The Centipede game: A MATLAB Approach

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INTRODUCTION

For this project I have chosen to explore the dynamic aspect of cooperation and profit seeking. To do this I make use of game theory, more specifically: the centipede game. The centipede game was first introduced by Rosenthal (1981) as a means of explaining the dynamics of cooperation and the ever-ongoing profit maximization incentives faced by rational individuals. In his famous paper, Rosenthal wanted to demonstrate what he believed to be the correct approach to game solutions. With this in mind, "the thesis of [his] paper [was] that finite, non-cooperative games possessing both complete and perfect information ought to be treated like one-player decision problems" (Rosenthal, 1981). Since its inception, the centipede game has been further studied by a number of economists including Binmore (1987), Kreps (1990), Reny (1988), and many others. In this paper, I discuss the history of the centipede game, its various versions, experimental results, and practical applications. The main focus of this paper is to provide a detailed analysis of the centipede game in the theoretical context of game theory using MATLAB as an experimental platform. The layout of this paper is as follows

1. Centipede Game Overview
2. Economic Model
3. Computational Model
4. Centipede game theoretical predictions and modified version
5. Experiments
    a. MATLAB experiment 1 and 2
    b. Earlier Experiments (McKelvey & Palfrey (1992) and Nagel and Tang (1996)
    c. MATLAB experiment 3, 4 and 5
6. Discussions

7. Applications

8. Conclusion

CENTIPEDE GAME OVERVIEW: ORIGINAL

The Centipede Game is an extensive-form game where two players have an alternating chance to claim the larger portion of a continuously increasing payoff. The original versions of the game consisted of a hundred-move sequence (hence the name "centipede") with linearly increasing payoffs. As soon as a player *takes*, the game ends with the players receiving their respective payoffs for that node (Bornsteina, Kuglera & Ziegelmeyer, 2002, pp 1-2). An illustration of the Rosenthal centipede game is given in his 1981 paper.

FIGURE 1. Derived from Rosenthal 1981

ECONOMIC MODEL

This project centers on the use of game theory as a means of understanding human behaviour. Game theory is an aspect of economics that deals with the use of 'games' to model the decisions faced by humans during strategic interactions. The details of these games vary depending on the scenario being modeled, but common traits include: the presence of rational players or agents (I will use these two words interchangeably to refer to the individuals in the game); the choice of action by the players; and the allocation of numerical payoffs for different combinations of strategies. Most, if not all, games posses all of the above listed attributes. Rational play is a strict assumption made in game theory. It implies that all players play in a way that is most beneficial to them. Meaning that all agents play to maximize their own payoff. The payoffs for each player include all the benefits that agent receives from a particular play. These payoffs can either be taken as cardinal or ordinal depending on the game being played.

The centipede game models a situation of complete information between 2 agents or players. It is a game of complete information and therefore insists that all players involved in the game are fully aware of the respective payoffs of each combination of actions or strategies played by the other agents, and therefore play accordingly. The centipede game, like mentioned above, is a dynamic game, meaning that one agent starts the game and the next agent plays after seeing the choice of action by the first agent. Sequential or dynamic games are often solved via a process called backward induction. Backward induction is a strategy of solving sequential games that involves starting at the end node, picking the dominant strategies of each player, and working our way backwards until we arrive at the root node. Backward induction is only applicable in situations of complete information like in the centipede game.

The effectiveness of game theory lies in the idea of Nash Equilibrium. A Nash equilibrium is described as a combination of strategies that yields the highest possible payoff for each individual provided what the other player(s) is playing. For my analysis I discuss the Nash Equilibrium of the centipede game computed using MATLAB, and compare my results to the theoretical hypotheses and previous experiments done on the game. I will also be testing the weight of the rationality assumption on the choice the agents make.

COMPUTATIONAL MODEL

For the purpose of this paper I make use of MATLAB to arrive at a Subgame Perfect Nash Equilibrium (SPNE). I adopt the theoretical assumption that the agents have complete information; meaning that they know the exact payoff associated with any given strategy and play to maximize their own payoffs. Using MATLAB I create two agents that exist in a world of pre-determined benefits/payoffs. The agents then take part in a game, both trying to maximize his or her own payoffs given what the other is playing. Here, using a computational method like MATLAB is very helpful to compute the best response of the players in an efficient and precise manner.

THE CENTIPEDE GAME: THEORETICAL HYPOTHESES AND MODIFIED VERSION

Consider *Figure 1*, by taking the far left node as the root node, and the choice at every node for each player to be either *Right* or *Down*, we can deduce that player 2 will always pick *Down* on the first move for all the Nash equilibriums in this game. How? Look again at *Figure 1*, using backward induction, we can see that player 2 would be better off playing *d* in the second to last period for a payoff of 11. This is higher than the payoff of 10 that she will receive if she continued on to the last round. Likewise, player 2 would be better of playing *D* in 8^{th} period for a payoff of 8 as opposed to 7, which is what his payoff would be given he lets player 2 continue to play. Thinking along the same vein we can decipher that principal equilibrium has both players selecting *Down* on their first move.

In the original version of the centipede game by Rosenthal (1981), the payoffs increased linearly (as we see above), while in the modified version of the game created by Aumann, in 1988, the (joint) payoff increases exponentially. For his game, Aumann also revised the choices faced by the players to be *Continue* and *Stop*. In this paper I will be making use of a modification of the six-legged game under the Aumann centipede game model.

