

Under this scenario, there has been widespread global technology sharing and Canada has moved away from oil sands extraction and other fossil fuel heavy industries. There would also have to be important low carbon transportation breakthroughs in Canada.

Results

The program is coded to return the Nash equilibrium and the value of S for each loop. That is, the output consists of what looks like a 3x1 matrix: the first two

columns give the decision choices of player 1 and player 2, and the third gives the corresponding value of S . The reader can then browse through the output, stopping where player 1's choice switches from 2 (defect) to 1 (co-operate). The value of S^* can be read from this point in the output.

Experiment 1: Set $C_a = \frac{\$3712 \text{ GDP/tonne}}{\$2348 \text{ GDP/tonne}} C_b = 1.581 C_b = 3$

Looking down the *Matlab* output, we see that $S^* = 2.00$. That is, a sanction of value 2.00 levied against Canada every time it defects from an IEA will create enough incentive to stop it from deferring. This of course assumes that the threat of a sanction is credible. See the Appendix or attached USB stick for *Matlab* code.

Experiment 2: Set $C_a = 2C_b = 4$

When it costs Canada twice as much to abate than the rest of the world, we can see from the *Matlab* output that $S^* = 4.00$. We can conclude that it takes a larger sanction to induce Canada to co-operate when it costs them more to abate. That is to say, the policy choices of today (e.g. the rate at which we develop the oil sands) can have an important effect on the teeth required of future IEAs.

Experiment 3: Set $C_b = C_a = 2$

Encouragingly, we see that $S^* = 0$ in this case. So when Canada and the rest of the world are on par for the cost of emissions reductions, sanctions are unnecessary. Again, the policy choices of today (e.g. the development of a green economy) effect how harsh future IEAs will have to be in order to work.

Discussion

Initially, my vision for this paper was very grand. I had plans to write a program that could accept any functional forms for costs and benefits then return neat graphs showing players' decision based on each possible value of S . Manipulating *Matlab* to return useful diagrams proved much more difficult than I had imagined (and much more difficult than generating roughly equivalent visuals in *Mathematica*). Although I am still happy with my algorithm's abilities, my initial experiences with *Matlab* have been less than optimal; in preparing my *Excel* and *Mathematica* projects, I can't think of any instances when I spent eight-plus hours in the lab with nothing to show for my efforts.

In summary, although the visual representations created by my algorithm are not very flashy, it still solves simple problems and provides some insight into how trade sanctions might make an IEA more enforceable, and how getting S right can avoid economic inefficiencies.

Part III: The Effect of Adding Natural Resources to a Computable General Equilibrium

Introduction

Nordhaus argues that natural resource do not impose any limits to growth, but instead create rather undesirable “growth drags” (Romer 2006). A growth drag is a reduction in the growth rate of the economy, which is not to be confused with a

“limit to growth,” which is a level effect that caps the size of output at some finite value. That is, a limit to growth need not have any effect on the growth rate, just as a growth drag does not in itself imply any upper bound on the scale of economic activity.

Critically, Nordhaus contends that the size of the drag on growth created by the finite nature of earth’s natural resources is very small (Romer 2006). Not negligible, but certainly not substantial either. This view has generated some protest from ecological economists and environmentalists, but continues to be taken very seriously by most “mainstream” economists (Romer 2006). In light of this, I have attempted to factor natural resources into a simple general equilibrium model of the economy in a very simple way. This modification to the model that was presented in class will, I hope, help to explain how Nordhaus settled on the conclusion that natural resources are not actually all that important.

Economic Model

Model 1: Simple General Equilibrium with Natural Resources

Since the general equilibrium model was presented in class and covered in the textbook, I will only take space here to explain the simple modifications I made to it. These were made following the lead of Brock and Taylor (2009), who build what they call the “Green Solow Model” in order to explain how monotonic convergence to the steady state leads to the generation of an Environmental Kuznets Curve (EKC) type relationship between income per capita and pollution. Although I do not plan

on modeling an EKC in this paper, the production function Brock and Taylor use is still relevant.

