TIME VARYING NONLINEAR FEEDBACK RULES IN CONTROL THEORY

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ABSTRACT

This paper develops a feedback control framework on a macroeconomic model, incorporating non-linear "handcrafted" rules for monetary and fiscal policy to offer policy suggestions aimed at deficit reduction and macrostabilization. The issue of the federal debt has in recent years dominated political rhetoric and polarized the nation. This paper, then, explores the various policy tools that the government has to reduce the deficit while maintaining macroeconomic stability. This research thus provides an argument for a feedback control framework with "handcrafted" rules and suggests policy approaches to reducing the deficit.

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ACRONYMS

FED The Federal Reserve Board

FCF Feedback Control Framework

TVFCF Time Varying Feedback Control Framework

FOMC Federal Open Market Committee

1

INTRODUCTION

The boom, not the slump, is the time for austerity

*— **John Maynard Keynes***

The issue of the federal debt has in recent years dominated political rhetoric and polarized the nation. Conservatives demand cuts to government spending, which they argue would bring the deficit down and ease the national debt. The multiplier model, however, explicitly reveals that cutting government expenditures would hurt GDP, the main indicator of economic success in the United States. With an already struggling economy, budget cuts would worsen economic growth and unemployment, causing further problems and worsening the country's credit rating, the perceived likelihood to pay its national debt. Conservatives also paradoxically suggest cutting taxes in the hope that this will stimulate economic activity and increase disposable income. However, it is trivial to show that fewer taxes mean less government revenue, which would further worsen government deficit.

In this paper, then, I explore various policy tools that the government has to reduce the deficit while maintaining macroeconomic stability. Using the Hall and Taylor dynamic non-linear model, I develop a feedback control framework composed of various "handcrafted" rules to attain desired outcomes.

1.1 THE GREAT RECESSION

In September 15, 2008 one of the largest investment banks in the U.S., *Lehman Brothers Holdings Inc.*, declared for bankruptcy. Although the **Great Recession**, as the downturn has been coined, officially started in December of 2007, it was the fall of *Lehman Brothers* that precipitated a run on the financial system and highlighted the gravity of the coming recession. In the following days, the Federal Reserve along with the Treasury and the Securities Exchange Commission took steps towards alleviating the crisis.

The banking crisis, on its own, necessitated immediate intervention to prevent total collapse. The financial meltdown led the *American International Group (AIG)*, an insurer of mortgage securities, to a **liquidity crisis** as massive defaults occurred. To prevent a domino effect on *AIG's* clients, the **FED's Board of Governors** authorized a credit of 85 billion dollars, to be paid for by U.S. taxpayers. This allowed financial institutions to stay afloat, and by the summer of 2009, financial markets were "more or less back to normal" (Krugman, 2012). Nevertheless, the worse was still coming for the general public.

The **unemployment** rate in the United States would rise from a pre-crisis level of 5% and peak at 10% in October 2009, marking an increase in 8 million unemployed Americans. **Housing prices**, an important part of household wealth, fell approximately 30% while the net worth of U.S. households fell by 22%. Facing a though transition, the *Obama Administration* set out to design a stimulus bill in order to help the economy. As noted above, while the banking sector saw its bail out in a matter of days, the stimulus package to help the larger American public would not come until 2009.

Signed into law on February 17, 2009, the **American Recovery and Reinvestment Act** consisted of tax cuts, extensions on unemployment benefits, help to sustain Medicaid, aid to state and local governments, and infrastructure spending at a price tag of $787 billion dollars. This bailout, however, would prove insufficient, as unemployment would keep rising and private spending remain stagnant.

A usual arm against recessions, the **FED's** target interest rate is called to action to stimulate private investment. As the interest is lowered, private actors have less of an incentive to save, and may opt for consumption and/or investment. This increase in consumption and investment has the effect of working as a stimulus for the economy, providing liquidity and enabling *GDP* to recover. During the **Great Recession**, however, the United States found itself in a *liquidity trap* as the **FED** lowered the interest rates to zero. A **liquidity trap** occurs when changes in interest rates fail to stimulate economic growth as investors fear insufficient aggregate demand. With private investment remaining unresponsive to the zero bound, a different solution is needed to stabilize economic output.