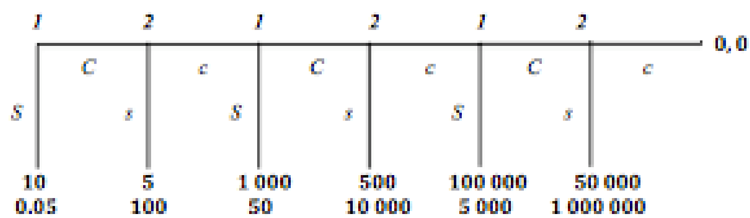

FIGURE 2. Two-person six-move centipede game (Aumann, 1992)

In the illustration above we can clearly see that passing at any point strictly decreases a player's payoff if the opponent player *stops* on the next move. If the opponent also continues, the two players are faced with the same choice situation with reversed roles and increased payoffs. Given the fact that the game has a finite number of moves, which is known in advance to both players (in our case 6), the standard argument of backwards induction leads to a unique Nash equilibrium prediction: the first player takes the larger pile on the first move (McKelvey & Palfrey, 1992). In support of Rosenthal, Binmore explained the thought behind this. In his paper, Binmore argued that by working from right to left and successively deleting non-optimal actions, the undeleted action at each node will be for each player to play *stop* at any given node they find themselves (1987 pp. 196). The pure strategies for the two players are therefore also dominant strategies of *S* for player 1 and *s* for player 2 (or more simply put, *stop* for both). These strategies also constitute the unique sub-game perfect equilibrium.

In my analysis I will be testing the theoretical predictions of the centipede game using MATLAB and comparing my results to other experiments done on the game

<u>**EXPERIMENTS AND RESULTS: CENTIPEDE GAME IN THE TEST ENVIRONMENT**</u>

For the discussion on earlier experiments we will focus on two main experiments, both conducted in the 1990's: One conducted by McKelvey & Palfrey in 1992; and the other by Nagel and Tang in 1998. As for my MATLAB, analysis I include five main experiments on the centipede game. To do this I use two modified versions of the Aumann game (described above), which I created in MATLAB. The versions constitute of a shorter and a longer version of Aumann's centipede game[1]. Like the Aumann game I adopt the choices available to the player to be *continue* or *stop.* I set *continue* to be choice 1 and *stop* to be choice 2. For my analysis, I run experiments on both versions to test the consistency of the Nash Equilibrium with respect to the duration of the game. In addition, I also relax the rationality assumption of the players and test the outcome of one player being irrational, and then the other. This helps provide a complete understanding of the behaviour of the agents in the real world. Below is the breakdown of the experiments I will be running using MATLAB.

Experiment	Rounds	Game Used	Rationality level
1	Round 1	Short Centipede game	Both players rational
2		Long Centipede game	Both players rational
3	Round 2	Short Centipede game	Second player is irrational
4		Long Centipede game	Second player is irrational
5	Round 3	Long Centipede	First player is irrational

FIGURE 3: Table showing experiments

[1] I will henceforth refer to these games as short and long centipede game respectively.

MATLAB EXPERIMENTS ROUND 1

For the first round of MATLAB experiments I will be testing the theoretical hypothesis of the centipede game

<u>EXPERIMENT 1 AND RESULTS:</u>

For Experiment 1, I replicated a shorter version of the Aumann game, with exponentially increasing payoffs (see Figure 3). For my analysis I broke the game done into 3 subgames, starting with the end node. In this game Player 1 has four strategies namely; (C, C), (C, S), (S, C), and (S, S) while Player 2 only has 2; c or s.

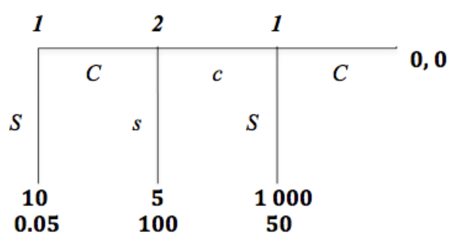

FIGURE 4:Game tree of a shortened version of Aumann centipede game

Using game theory logic and MATLAB I created a system of finding the Nash equilibrium of the game using back ward induction. First, I created the payoff matrix for both players. These payoff matrices show the payoff choices for each player at each individual subgame node. Again, because I am using backward induction, I begin at the end node.

```
P1=[0 1000;     P2=[0 50;
    5 5;            100 100;
    10 10;          0.05 0.05;
    10 10];         0.05 0.05];
```

FIGURE 5: Payoff matrices for the short centipede game

Second, I broke the game down into subgames, and identified the choices and payoffs that each player faces in those subgames. Third, I singled out the choices faced by player one in the last subgame and set him to choose a strategy that best maximizes his payoffs. This step is in accordance with the assumption that every player is rational and therefore plays in order to attain the highest possible payoff. From this point I can compute the outcome faced by player two in the second subgame, based on player one's best response in the previous subgame. The forth step is to repeat what I did for player one, for player two; identifying the choices and payoffs and setting player two to be rational thus maximizing her benefit. From here I arrive at the last outcome, and consequently player one's first choice decision. Player one, like the rational player he is, then plays in a way that is most beneficial to him[2]. My final outcome, as well as the best responses of each player aligns with the theoretical hypotheses that each player will play *stop* at the very first node. So playing *stop* becomes the Subgame Nash Equilibrium.

Outcome	
SPNE=[(bri11 sgbri11); sgbri21)]	**SPNE=[(2 2); 2)]**

EXPERIMENT 2 AND RESULTS:

For experiment 2, I expanded the centipede game to include 4 subgames, to test the consistency of the previously found subgame Nash Equilibrium. The game becomes more complicated with the addition of another subgame.