Our production function will look as follows:

$$Y_t = (1 - \theta) F(K_t, B_t L_t, N_t)$$

$$Y_t = (1 - \theta) K_t^{1-a-c} N_t^c (B_t L_t)^a = b_t (1 - \theta) K_t^{1-a-c} N_t^c L_t^a$$

where θ is the level of abatement, and B_t is the productivity of labour, K_t is capital, L_t is labour, and N_t is natural resources. Including θ (between 0 and 1) imposes costs to abatement, which are very high for values of θ close to 1. The second line of the equation applies a Cobb-Douglas production specification, which follows the example set by the simple general equilibrium model presented in class. Notice that we maintain constant returns to scale, and that the factor share paid to labour is a , natural resources is c , and capital is $(1 - a - c)$.

As a result of this adjustment to the goods supply equation, we must now specify equations for a natural resource market (in order for *GAMS* to solve the new general equilibrium). For this model, I assume we are looking at the very short term (i.e. and instant in time), so I fix the supply of natural resources as follows:

$$ns = \theta(nsbar)$$

Notice that θ shows up in order to reflect the level of abatement. This means that as θ increases, the cost of abatement goes up, but so does the supply of natural resources. A story we might tell to justify this representation is that when we choose to abate (i.e. use fewer resources, in this case) we must develop technological substitutes, which divert research and development funds from uses that might increase output or economic growth.

Next, we must specify a demand function for natural resources. I won't go into any detail here because I simply follow the lead of the general equilibrium model presented in class and define demand for resources similarly to how demand for capital and labour are specified:

$$nd = c \times Y_t \times \left(\frac{p}{f}\right)$$

Where p is price of the good produced, and f is the unit price of natural resources.

Next, we simply run *GAMS* and find the optimum level of income.

Model 2: Johansen General Equilibrium with Natural Resources

I made some very simple modifications to the Johansen model in order to explore the effects that natural resources have on growth rates. Experiments with the simple general equilibrium model should give a nice reflection of the level effects of modeling resources as shown above, but can only give linear approximations of the effects of resources on growth rates (and would require at least two runs of *GAMS* for each linear approximation).

Since I modified the production function by following the lead of Brock and Taylor (as shown previously under Model 1), the growth rate of production will also change from the one presented in class. This is illustrated below:

$$\ln Y_t = \ln(1 - \theta) + (1 - a - c) \ln K_t + c \ln N_t + a \ln L_t$$

$$\frac{\dot{Y}_t}{Y_t} = (1 - a - c) \frac{\dot{K}_t}{K_t} + c \frac{\dot{N}_t}{N_t} + a \frac{\dot{L}_t}{L_t}$$

Notice the absence of the $(1 - \theta)$ term in the growth rate of output/goods supply function. This is because θ does not vary with time, and so is a level effect only (as explained earlier). This will facilitate the Nordhaus-type conclusions that we are aiming to model.

Next, we simply write equations for the growth rate of natural resource supply and natural resource demand, following the method already used in the model for capital and labour. This will allow us to vary the growth in resources—more on this later in the experiments section. Finally, we run *GAMS* and find the optimum growth rate of income.

Computational Method

If any of the computational method remains unclear after reading the model descriptions above, I invite the reader to investigate the *GAMS* code provided (on

the USB stick). Our method for solving the general equilibrium is equivalent to the one used in class, so no further comment is necessary in this section.

Data

The issue of what factor shares to put in labour, natural resources, and capital is an important one. I use the Mankiw, Romer, and Weil (1992) values for capital and labour (1/3 and 2/3, respectively, which assumes that labour is technology-augmented). The question of what factor share to put on natural resources is potentially more contentious, but I use the Nordhaus value of approximately 0.05. Many would argue that this value is too small, but for the sake of convenience and because of the empirical rigor which Nordhaus arguably employed to settle on this value, I'll use it anyways.