At the same time, the discussion in Washington changed from a focus on unemployment to the debt and deficit. Congressional Republicans bemoaned the rising deficits and the effects that debt could have on growth. President Obama himself was caught up in this rhetoric, proposing spending cuts instead of further stimulus. **Austerity Economics** took hold of the public debate and was reinforced by dubious *academic* research (Rogoff, Reinhart, 2009) that spelled disaster for high debt economies. The result is a generalized neglect of fiscal policy as a stabilization mechanism for the economy, where instead austerity and deficit reduction are being used to attempt to solve the deficit problem.

Thus we find the United States economy in its current situation. The last couple of years have seen a modest recovery at best, with an unemployment rate of 7.5% and a slugging GDP growth of 2.5%. The political environment is dominated by *deficit hawks* who keep pushing for austerity measures and take hostage any policy aimed at stimulus spending. Other economists argue differently; namely, that we must focus first on unemployment and then on deficit reduction. In this spirit, it becomes necessary to develop a framework with which we can gain an understanding of how the economy works and what policies we can use to solve the unemployment and the deficit without causing further strains in the economy.

1.2 THE HALL/TAYLOR MODEL

Many of the undergraduate level Economics courses and indeed much the Economics field in general analyze the economy and policies in two time periods: the **short-run** and the **long-run**. In this framework, the *short-run* is understood as the immediate effect of a policy measure, i. e., the price level and expectations have not fully adjusted to the state of the economy. On the other hand, in the *long run* the economy has reached an equilibrium, and barring any other shocks, it stays that way. Although this framework is useful as an introductory idea with instructional value, we mustn't forget that an economy is inherently dynamic, and the section between the short-run and the long-run too important to neglect.

> *The long run is a misleading guide to current affairs. In the long run we are all dead. Economists set themselves too easy, too useless*

a task if in tempestuous seasons they can only tell us that when the storm is past the ocean is flat again.

–John Maynard Keynes

In this interest, we want to use a framework that allows us to see the effects, quarter by quarter, of policy experiments. With a dynamic model, the picture of causality and magnitude will become clearer and provide better insights.

The **Hall and Taylor** dynamic model is established in (Mercado et al., 1998). It consists of an IS-LM open economy sub-model for aggregate demand, an "expectations augmented" Phillips curve for aggregate supply, and definitions for government deficit and unemployment. The model is defined by the following equations:

GDP Identity	$Y_t = C_t + I_t + G_t + X_t$	(1a)
Disposable Income	$Y_t^d = (1 - \tau_t)Y_t$	(1b)
Consumption	$C_t = a + bY_t^d$	(1c)
Investment	$I_t = e - dR_t$	(1d)
Money Demand	$\frac{M_t}{P_t} = kY_t - hR_t$	(1e)
Expected Inflation	$\pi_t^e = \alpha\pi_{t-1} + \beta\pi_{t-2}$	(1f)
Inflation Rate	$\pi_t = \pi_t^e + f\left(\frac{Y_{t-1} - Y_N}{Y_N}\right)$	(1g)
Price Level	$P_t = P_{t-1}(1 + \pi_t)$	(1h)
Real Exchange Rate	$\frac{E_t P_t}{P_w} = q + vR_t$	(1i)
Net Exports	$X_t = g = mY_t - n\frac{E_t P_t}{P_w}$	(1j)
Government Deficit	$G_t^d = G_t - \tau_t Y_t$	(1k)
Unemployment Rate	$U_t = U_N - \mu\left(\frac{Y_t - Y_N}{Y_N}\right)$	(1l)

ENDOGENOUS VARIABLES		POLICY VARIABLES	
C	Consumption	**G**	Government Expenditures
E	Nominal Exchange Rate	**M**	Money Stock
$\mathbf{G^d}$	Government Deficit	τ	Average Tax Rate
I	Investment		
P	Domestic Price Level		
R	Real Interest Rate		EXOGENOUS VARIABLES
U	Unemployment Rate	$\mathbf{P_w}$	Foreign Price Level
X	Net Exports	$\mathbf{U_N}$	"Natural" Rate of Unemployment
Y	GDP	$\mathbf{Y_N}$	Potential GDP
$\mathbf{Y^d}$	Disposable Income		
π	Inflation Rate		
π^e	Expected Inflation		

Parameters

$a = 220$; $b = 0.7754$; $e = 1000$; $f = 0.8$; $h = 1000$; $k = 0.1583$; $m = 0.1$; $n = 100$; $q = 0.75$; $v = 5$; $\alpha = 0.4$; $\beta = 0.2$; $\mu = 0.33$.