[2] For a detailed breakdown of the code see Appendix 2a

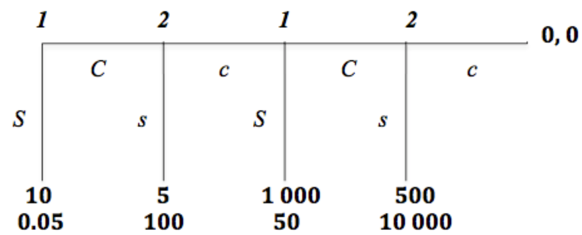

FIGURE 6:Game tree of a longer version of Aumann centipede game

The players now have 4 strategies each and 8 possible outcomes. I approach this game similarly to the way I did the first experiment, beginning with the payoff matrices and then computing the best response of each player given their possible payoffs.

```
P1=[0 500;          P2=[0 10000;
1000 1000;          50 50
5 5;                100 100;
5 5;                100 100;
10 10;              0.05 0.05;
10 10;              0.05 0.05;
10 10;              0.05 0.05;
10 10];             0.05 0.05];
```

FIGURE 7: Payoff matrices for the long centipede game

For this experiment I set up each player to take into consideration the decision of the player before him or her. By using the backward induction process I set up in MATLAB, I was able to find the best response of both players given the expected play of the other player at each subgame, and eventually for the whole game. Once again I was able to replicate the theoretical hypotheses.[3]

Outcome	
`SPNE=[(bri1 sgbri11);(sgbri1 sgbri111)]`	`SPNE=[(2 2);(2 2)]`

DISSCUSSION OF EXPERIMENT 1 & 2

Using MATLAB I was able to arrive at a Subgame Nash Equilibrium in which both players play stop at their first decision node for an outcome of (10, 0.05). This means that my findings are inline with the theoretical hypotheses of the centipede game. Since I achieved the same results when I ran the experiment on both versions of the centipede game I can conclude that our predicted response corresponds to the Nash equilibrium of the Rosenthal game. Therefore, we thus have a situation where there is an unambiguous prediction made by game theory. Yet, despite this unambiguous prediction, game theorists have not seemed too comfortable with the above analysis of the game. They wonder whether it really reflects the way in which anyone would actually play such a game (Binmore (1987), Aumann (1988)). Binmore argues that the above way of playing the game may not always be sub game perfect. He makes the point that if an individual suddenly finds herself at node 50 in a 100-step centipede game, should she then adopt the view that arriving at this point was due to some series of uncorrelated flukes? It would seem highly unlikely, and so what then is the cause and how should the player involved proceed? Bimore states, “the answer depends on the environment in which the game is played” (Binmore 1987, 196). In the same way, Aumann (1988) makes a point of known rationality being a key

[3] For a detailed breakdown of the code see Appendix 2b

factor in the determining the actual sequence of play in the "real world" (Rapoport 2006 p.3). From this, we can say that although we believe and accept the Nash equilibrium of the game to be *down* or *stop*, as the case may be, we know that the state of the environment may have an affect on the play. This point brings us to results found by earlier experiments on the game.

EARLIER EXPERIMENTS

MCKELVEY & PALFREY (1992)

McKelvey and Palfrey made use of the Aumann centipede game of exponentially increasing payoffs. They reported the results of multiple sessions of two carefully conducted centipede game experiments. The participants were undergraduate students who were paid in accordance with their performance, none of who had any previous experience with the centipede game. Each game started with a total pot of 50 cents, divided into a 'large' pile of 40 cents and a 'small' pile of 10 cents. Each time a player *continued*, both piles were doubled in value and the roles of the two players were interchanged. Each experimental session included 20 or 18 participants who were divided into two groups (player 1 and player 2) at the beginning of the session for a total of 7 sessions. Player roles (types) remained fixed during the session. Each participant played one game with each of the participants assuming the opposite role. Thus, no participant was ever matched with another participant more than once thus eliminating the possibility of cooperation. All of the above was made common knowledge. The two games were a four move game and a six move game.

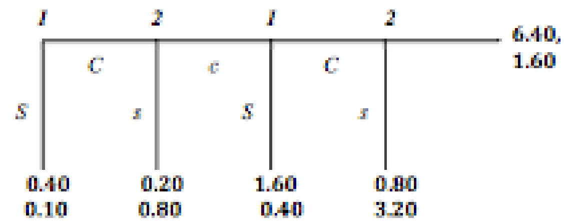

Figure 8. Two-person four-move centipede game (Mckelvey & Palfrey, 1992)

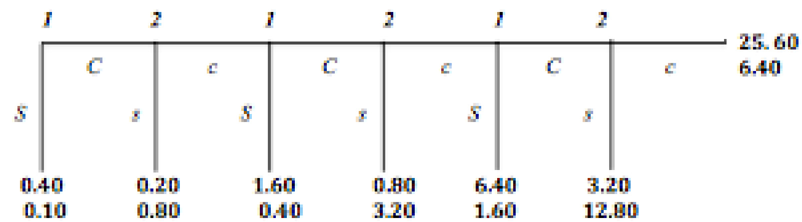

FIGURE 9. Two-person six-move centipede game (Mckelvey & Palfrey, 1992)

These two games were designed to study the descriptive power of the backward induction solution. Under the Nash equilibrium solution, all the games should end at the first terminal node. On the other hand, if the two players fully cooperate by always continuing, all the games end in the final terminal node. The results of the experiment did not correspond with any of the above. Mckelvey & Palfrey found that only 37 of 662 games end with the first player taking the large pile on the first move, while 23 of the games end with both players passing at every move with the rest of the games possessing a scattered distribution of end game nodes. Nevertheless, the experiment illuminated three patterns in behaviour of the university students. I) The probability of stopping increases, as we get closer to the last move. II) As subjects gain more experience with the game their plays become more 'rational'. Rational in this cause refers to the behaviour predicted under the Nash equilibrium. III) The third pattern was that there appeared to be a higher probability of stopping in the four-move game than in the corresponding node in six move game even though the payoff were the same. Mckelvey & Palfrey attributed this phenomenon to the increase in the possible payoffs under the six-move game. The experiment yielded one other interesting result; there were individuals that always played *continue*. Mckelvey & Palfrey found around 5% of the subject pool to always play *continue*. They described these individuals as altruistic.