Experiments

Model 1: Simple General Equilibrium with Natural Resources

Experiment 1.1: Abatement Effort

Since the proportion θ scales up the amount of resource we have and scales down our level of production, we can think of it as the amount of abatement effort society is exerting. At high levels of θ , we are spending a large portion of GDP on developing abatement technology, and as a result we have greater access to natural resources (i.e. natural resource supply is high). At low level of θ , we are only spending a small portion of GDP on abatement, but natural resource supplies are being impacted

harshly. So there is a trade-off between natural resource supplies and the cost of abatement. We will experiment with this relationship by taking three levels of θ (a low, medium, and high value) and seeing what the effect is on income.

Experiment 1.2: Natural Capital Pool

Many economists have begun to think of natural resources as a kind of capital. We will embrace this concept and examine the effects of exogenous resource “shocks.” That is, we will run two different levels of natural resource supply (n_s) and see how income is affected.

Model 2: Johansen General Equilibrium with Natural Resources

Experiment 2.1: Resource Regeneration

By definition, renewable resources have the potential to regenerate, while non-renewable resources either do not regenerate, or regenerate on time scales that are too large to be significant to society. We will model this with three experiments: the first will impose a positive level of resource growth (this would apply in a case where society was more than sustainable; we are using resources at a rate lower than they are growing back); the second will have zero resource growth (in this case, we have nailed sustainability spot on); and the third case will have negative resource growth, or resource degradation.

Results

Model 1: Simple General Equilibrium with Natural Resources

Experiment 1.1: $\theta = 0.10, 0.50, 0.90$

theta	0.10	0.50	0.90
good supply/demand	0.798	0.754	0.183
labour supply/demand	2	2	2
capital supply/demand	1	1	1
resource supply/demand	0.2	1	1.8
price	1	1	1
wage	0.132	0.124	0.03
cost of capital	0.271	0.256	0.062
resource cost	1.317	0.249	0.034
income	0.798	0.754	0.183

Table 1. How Theta Affects Variables Including Income

As we can see, income is highest when theta is 0.10, and lowest when it is 0.90. This suggests that the tradeoff between natural capital levels and abatement costs lies in favour of avoiding abatement costs and having fewer natural resources. This conclusion seems appropriate given our Nordhaus type approach. We also notice that resource supply (and thus, demand) is highest when we do the most abatement—this is encouraging. Finally, we discover that the price of resources is lowest when we abate the most and they are the most abundant. This also seems consistent with what we might have predicted.

Experiment 1.2: ns = 1, 2, 5

nsbar	1	2	5
good supply/demand	0.635	0.798	1.08
labour supply/demand	2	2	2
capital supply/demand	1	1	1
resource supply/demand	0.1	0.2	0.5
price	1	1	1
wage	0.105	0.132	0.178
cost of capital	0.216	0.271	0.367
resource cost	2.095	1.317	0.713
income	0.635	0.798	1.08

Table 2. How nsbar Affects Variables Including Income

Encouragingly, income is highest when nsbar is 5. We should keep in mind that nsbar is the *unadjusted* resource supply, which is multiplied by theta to obtain the actual resource supply/demand (which is presented in the table). Notice also that the price of resources goes down as they become more abundant. This is a good sign.

Model 2: Johansen General Equilibrium with Natural Resources

Experiment 2.1: Gns = 0.25, 0, -0.25

Growth rates	x100 for Percent		
ns	0.25	0	-0.25
good supply/demand	0.013	0	-0.013
labour supply/demand	0	0	0
capital supply/demand	0	0	0
price	0	0	0
wage	0.013	0	-0.013
cost of capital	0.013	0	-0.013
resource cost	-0.237	0	0.237
income	0.013	0	-0.013

Table 3. How the Growth Rate of ns Affects Variables Including Income

Since capital accumulation is assumed to be nonexistent and it is assumed that population is stable, all growth is generated from growth in natural resources. As a result, we can see exactly the effect that resources have on growth. As expected, we see modest growth when resources are growing continuously at a rate of 25%. Also encouraging is the fact that income growth is zero when growth in ns is zero, and that income falls when resources are depleted at a rate of 25%. These results are in line with the Nordhaus school of thought, because even very large swings in natural resources cause only modest changes in the growth of income. This is due to the fact that we gave resources s factor share of only 0.05 – a value Nordhaus would probably think of as very generous.