1.3 MODIFICATIONS ON INVESTMENT

The starting model, referred to as ORIG in the graphs, is a basic non-linear dynamic representation of the economy for 20 quarters. An important point of departure from the Hall and Taylor model lies in Equation 1d, the Investment identity. The Hall and Taylor model provides a simple understanding of the dynamics of investment with the equation given by:

$$I_t = e - dR_t \tag{2}$$

This equation defines the investment identity as a function of the negative value of the interest rate, signifying the incentive of capital holders to increase investment when interest rates are low and to decrease investment when interest rates are high. This identity, however, overlooks the positive effect that economic growth has on investors' confidence. When an economy is booming, capital holders see the incentive to invest in order to generate a profit; on the other hand, when the economy is in a slump, capital holders are hesitant to risk their money. This simple idea can be captured by the addition of a growth variable multiplied by a coefficient to the investment identity as follows:

$$I_t = e - dR_t + \gamma(Y_t - Y_{t-1}) \tag{3}$$

The modification of the investment identity appears to increase the model's volatility, although as seen in Figure 1, the fluctuations are minor and the model still converges with $\gamma = 0.3$.

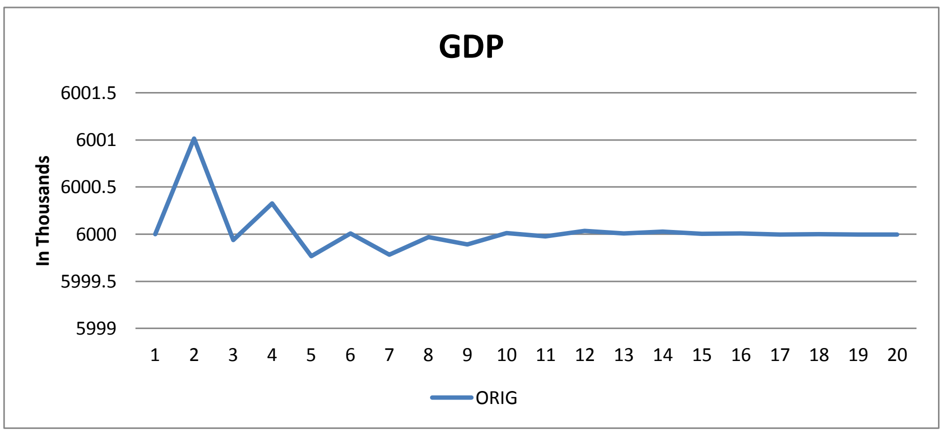

Figure 1: GDP Fluctuations (ORIG)

A major characteristic of this model is a consistent government deficit that reflects the problem the U.S. economy faces. As Figure 2 shows, the government of this model runs a deficit of about 75 units every quarter.

In Part i, we will develop a nonlinear feedback rule that forces the model to converge to zero deficit. Analyzing the results, we will find it useful to further use nonlinear feedback rules to attenuate some "negative externalities." In Part ii we will use the results of Part i and the