CRITICAL EVALUATION OF DISCREPENCIES BETWEEN THEORETICAL PREDICITIONS AND RESULTS

In their paper, Mckelvey & Palfrey account for the above results with a model that had two main features; the possibility of errors in actions and the possibility of errors in beliefs. These two features serve to explain the inconsistencies in the data. Mckelvey & Palfrey's 'error in action' hypothesis attributes irrational play to be as a result of subjects experimenting with different strategies; pressing the wrong key; misunderstanding which player they were; failing to notice that it was the last round; or by some other random event. The authors noticed that sophisticated players, knowing that other players may make mistakes, exploited the situation to delay take and increased their own payoffs (Mckelvey & Palfrey, 1992, p 815). On the other hand, Mckelvey & Palfrey's "error in beliefs" hypothesis takes the view that the subject pool contained a certain proportion of altruists, who placed a positive weight of their utility function on the payoff of their opponents.

NAGEL & TANG (1998)

Nagel and Tang took the centipede game as a reduced normal form game. They collected the results of 5 sessions of play. Both the first end node and the last end node have the same payoff vector as in the 6-move game in McKelvey and Palfrey (1992). Each player has seven choices; player A chooses an odd number from 1 to 13 and player B an even number from 2 to 14[4]. Each session involved 12 university students (six for type A and six for type B). Each participant held the same role through out the duration of the experiment, which was done via computer terminals. The one-shot game was repeated a hundred times, and each time a subject was matched with another student in a different group. All participants were informed of such (complete information). Many patterns of the behaviour found in the extensive-form game study by McKelvey and Palfrey (1992) can also be recognized in Nagel and Tang's study. The results of the study were as follows:

1. All strategies, but 1 and 3 in session 4, have strictly positive relative frequencies.
2. The weakly dominated choice 14 is selected with positive probability 7.70 across all sessions.
3. Choice 1 is chosen 0.50 of time in our game, compared with 0.70 in McKelvey and Palfrey 6-move games and 70 in 4-move games.
4. Modal behaviour is concentrated at the middle choice 7 and choice 9 for players A, or middle choice 8 and choice 10 for players B.
5. There is no clear trend at the sessional level.

[4] See Appendix 1 for the reduced stage normal game payoff matrix

CRITICAL EVALUATION OF DISCREPENCIES BETWEEN THEORETICAL PREDICITIONS AND RESULTS

Like McKelvey and Palfrey, Nagel and Tang obtained data that did not align with the theoretical predictions. But unlike the previous experimentalists, Nagel and Tang focused their reason on the learning aspect of the experiment. They concluded that that most subjects conform on average to the qualitative learning theory. When taking a qualitative outlook, it is assumed that students learn cumulatively, interpreting and incorporating new material with what they already know, their understanding progressively changing as they learn. Thus, comprehension of taught content is gradual and cumulative. So, as each round progresses, each player learns something about their opponent, and as each game passes each player will be learning something about the group they are playing with. This accumulated knowledge may lead to players making decisions not only based on current payoffs but also on knowledge of past results. Acknowledging that there may be deviations from the Nash Equilibrium leads us to our second round of experiments

MATLAB EXPERIMENTS ROUND 2

Round 2 of the MATLAB experiments tests situations that lead to outcomes other than the expected theoretical Nash equilibrium.

<u>EXPERIMENT 3 AND RESULTS:</u>

For experiment 3 I used the short version of the centipede game I used in experiment 1, but this time I set my player two to play irrationally. For simplicity sake, I use the most extreme form of irrationality and set player two to minimize her benefits rather than maximize them. Other than the change in player two's rationality, the set up of the game is the same as the previous experiments[5]. My results from this experiment show that when player two minimizes her payoffs, player one, having complete information, would play in a way that is of best interest to him. I found that provided that player one is rational he will end up playing *stop* in the third node for a payoff of 1000. This happens because player one knows that an irrational player 2 will play *continue* in the first round of the game, and that he, player one, would be better of playing *stop* in the round after that, for a higher payoff. Thus the outcome of the game becomes (1000, 50).

Outcome	
`SPNE=[(bri11 sgbri11); sgbri21)]`	**`SPNE=[(1 2); 1]`**

EXPERIMENT 4 AND RESULTS:

For experiment 4 I used the longer version of the centipede game I used in experiment 2, and again set player two to be irrationally. Once again irrationality here, leads to player two minimizing her benefits rather than maximizing them. Other than the change in rationality of player two the set up of the experiment is the same as the previous experiments[6]. My results show that if player two plays to minimize her benefits that the outcome of the game becomes (1000, 50), with player one playing stop at the forth node just like in experiment 4.

[5] For a detailed breakdown of the code see Appendix 2c
[6] For a detailed breakdown of the code see Appendix 2d

Outcome	
`SPNE=[(bri1 sgbri11);(sgbri1 sgbri111)]`	`SPNE=[(1 2);(1 1)]`

DISSCUSSION OF EXPERIMENT 3 & 4

In my experiments I described irrationality as playing to minimize ones benefits, as unrealistic as this may sound it is not uncommon. Earlier experiments like that of Mckelvey & Palfrey (1992) show that individuals may play to minimize their own payoff for variety reasons. Mckelvey & Palfrey describes these individuals describes as being “altruistic”. They also found the presence of selfish players, who believing that there was a probability that other the players were altruistic had an incentive to mimic this behavior by passing, until they found it convenient to stop . As argued by McKelvey and Palfrey, “these incentives to mimic are very powerful, in the sense that a very small belief that altruists are in the subject pool, can generate a lot of mimicking, even with a very short horizon” (McKelvey and Palfrey, 1992, p. 805). This behavior is captured in experiments 3 and 4.