Discussion

What I have presented in this paper is a very simple way to model how natural resource levels interact with production. It is alarming to observe that even the “cutting edge” in growth theory is not much more sophisticated in its treatment of natural resources. Nordhaus uses some relatively advanced econometrics to prove that the factor share of resources are very small (and have actually been dropping in the recent past), but provides very little evidence that this trend will continue. He also does little to explain how technology, labour, and capital can be substituted indefinitely for natural resources. That is to say, he subtly makes the claim that as long as we have enough technology, we really do not need any natural resources whatsoever.

This paper makes the claim that although Nordhaus’ findings can be confirmed using a general equilibrium computational approach, they still may be based on some very shaky assumptions. I would be interested to see future research that modeled the substitutability of technology (or labour or capital) for resources as having decreasing returns *once a certain point is reached*, or even approaching zero at some point.

Appendix

Part I:

A1 - Code for Model 1 (Textbook Solow)

```

SetAttributes[n, Constant]
SetAttributes[g, Constant]
SetAttributes[δ, Constant]
SetAttributes[α, Constant]
SetAttributes[γ, Constant]

Clear[n, g, δ, α, β, γ, k]

Manipulate[
  Show[Plot[
    {Tooltip[(δ + n + g) k, "break-even investment, (δ + n + g)k"],
      Tooltip[k^α, Row[{"output per worker, ", Style["f(k)", Italic]}]],
      Tooltip[γ k^α, Row[{"investment, ", Style["sf(k)", Italic]}]]}, {k, 0, 20},
    PlotStyle → {{Thickness[.005], Blue}, {Thickness[.005], Green}, {Thickness[.005], Red}},
    PlotRange → {{-3.5, 20}, {- .8, 4}},
    Filling → {1 → {3}},
    ImageSize → 550,
    Ticks → {
      {{{(γ / (δ + n + g))^(1/(1-α))}, Style["k*", {12, Bold}]},
      {
        {(α / (δ + n + g))^(1/(1-α))}, Style["k*_GR", {12, Bold}]},
      {None}}}],
    {{γ, .5, "rate of saving"}, 0, .9, Appearance → "Labeled"},
    {{δ, .1, "depreciation"}, 0.0, .5, Appearance → "Labeled"},
    {{n, .03, "labor-force growth"}, 0.001, .2, Appearance → "Labeled"},
    {{g, .03, "rate of technical progress"}, 0.001, .2, Appearance → "Labeled"},
    {{α, 0.33, "capital's factor share"}, 0.001, 0.75, Appearance → "Labeled"}]