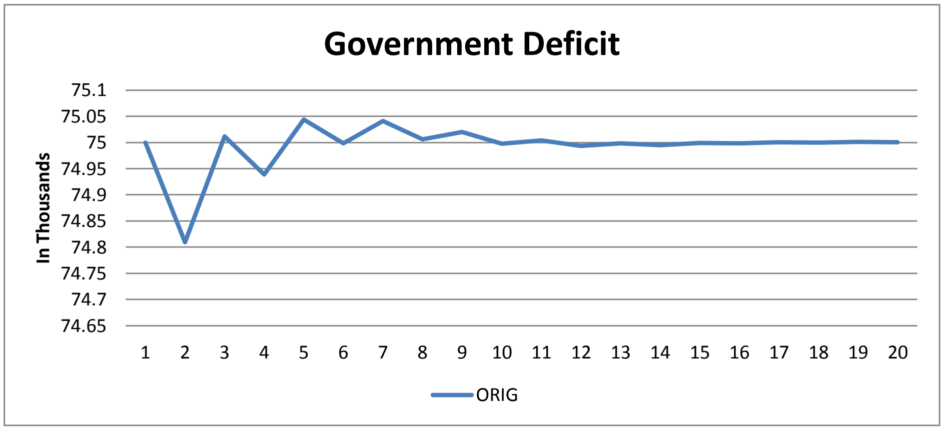

Figure 2: The Path of Deficit (ORIG)

intuition obtained from it to develop a general model of feedback control that is flexible to various priorities in the use of monetary and fiscal policy. Part iii provides an **Appendix** with the source code, including commentary, used for the various models that appear throughout this paper.

Part I

FEEDBACK CONTROL

2

NONLINEAR FEEDBACK MODEL

Before developing a feedback control model, we must analyize a particular kind of equations that will help us achieve our goals.

2.1 RECURRENCE EQUATIONS

A **recurrence relation** is an equation that recursively defines a sequence based on an initial value.

2.1.1 *Gemetric Sequences*

A simple kind of recurrence relation is a **geometric sequence** of the form

$$u_n = \alpha u_{n-1} \tag{4}$$

where α is a constant value defined as the **geometric ratio**. We can assume the solution is of the form

$$u_n = \beta^n \tilde{u} \text{ for some } \beta, \tilde{u} \tag{5}$$

From this, we also have the identity $u_{n-1} = \beta^{n-1}\tilde{u}$, which if substituted into Equation 4 along with Equation 5 gives us the following result

$$\beta^n \tilde{u} = \alpha \beta^{n-1} \tilde{u} \tag{6}$$

Cancelling like terms gives us the result that $\alpha = \beta$ such that the general solution is

$$u_n = \alpha^n \tilde{u} \tag{7}$$

To find the particular solution to our equation, we simply set $n = 0$ to find

$$u_0 = \alpha^0 \tilde{u} = 1\tilde{u} = \tilde{u} \tag{8}$$

Thus, the particular solution to the recurrence equation is

$$u_n = \alpha^n u_0 \tag{9}$$

2.1.2 *Geometric Sequences with Varying Coefficients*

If we allow for the geometric sequence to vary with time, a different solution arises. The recurrence equation is given by

$$u_n = \alpha_n u_{n-1} \tag{10}$$

From this, we know that $u_1 = \alpha_1 u_0$ and using an inductive argument, we can show that

$$u_n = u_0 \prod_{i=1}^{n} \alpha_i \tag{11}$$

Proof If we assume that the solution is correct for n, we must show that it is also correct for $n+1$. The equation is

$$u_{n+1} = u_0 \prod_{i=1}^{n+1} = u_0 \alpha_{n+1} \prod_{i=1}^{n} \alpha_i = \alpha_{n+1} \left(u_0 \prod_{i=1}^{n} \alpha_i \right) = \alpha_{n+1}(u_n)$$

which is the recurrence definition of u_{n+1}.

2.1.3 *Convergence of Geometric Sequences*

It is easy to show that geometric sequences converge when the *geometric ratio* is $0 \leq |\alpha| < 1$ and $\alpha = 1$. When $\alpha = 1$, we can show that $\lim_{n\to\infty} u_n = u_0 \sum_{n=1}^{\infty} \alpha^n = u_0$. Similarly, when $\alpha = 0$, $\lim_{n\to\infty} u_n = u_0 \sum_{n=1}^{\infty} \alpha^n = 0$. Now, it is left to show that this equation converges when $|\alpha|$ is between zero and one. The important part is $\sum_{n=1}^{\infty} \alpha^n$, as it scales the initial value u_0. This equation expands to

$$\sum_{n=1}^{\infty} \alpha^n = \alpha^1 + \alpha^2 + \alpha^3 \ldots = s$$

where s is the sum of these coefficients. If we factor out an α after the first term, we obtain

$$s = \alpha + \alpha(\alpha^1 + \alpha^2 + \ldots) = \alpha + \alpha s \tag{12}$$