MATLAB EXPERIMENTS ROUND 3

For my last experiment I explore the possibility of achieving the theoretical outcome, with the presence of irrationality.

EXPERIMENT 5 AND RESULTS:

For my last experiment, I used the longer version of the centipede game, but this time I set player one to be irrational[7]. My results show that if player one plays to minimize his benefits that the outcome of the game becomes the theoretical Nash (10, 0.05), with player one playing stop at the first node.

Outcome	
SPNE=[(bri1 sgbri11);(sgbri1 sgbri111)]	**SPNE=[(2 1);(1 2)]**

The results show us that, the Subgame Nash Equilibrium of Experiment 5 is different from that of Experiment 2, but arrives at the same Nash outcome. Although the outcome of experiment 5 is the same as the theoretical Nash equilibrium, the intuition is quite different. In this experiment player one stops at the first node because he knows that if he was to try and minimize his benefits by playing continue player two, being rational, would stop in the forth node, causing player one (and player two) to have a higher payoff, which in this case player one does not want. The results of this last experiment show that rationality is indeed a factor in the determination of the theoretical Subgame Nash equilibrium, but is not necessary to achieve the hypothesized Nash outcome of (10, 0.05).

[7] For a detailed breakdown of the code see Appendix 2e

DISCUSSION AND LIMITATIONS

After looking at the studies done by Mckelvey & Palfrey etc. we can make some fairly learned assumptions as to the cause of the discrepancies between the human experiment results and the theoretical Nash equilibrium. I adopt the belief that the true explanation is some combination of the "errors in beliefs" hypothesis by Mckelvey & Palfrey (1992) and the learning theory by Nagel and Tang (1998).

Like mentioned above, the rational play would be to always play *stop* at any given time. Knowing this, we can make the argument that if any player plays *continue,* the other players could rightly consider them irrational and therefore, play accordingly (This is shown in experiment 3 and 4). Despite the inclusion of irrationality, the game remains of complete information, provided that all the players are fully aware each players rationality levels. However, if a player is uncertain about the rationality of another player, we can no longer consider the game to be one of complete information. Take for example a player assigned the role of player two, finding herself at the second node and being given a choice of *continue* or *stop*; she knows that the very fact that she has been given a choice of play means that player one played *continue* and therefore, by theory, must be irrational. She is then faced with the choice of her rational choice of *stop* or the choice to take advantage of player 1's proposed irrationality and play *continue*. This idea can be applied to player one. If he knows that by *continuing* he may be perceived as irrational by player 2, thereby causing her to play irrationally, he might chose to do so because he will then be given the the possibility of taking a larger "pile" of money. We can then explain any continuation of play in the game as being attributed to some level of uncertainty of a player about the rationality of the other player. This uncertainty in rationality is more descriptive of the "real world".

Unfortunately, a limitation of this paper is that the MATLAB experiments did not capture this uncertainty between players. The MATLAB experiments adhered to the notion of complete information; player one knew if player two was rational or not, and although, the experiments showed that a change in the rationality of the agent causes a change in the predicted outcome of the game, they did not explore the effect of incomplete information on outcome of the game. One would expect a game, where the agents involved did not know whether or not their opponents were rational to affect the final outcome. A better experiment would be to relax both the rationality and complete information assumptions and compare the outcomes from that experiment to the others. In the same breath, we can deduce that reason experiments 1 and 2 yielded the theoretical Nash equilibrium was because they strictly enforced rationality and adhered to the idea of complete information. Complete information and rationality are therefore necessary assumptions to arrive at the Subgame Nash equilibrium, although they are not realistic assumptions to make in the real world. I therefore believe that the cause of the discrepancies between my MATLAB results and the results found by Mckelvey & Palfrey and Nagel & Tang to be based on common knowledge and rational.

LESSONS AND APPLICATIONS

From the above experiments we see that there are many facets to individual behaviour and players may not always seem to play rationally for a variety of reasons. As mentioned before these reasons include the uncertainty of rationality, the differences in experience levels with strategic games and the presence of optimistic or altruistic players in the game. When viewed in isolation the centipede game may only hold the interest of theorists, however, when given context, it is possible to draw conclusions using the centipede game in regards to both the nature of cooperation and the conditions necessary for it to remain stable within a business framework. For example consider the following scenario:

> *Suppose a game begins with one player being able to post a legal bond (i.e. putting a certain amount of money in a trust managed by an independent party). The enforceable terms of this bond stipulate that 1) if the player posting the bond ever plays "Grab" the amount of the bond will be given to the other player; and 2) if the player posting the bond never plays Grab, the money will be returned to her at the conclusion of the game.*

Given information provided by the story above we are now capable of generating the minimum legal commitment required for both firms to maintain a weak preference to pass play until the final node if that is the desired result. Within any given industry two working firms are capable of generating a framework of co-operation using the model produced by the application of the centipede game. In industries where foresight is not usually a priority the centipede game becomes a useful tool in designing an agreement that forces firms to cooperate until the desired node.