```

A2 – Code for MRW Solow with Human Capital

```

SetAttributes[n, Constant]
SetAttributes[g, Constant]
SetAttributes[δ, Constant]
SetAttributes[α, Constant]
SetAttributes[β, Constant]
SetAttributes[γ, Constant]
SetAttributes[φ, Constant]

Clear[n, g, δ, α, β, γ, φ, k, h]

Manipulate[
Show[Plot3D[
{Tooltip[(n + g + δ) k, "break-even physical investment, (n+g+δ)k"],
  Tooltip[(n + g + δ) h, "break-even human investment, (n+g+δ)h"],
  Tooltip[k(α/(1-α-β)) hβ/(1-α-β),
    Row[{"output per effective worker,", Style["f(k,h)", Italic]}]},
  Tooltip[γ k(α/(1-α-β)) hβ/(1-α-β),
    Row[{"investment in physical capital," Style["γf(k,h)", Italic]}]},
  Tooltip[φ k(α/(1-α-β)) hβ/(1-α-β),
    Row[{"investment in human capital," Style["φf(k,h)", Italic]}]}], {k, 0, 20},
{h, 0, 20}, PlotStyle → {{Thickness[0.005], Green}, {Thickness[0.005], Darker[Green, 0.3]},
  {Thickness[0.005], Red}, {Thickness[0.005], Blue},
  {Thickness[0.005], Darker[Blue, 0.3]}}, PlotRange → {{0, 20}, {0, 20}, {0, 80}},
ImageSize → 550,
Ticks → {{{{ $\left(\frac{\gamma^{1-\beta} \phi^\beta}{n+g+\delta}\right)^{1/(1-\alpha-\beta)}$ , Style["\!\(\(*SuperscriptBox[\(k\), \(*\)]\)\"", {16, Bold}]}]},
  {{{ $\left(\frac{\gamma^\alpha \phi^{1-\alpha}}{n+g+\delta}\right)^{1/(1-\alpha-\beta)}$ , Style["\!\(\(*SuperscriptBox[\(h\), \(*\)]\)\"", {16, Bold}]}]},
  None}}],
{{γ, 0.3, "rate of savings on physical capital"}, 0, 0.9, Appearance → "Labeled"},
{{φ, 0.3, "rate of savings on human capital"}, 0, 0.9, Appearance → "Labeled"},
{{α, 0.30, "physical capital's factor share"}, 0.1, 0.5, Appearance → "Labeled"},
{{β, 0.30, "human capital's factor share"}, 0.1, 0.5, Appearance → "Labeled"},
{{n, 0.03, "labor-force growth"}, 0.001, 0.2, Appearance → "Labeled"},
{{g, 0.03, "rate of technological progress"}, 0.001, 0.2, Appearance → "Labeled"},
{{δ, 0.1, "depreciation of physical or human capital"}, 0.033, 0.5, Appearance → "Labeled"}]

```


Part II:

B1 – Code for MATLAB

```

1      %International Environmental Agreements and Trade Sanctions
2
3      %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
4      %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
5      %   Players: Player 1 (Canada) and Player 2 (rest of the world)   %
6      %   Two strategies each: C1/C2 co-operate or accede to agreement   %
7      %   D1/D2 defect or go against agreement                           %
8      %   Payoffs: P1(C,C)=B*(Xa+Xb)-Ca*Xa, P1(C,D)=B*Xa-Ca*Xb,        %
9      %   P1(D,C)=B*Xb-S, P2(D,D)=0-S                                   %
10     %   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
11     %   P2(C,C)=B*(Xa+Xb)-Cb*Xb, P2(C,D)=B*Xa,                        %
12     %   P2(D,C)=B*Xb-Cb*Xb, P2(D,D)=0                                  %
13     %   Matrix:                                                         %
14     %   P2                                                                %
15     %   C           D           %
16     %   C  B*(Xa+Xb)-Ca*Xa, B*(Xa+Xb)-Cb*Xb  B*Xa-Ca*Xb, B*Xa  %
17     %   P1                                                                %
18     %   D           B*Xb-S, B*Xb-Cb*Xb           0-S, 0           %
19     %   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
20     %   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
21
22     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
23     %Now we need pay-off matrices. P1 is player 1's pay-off matrix and %
24     %P2 is player 2's.                                                 %
25     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
26
27     for S=0:0.1:5
28
29     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
30     %The S loop (i.e. "for S=0:0.1:5") replays the game                %
31     % (endval-initval)/increment number of times, each time adding the %
32     % increment value to S. In our example, S starts at 0 and increases by %
33     % 0.1 units until it reaches 5.                                     %
34     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
35
36     B=2; %B scales up the benefits from emissions reduction
37     Ca=3; %Ca scales up the costs of emissions reductions for country a (Canada)
38     Cb=1.90; %Cb scales up the costs of emissions reduction for country b (world)
39     Xa=2; %Xa is the amount of emissions reduction done by country a (Canada)
40     Xb=2; %Xb is the amount of emissions reduction done by country b (world)
41
42     P1=[B*(Xa+Xb)-Ca*Xa B*Xa-Ca*Xb;
43         B*Xb-S 0-S];
44
45     P2=[B*(Xa+Xb)-Cb*Xb B*Xa;
46         B*Xb-Cb*Xb 0];