A little algebraic manipulation gives us the result $s - \alpha s = s(1-\alpha) = \alpha$. Thus, we conclude that

$$s = \sum_{n=1}^{\infty} \alpha^n = \frac{1}{1-\alpha} \tag{13}$$

Now, when the *geometric ratio* is time variant, the analysis is not as straightforward, and we must make a couple of assumption. First, it is quite obvious that $\prod_{n=1}^{\infty} \alpha_n$ diverges if all of the α_i's are greater than one, and remains stationary if they are exactly equal to one. If we want to specify a geometric sequence that converges to a steady state, then, we must construct a geometric ratio that goes to one as n increases. Let us assume that there exists a finite number m of α_i's that are not equal to one. If we multiply these first, we can partition our product into $\prod_{i=1}^{m} \alpha_i \prod_{j=m+1}^{\infty} \alpha_j$. Because the α_j's are all equal to one and α_i's form a finite product, we can deduce that the product converges.

With these results, we can now use the intuition from geometric sequences to construct nonlinear feedback models. The idea will be to construct a sequence of coefficients that converge to one.

2.2 THE TAX MODEL

The approach to trying to balance the budget consists of increasing the tax rate in order to increase revenue. For this purpose, we construct the tax rate as an endogenous variable and set up the following feedback rule:

$$\tau_t = \tau_{t-1} \left(\frac{G_t}{\tau_t Y_t} \right) \tag{14}$$

This rule ensures that, as long as the ratio of government expenditures to government revenue is positive, the tax rate will increase by a factor of that ratio. In the long run, as deficits decrease and the budget becomes more balanced, this ratio becomes smaller and converges to a value of 1. This rule helps ease the policy into the economy to prevent undesired shocks or too strong policy measures that could overshot and destabilize the model. An important note to make is the steady state of this rule, when $\tau_t = \tau_{t-1} = \tau$. Simplifying, we obtain $\tau = G_t/Y_t$, an important conclusion that suggests that the tax rate should be the ratio of government expenditures to income in order to have a balanced budget.

The model provides positive results for the reduction and eventual elimination of a government deficit, doing so by quarter 7 as shown in Figure 3. The steady state tax rate needed for the reduction of government deficit converges to about $\tau = .2$, or 20%

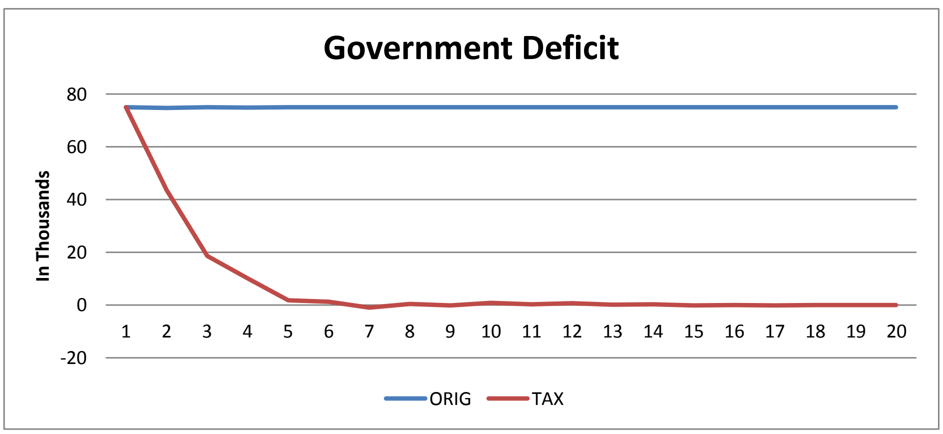

Figure 3: The Path of Deficit (TAX)

It is important, however, to see what other effects that were unaccounted for by the rule give us. GDP, the most important indicator, takes a hit during the first few quarters of tax increase, and takes some time to readjust.