CONCLUSION

The centipede game has extended and intensified the discussion of fundamental concepts of the evolving discipline of game theory. There is a need for further theoretical work on the concepts of rationality, common knowledge, backward induction, and beliefs in interactive decision-making. We have seen data from different experiment sets all with similar payout structures. All these experiments seem to lead to similar conclusions, if only by a different means. In a theoretical examination of the game, when the game is true to form, we see rational players exiting in the first round in order to achieve rational, utility-maximizing, payouts. The same is true for the results of experiment 1 and 2. The human experiments themselves however rarely demonstrate these types of theoretically accurate results. Experiments 3 and 4 showed us that by relaxing the rationality assumption, we could predict some-what similar outcomes as Mckelvey & Palfrey and Nagel & Tang did, in their experiments. I discussed a few explanations that may account for this divergence between theory and data. The first assumes that the subject pool contains a certain proportion of altruists who place a positive weight in their utility function regarding the payoff of their opponent. The second explanation considers the possibility of action errors. Errors in action may result from subjects experimenting with different strategies or simply from subjects pressing the wrong key. Last, I discussed applications of the centipede game.

APPENDIX

1.

		Player B (even numbers) 2	4	6	8	10	12	always "pass" 14
Player A	1	4 * 1	4 1	4 1	4 1	4 1	4 1	4 1
	3	2 5	8 2	8 2	8 2	8 2	8 2	8 2
(odd numbers)	5	2 5	3 11	16 4	16 4	16 4	16 4	16 4
	7	2 5	3 11	6 22	32 8	32 8	32 8	32 8
	9	2 5	3 11	6 22	11 45	64 16	64 16	64 16
	11	2 5	3 11	6 22	11 45	22 90	128 32	128 32
always "pass"	13	2 5	3 11	6 22	11 45	22 90	44 180	256 64

FIGURE 4. Reduced normal form payoff matrix of the centipede game (= Nash-equilibrium payoffs) (Nagel and Tang 1998)*

2.

a. EXPERIMENT 1 CODE:

```
% ONOSETALE OKHIRIA  V00698540

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 1: Identify the payoffs of each subgame
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

P1=[0 1000;
5 5;
10 10;
10 10];

P2=[0 50;
100 100;
0.05 0.05;
0.05 0.05];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 2: Break the sequential game into subgames.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%Given P1, P2 = {1,1}
SG11=[P1 (1,:)' P2 (1,:)'];
%Given P1, P2 = {1,2}
SG12=[P1 (2,:)' P2 (2,:)'];
%Given P1, P2 = {2,1}
SG13=[P1 (3,:)' P2 (3,:)'];
%Given P1, P2 = {2,2}
SG14=[P1 (4,:)' P2 (4,:)'];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 3: Define player one's subgame isolated payoff vectors (SGIPV's)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player one's IPV given P1, P2 = {1,1}
SGIPV11=SG11 (:,1);
%Player one's IPV given P1, P2 = {1,2}
SGIPV12=SG12 (:,1);
%Player one's IPV given P1, P2 = {2,1}
SGIPV13=SG13 (:,1);
%Player one's IPV given P1, P2 = {2,2}
SGIPV14=SG14 (:,1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 4: Define player 1's best action, given player 1 and 2's previous
choice of initial actions.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player two's best action given P1, P2 = {1,1}
[Sgbr11 sgbri11]=max(SGIPV11);
%Player two's best action given P1, P2 = {1,2}
[Sgbr12 sgbri12]=max(SGIPV12);
%Player two's best action given P1, P2 = {2,1}
[Sgbr13 sgbri13]=max(SGIPV13);
%Player two's best action given P1, P2 = {2,2}
[Sgbr14 sgbri14]=max(SGIPV14);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 5: State the outcomes for both players
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player one and two's outcomes given P1,P2 = {1,1}
Outcome11=[sgbr11 P2(1,sgbri11)];
%Player one and two's outcomes given P1,P2 = {1,2}
Outcome12=[sgbr12 P2(2,sgbri12)];
%Player one and two's outcomes given P1,P2 = {2,1}
Outcome13=[sgbr13 P2(3,sgbri13)];
%Player one and two's outcomes given P1,P2 = {2,2}
Outcome14=[sgbr14 P2(4,sgbri14)];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 6: Outcome matrix for the first subgame
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

OM1=[Outcome11;
Outcome12;
```

```
Outcome13;
Outcome14];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 7: Calculate the second set of subgames
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Given P1 plays action 1
SG21=OM1(1:2,:);
%Given P1 plays action 2
SG22=OM1(3:4,:);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 8: Define player two's subgame isolated payoff vectors (SGIPV's)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player two's IPV given P1 plays 1
SGIPV21=SG21(:,2);
%Player two's IPV given P1 plays 2
SGIPV22=SG22(:,2);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 9: Determine player 2's best action, given player 1's choice of initial
action.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player two's best action given P1 plays 1
[sgbr21 sgbri21]=max(SGIPV21);
%Player two's best action given P1 plays 2
[sgbr22 sgbri22]=max(SGIPV22);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 10: Each player's outcome must be determined for each subgame as was
done with
%the initial set of subgames.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player one and two's outcomes given P1 plays 1
Outcome21=[SG21(sgbri21,1) sgbr21];
%Player one and two's outcomes given P1 plays 2
Outcome22=[SG22(sgbri22,1) sgbr22];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 11: The second outcome matrix is now created.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

OM2=[Outcome21;
Outcome22];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 12: Player one's best initial action is determined by finding the
%maximum values in the first row of the outcome matrix.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
[br11 bri11]=max(OM2(:,1));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 12: Given P1's best initial action is bri11, can now determine P2's
action.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for k=1:2;
if (bri11==k)
bri21=eval(sprintf('sgbri2%d',k));
end;
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 13: Given P1's best initial action is bri11 and P2's best action is
bri21, can
%now determine P1's best response action.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if (bri11==1)
if (bri21==1)
bri12=sgbri11;
elseif (bri21==2)
bri12=sgbri12;
end;
elseif (bri11==2)
if (bri21==1)
bri12=sgbri13;
elseif (bri21==2)
bri12=sgbri14;
end;

end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 14: The proper SPNE is now determined.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

SPNE=[bri11 sgbri11 sgbri21];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 15: This is the final outcome of the game.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

SPNE
```