```

```

47
48 *****
49 %Created an isolated payoff vector (IPV) for each player and each
50 %strategy. Player one will have two IPVs, C1 and D1, and player two will
51 %have two IPVs, C2 and D2. Each IPV takes the other player's strategy as
52 %given and then returns the payoffs corresponding to each strategy.
53 *****
54
55 %Player 1 (Canada):
56 - C1=P1(:,1);
57 - D1=P1(:,2);
58
59 %Player 2 (world):
60 - C2=P2(1,:);
61 - D2=P2(2,:);
62
63
64 *****
65 %Now we need the best response functions for each player:
66 *****
67
68 %Player 1
69 - [br11 bri11]=max(C1); %Best response for player 1 given player two plays C
70 - [br12 bri12]=max(D1); %Best response for player 1 given player two plays D
71
72 %Player 2
73 - [br21 bri21]=max(C2); %Best response for player 2 given player one plays C
74 - [br22 bri22]=max(D2); %Best response for player 2 given player one plays D
75
76 *****
77 %Find the Nash Equilibrium(s)
78 *****
79
80 - if (br11==bri12) %P1 has a strictly dominant strategy
81 -     if (bri21==bri22) %P2 has a strictly dominant strategy
82 -         nash=[bri11 bri21];
83 -     elseif (bri21~=bri22) %P2 does NOT have a strictly dominant strategy
84 -         if (br11==1)
85 -             nash=[bri11 bri21];
86 -         elseif (br11==2)
87 -             nash=[bri11 bri22];
88 -         end;
89 -     end;

```

```
90 - elseif (bri11~=bri12) %P1 does NOT have a strictly dominant strategy
91 -     if (bri21==bri22) %P2 has a strictly dominant strategy
92 -         if (bri21==1)
93 -             nash=[bri11 bri21];
94 -         elseif (bri21==2)
95 -             nash=[bri12 bri21];
96 -         end;
97 -     elseif (bri21~=bri22) %P2 does NOT have a strictly dominant strategy
98 -         if (bri11==bri21)
99 -             if (bri11==1)
100 -                 nash1=[bri11 bri21];
101 -             elseif (bri11==2)
102 -                 if (bri12==1)
103 -                     nash1=[bri12 bri21];
104 -                 elseif (bri12==2)
105 -                     nash1=[0 0];
106 -                 end;
107 -             end;
108 -         elseif (bri11~=bri21)
109 -             nash1=[0 0];
110 -         end;
111 -
112 -     if (bri12==bri22)
113 -         if (bri12==2)
114 -             nash2=[bri12 bri22];
115 -         elseif (bri12==1)
116 -             if (bri11==2)
117 -                 nash2=[bri11 bri22];
118 -             elseif (bri11==1)
119 -                 nash2=[0 0];
120 -             end;
121 -         end;
122 -     elseif (bri12~=bri22)
123 -         nash2=[0 0];
124 -     end;
125 -
126 -     if (nash1==nash2)
127 -         nash=nash1;
128 -     elseif (nash1~=nash2)
129 -         nash=[nash1;
130 -             nash2];
131 -     end;
132 - end;
133 - end;
134 -
```

```
135
136 *****
137 %Display the Nash Equilibrium output. If one unique equilibrium then
138 %displays a scalar, if multiple equilibriums then displays a vector.
139 *****
140
141 - decision = [nash, S];
142
143 - decision
144
145
146 - end
147
```

Part III:

Please find the *GAMS* code used to generate the findings presented in this report on an attached USB stick. You will also find the list files for each experiment, filed in separate folders for experiments 1 and 2.

References

Part I:

Mankiw, N. G., Romer, D., & Weil, D. N. (1992). A contribution to the empirics of economic growth. *The Quarterly Journal of Economics*, 107(2), pp. 407-437.

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CO₂ Emissions from fuel combustion: Highlights (2010). *International Energy Association*. < <http://www.iea.org/co2highlights/CO2highlights.pdf>>

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Romer, D. (2006). *Advanced macroeconomics*. New York: McGraw-Hill