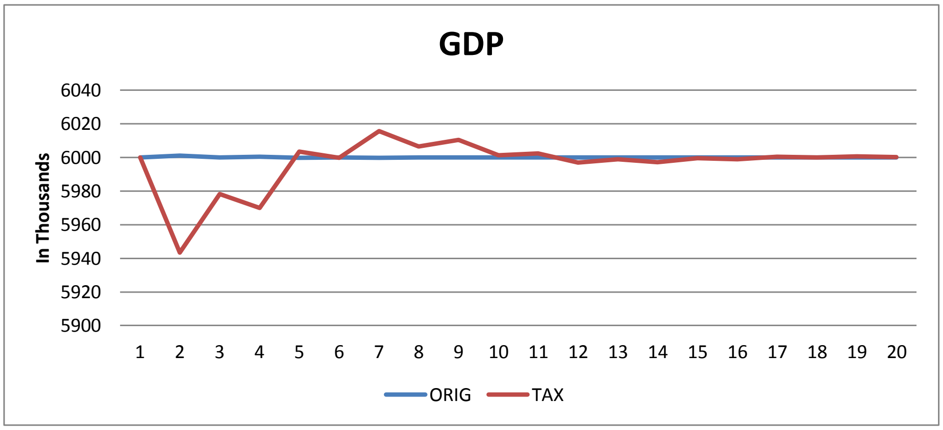

Figure 4: GDP Fluctuations (TAX)

The increase in taxes in the first few quarters created a contraction of disposable income, thereby causing consumption and subsequently, GDP to fall. A decrease in GDP, however, led to deflation and lower price levels, which lead to increased real money stock and a decrease in interest rates. This decrease in interest rates is the stabilizing factor for the economy, as it led to increases in investment and therefore increases in GDP. The dynamics of inflation and interest rate form the basis for the stabilization of the Hall and Taylor model, and appear quite strongly in these results.

Two other important indicators of economic well being are the rate of unemployment and price stability. Given the current economic conditions that the U.S. faces, I want to ensure that the policy suggestions presented in this paper do not have negative effects on an already high unemployment rate. Price stability, on the other hand, ensures more control over interest rates by stabilizing money demand.

As we can see from Figure 5, unemployment rises during the first quarters of tax increase. This is because as consumption goes down, GDP also goes down, signaling a decrease in production and a rise in unemployment. However, the same mechanisms that stabilize GDP help unemployment come back to the natural rate.

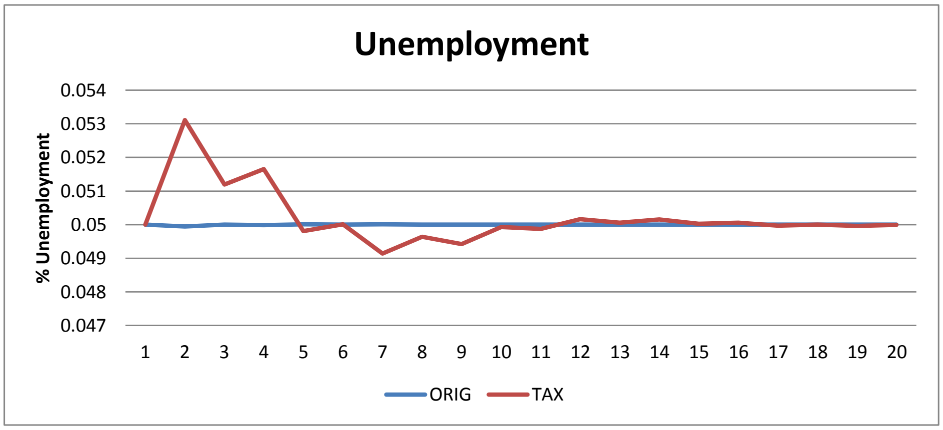

Figure 5: Fluctuations in Unemployment (TAX)