b. EXPERIMENT 2 CODE:

```
% ONOSETALE OKHIRIA  V00698540
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 1: Identify the payoffs of each subgame
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
P1=[0 500;
1000 1000;
5 5;
5 5;
10 10;
10 10;
10 10;
10 10];


P2=[0 10000;
50 50
100 100;
100 100;
0.05 0.05;
0.05 0.05;
0.05 0.05;
0.05 0.05];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 2: Divide the sequential game into subgames.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Given P1,P2,P1 = {1,1,1}
SGF111=[P1(1,:)' P2(1,:)'];
%Given P1,P2,p1 = {1,1,2}
SGF112=[P1(2,:)' P2(2,:)'];
%Given P1,P2,p1 = {1,2,1}
SGF121=[P1(3,:)' P2(3,:)'];
%Given P1,P2,p1 = {1,2,2}
SGF122=[P1(4,:)' P2(4,:)'];
%Given P1,P2,p1 = {2,1,1)
SGF211=[P1(5,:)' P2(5,:)'];
%Given P1,P2,P1 = {2,1,1}
SGF212=[P1(6,:)' P2(6,:)'];
%Given P1,P2,P1 = {2,2,1}
SGF221=[P1(7,:)' P2(7,:)'];
%Given P1,P2,P1 = {2,1,1}
SGF222=[P1(8,:)' P2(8,:)'];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 3:Define Player 2's subgame isolated payoff vectors (SGIPV's)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player two's IPV given P1,P2 = {(1,1),1}
SGIPV111=SGF111(:,2);
%Player two's IPV given P1,P2 = {(1,1),2}
SGIPV112=SGF112(:,2);
%Player two's IPV given P1,P2 = {1,2,1}
SGIPV121=SGF121(:,2);
%Player two's IPV given P1,P2 = {1,2,2}
SGIPV122=SGF122(:,2);
%Player two's IPV given P1,P2 = {2,1,1}
SGIPV211=SGF211(:,2);
%Player two's IPV given P1,P2 = {2,1,2}
SGIPV212=SGF212(:,2);
```

```
%Player two's IPV given P1,P2 = {2,2,1}
SGIPV221=SGF221(:,2);
%Player two's IPV given P1,P2 = {2,2,2}
SGIPV222=SGF222(:,2);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 4: Define Player 2's best action, given Player 1 and 2's previous
choice of initial actions.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player 2's best action given P1,P2 = {(1,1),1}
[sgbr111 sgbri111]=max(SGIPV111);
%Player 2's best action given P1,P2 = {(1,1),2}
[sgbr112 sgbri112]=max(SGIPV112);
%Player 2's best action given P1,P2 = {(1,2),1}
[sgbr121 sgbri121]=max(SGIPV121);
%Player 2's best action given P1,P2 = {(1,2),2}
[sgbr122 sgbri122]=max(SGIPV122);
%Player 2's best action given P1,P2 = {(2,1),1}
[sgbr211 sgbri211]=max(SGIPV211);
%Player 2's best action given P1,P2 = {(2,1},2}
[sgbr212 sgbri212]=max(SGIPV212);
%Player 2's best action given P1,P2 = {(2,1),1}
[sgbr221 sgbri221]=max(SGIPV221);
%Player 2's best action given P1,P2 = {(2,2),2}
[sgbr222 sgbri222]=max(SGIPV222);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 5: Calculate the outcomes for both players, given player 2's best
%action
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player one and two's outcomes given P1,P2 = {(1,1),1}
Outcome111=[P1(1,sgbri111) sgbr111];
%Player one and two's outcomes given P1,P2 = {(1,1),2}
Outcome112=[P1(2,sgbri112) sgbr112];
%Player one and two's outcomes given P1,P2 = {(1,2),1}
Outcome121=[P1(3,sgbri121) sgbr121];
%Player one and two's outcomes given P1,P2 = {(1,2),2}
Outcome122=[P1(4,sgbri122) sgbr122 ];
%Player one and two's outcomes given P1,P2 = {(2,1),1}
Outcome211=[P1(5,sgbri211) sgbr211];
%Player one and two's outcomes given P1,P2 = {(2,1),2}
Outcome212=[P1(6,sgbri212) sgbr212];
%Player one and two's outcomes given P1,P2 = {(2,2),1}
Outcome221=[P1(7,sgbri221) sgbr221];
%Player one and two's outcomes given P1,P2 = {(2,2),2}
Outcome222=[P1(8,sgbri222) sgbr222 ];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 6: The first outcome matrix is formed.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

OM1=[Outcome111;
Outcome112;
Outcome121;
```

```
Outcome122;
Outcome211;
Outcome212;
Outcome221;
Outcome222];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 7: Calculate the second set of subgames
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Given P1, P2 plays action (1,1)
SGS11=OM1(1:2,:);
%Given P1,P2 plays action (1,2)
SGS12=OM1(3:4,:);
%Given P1,P2 plays action (2,1)
SGS21=OM1(5:6,:);
%Given P1,P2 plays action (2,2)
SGS22=OM1(7:8,:);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 8: Define player 1's subgame isolated payoff vectors (SGIPV's)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player 1's IPV given P1,P2 = {1,1}
SGIPV11=SGS11(:,1);
%Player 1's IPV given P1,P2 = {1,2}
SGIPV12=SGS12(:,1);
%Player 1's IPV given P1,P2 = {2,1}
SGIPV21=SGS21(:,1);
%Player 1's IPV given P1,P2 = {2,2}
SGIPV22=SGS22(:,1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 9: Player one's best action must now be determined, given player 2's
%choice of play
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player one's best action given P1,P2 = {1,1,1}
[sgbr11 sgbri11]=max(SGIPV11);
%Player one's best action given P1,P2 = {1,2}
[sgbr12 sgbri12]=max(SGIPV12);
%Player one's best action given P1,P2 = {2,1}
[sgbr21 sgbri21]=max(SGIPV21);
%Player one's best action given P1,P2 = {2,2}
[sgbr22 sgbri22]=max(SGIPV22);

%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 10:  Calculate the outcomes for both players, given player 1's best
%action, given player's 2 choice of play
%%%%%%%%%%%%%%%%%%%%%%

%Player one and two's outcomes given P1,P2 plays (1,1)
Outcome11=[sgbr11 SGS11(sgbri11,2)];
%Player one and two's outcomes given P1,P2 plays (1,2)
Outcome12=[sgbr12 SGS12(sgbri12,2)];
```

```
%Player one and two's outcomes given P1,P2 plays (2,1)
Outcome21=[sgbr21 SGS21(sgbri21,2)];
%Player one and two's outcomes given P1,P2 plays (2,2)
Outcome22=[sgbr22 SGS22(sgbri22,2)];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 11: With these outcome vectors the second outcome matrix is formed.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

OM2=[Outcome11;
Outcome12;
Outcome21;
Outcome22];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 12: Calculate the third set of subgames
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Given P1 plays action 1
SGS1=OM2(1:2,:);
%Given P1 plays action 2
SGS2=OM2(3:4,:);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 13: Next, player two's subgame isolated payoff vectors (SGIPV's) are
%calculated.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player two's IPV given P1 plays 1
SGIPV1=SGS1(:,2);
%Player two's IPV given P1 plays 2
SGIPV2=SGS2(:,2);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 14: Player two's best action must now be determined, given player 1's
%choice of initial action.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player two's best action given P1 plays 1
[sgbr1 sgbri1]=max(SGIPV1);
%Player two's best action given P1 plays 2
[sgbr2 sgbri2]=max(SGIPV2);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 15: Each player's outcome must be determined for each subgame as was
done with the initial set of subgames.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player one and two's outcomes given P1 plays 1 and Player 2 plays best
%action
Outcome1=[SGS1(sgbri1,1) sgbr1];
%Player one and two's outcomes given P1 plays 2 and Player 2 plays best
%best action
Outcome2=[SGS2(sgbri2,1) sgbr2];
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 17: With these outcome vectors the third outcome matrix is formed.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
OM3=[Outcome1;
Outcome2];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 18: Player one's best initial action is determined by finding the
maximum values in the first row of the outcome matrix.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[br1 bri1]=max(OM3(:,1));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 21: The proper SPNE is now determined.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

SPNE=[bri1 sgbri11 sgbri1 sgbri111];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 22: The equilibrium strategy vector is now calculated. This is the
final outcome of the game.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
SPNE
```