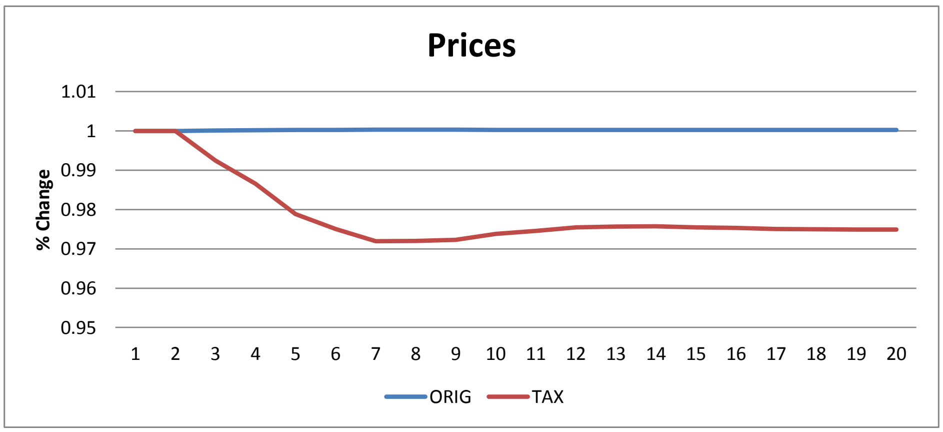

Figure 6: Price Level (TAX)

The contraction in GDP drives prices down, making interest rates go down as well. Stabilization of GDP is ultimately driven by a permanent increase in investment. Although GDP is stabilized, it is still important to keep prices stable so as to encourage consumption. Being such important indicators of macroeconomic as well as social stability, the next stage of my project is to reduce or remove these negative externalities.

2.3 THE FEEDBACK CONTROL MODELS

An important result of monetary policy research in the 1990's was the Taylor rule, developed by John B. Taylor. Since then, other economists have researched into a Taylor rule for fiscal policy, (Amman and Kendrick, 2011) and (Kliem and Kriwoluzky, 2010) to name a few. The Taylor rule for monetary policy as well as the feedback rules for fiscal policy suggested in the aforementioned papers, however, are derived from the quadratic linear optimization problem and are thus linear. For this paper, in contrast, I will develop "handcrafted" non-linear fiscal and monetary policy rules using a feedback control framework (FCF) to attenuate the negative externalities caused by the TAX model.

Fiscal policy consists of targeted government spending or austerity aimed at correcting undesired results in economic indicators. For the purposes of this model, I have used the unemployment rate as the control variable for fiscal policy. The rule is as follows:

$$G_t = G_{t-1}\left(\frac{U_t}{U_N}\right) \tag{15}$$

In words, this rule specifies an expansionary fiscal policy when unemployment is higher than the natural rate of unemployment and a contractionary policy when unemployment is lower. At the steady state and when the desired rate of unemployment is achieved, the rule converges to a constant level of government expenditures. This rule is based on the dynamics that by increasing government expenditures, GDP also increases. As GDP increases, it gets closer to the level of potential GDP, thereby reducing unemployment.

Monetary policy is regulated by the Federal Reserve System and it consists of controlling the supply of money to promote economic stability and growth. In the model, I use price stability as a control variable for monetary policy. The rule is:

$$M_t = M_{t-1}\left(\frac{P_{t-1}}{P_t}\right) \tag{16}$$

This rule specifies an expansionary policy when prices are falling in order to ensure a higher money demand, lower interest rates, and increased investment. This causes GDP to rise, promoting inflation and a rise in prices. When prices stabilize, the growth coefficient converges to 1 and the money supply becomes a constant.

A second feedback control model relies on a stronger Federal Reserve System assumption, that the FED can target its policy towards unemployment as well as price stability. The idea is that, keeping prices constant, when unemployment is high the money supply is incremented, causing a higher money demand, lower interests rates, and increased investment, which pulls up GDP and provides more jobs in the economy. This monetary policy rule is the following:

$$M_t = M_{t-1}\left(\frac{P_{t-1}}{P_t}\right)\left(\frac{U_t}{U_N}\right) \tag{17}$$

This rule reaches a steady state only when both prices and unemployment have reached desired goals.

The Models, and their subsequent equations, are summarized below.

FEEDBACK RULES	TAX	FCF	FCF2
Tax Rule	Equation 14	Equation 14	Equation 14
Fiscal Rule	None	Equation 15	Equation 15
Monetary Rule	None	Equation 16	Equation 17

Table 1: The Feedback Control Framework Models

3

COMPARATIVE ANALYSIS

Before analysing the results, it will serve our purpose to summarize the composition of each model.