c. EXPERIMENT 3 CODE:

For experiment 3, I used the same code as experiment 3, but changed step 9

```
% ONOSETALE OKHIRIA V00698540
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 9: Determine player 2's best action, given player 1's choice of initial
action.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player two's best action given P1 plays 1
[Sgbr21 sgbri21]=min(SGIPV21);
%Player two's best action given P1 plays 2
[sgbr22 sgbri22]=min(SGIPV22);
```

d. EXPERIMENT 4 CODE:

For experiment 4 I used the same code as experiment 2, but changed step 4 and 14

```
% ONOSETALE OKHIRIA V00698540
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%Step 4: Define Player 2's best action, given Player 1 and 2's previous
choice of initial actions.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player 2's best action given P1, P2 = {(1,1), 1}
[Sgbr111 sgbri111]=min(SGIPV111);
%Player 2's best action given P1,P2 = {(1,1),2}
[sgbr112 sgbri112]=min(SGIPV112);
%Player 2's best action given P1,P2 = {(1,2),1}
[sgbr121 sgbri121]=min(SGIPV121);
%Player 2's best action given P1,P2 = {(1,2),2}
[sgbr122 sgbri122]=min(SGIPV122);
%Player 2's best action given P1,P2 = {(2,1),1}
[sgbr211 sgbri211]=min(SGIPV211);
%Player 2's best action given P1,P2 = {(2,1},2}
[sgbr212 sgbri212]=min(SGIPV212);
%Player 2's best action given P1,P2 = {(2,1),1}
[sgbr221 sgbri221]=min(SGIPV221);
%Player 2's best action given P1,P2 = {(2,2),2}
[sgbr222 sgbri222]=min(SGIPV222);


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 14: Player two's best action must now be determined, given player 1's
%choice of initial action.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player two's best action given P1 plays 1
[sgbr1 sgbri1]=min(SGIPV1);
%Player two's best action given P1 plays 2
[sgbr2 sgbri2]=min(SGIPV2);
```

e. EXPERIMENT 5 CODE:

For experiment 5 I used the same code as experiment 2, but changed step 9 and 18

```
% ONOSETALE OKHIRIA  V00698540

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 9: Player one's irrational action must now be determined, given player
2's choice of play
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Player one's best action given P1,P2 = {1,1,1}
[sgbr11 sgbri11]=min(SGIPV11);
%Player one's best action given P1,P2 = {1,2}
[sgbr12 sgbri12]=min(SGIPV12);
%Player one's best action given P1,P2 = {2,1}
[sgbr21 sgbri21]=min(SGIPV21);
%Player one's best action given P1,P2 = {2,2}
[sgbr22 sgbri22]=min(SGIPV22);
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Step 18: Player one's best initial action is determined by finding the
%maximum values in the first row of the outcome matrix.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[br1 bri1]=min(OM3(:,1));
```

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