CONTROLS (STATES)	TAX	FCF	FCF2
Tax Rate (Deficit)	X	X	X
Government Expenditures (Unemployment)	No	X	X
Money Supply (Inflation)	No	X	X
Money Supply (Unemployment)	No	No	X

Table 2: The Feedback Control Framework Models: Properties

To compare these models, I will first start with the original goal, which was getting rid of government deficits.

As Figure 7 shows, all of the models are able to effectively get rid of government deficits by at most quarter 7. The TAX model seems to have converged more quickly, but it should be remembered that this model does not control for unemployment or inflation. The FCF model appears to have converged the slowest, while the FCF2 model lies in the middle

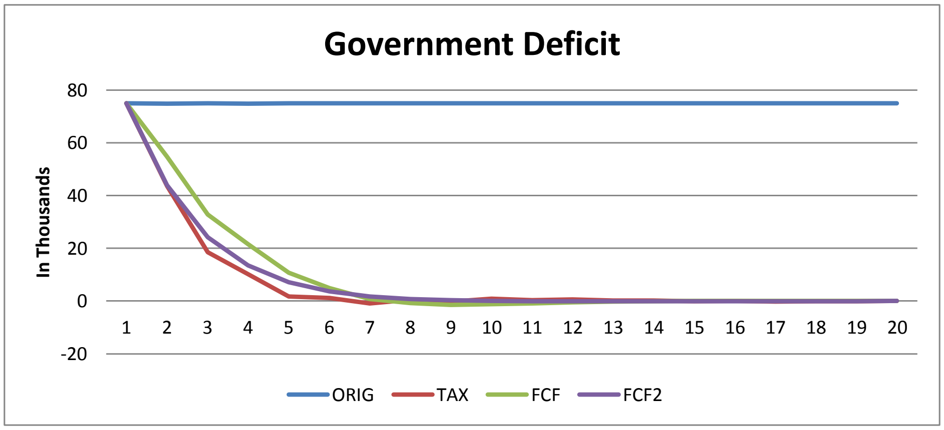

Figure 7: The Path of Deficit (FCF)

of these cases. Having reduced government deficits to zero, I will now analyze the effects on GDP and other economic indicators.

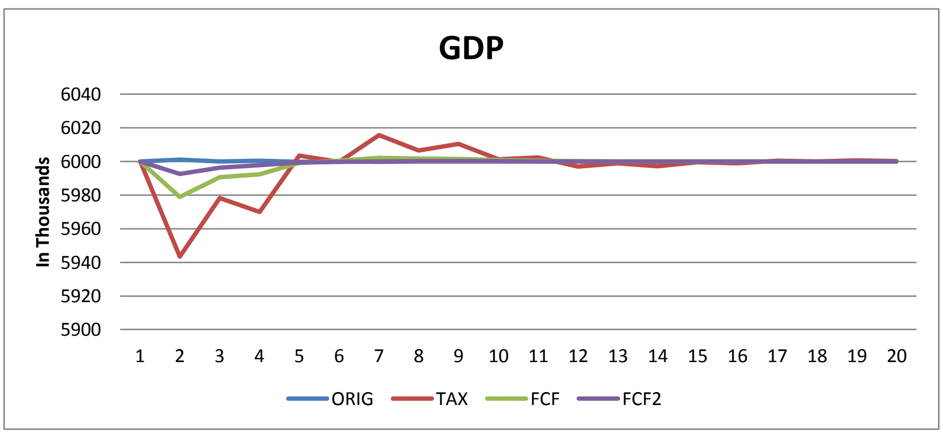

Figure 8: GDP Fluctuations (FCF)

Attaining a balanced budget seems to have come at a cost for all the models. The TAX model, in particular, appears to have the most fluctuations and takes longer to adjust. This is because, as the tax rate increased, disposable income decreased and thus consumption also decreased (Figure 8). As consumption decreased, GDP decreased as well, having other economic effects that will be analyzed later. The first correcting model, FCF, appears to have fewer fluctuations than TAX and adjusts more rapidly to potential. However, the second correcting model, FCF2, behaves even better than the first one and converges to potential faster.

GDP stability, nevertheless, should not be considered as our only factor in judging the effectiveness of these two models, as they were constructed with specific goals in mind. Therefore I will now analyze the extent to which these goals were met in the different models.

3.1 GOALS

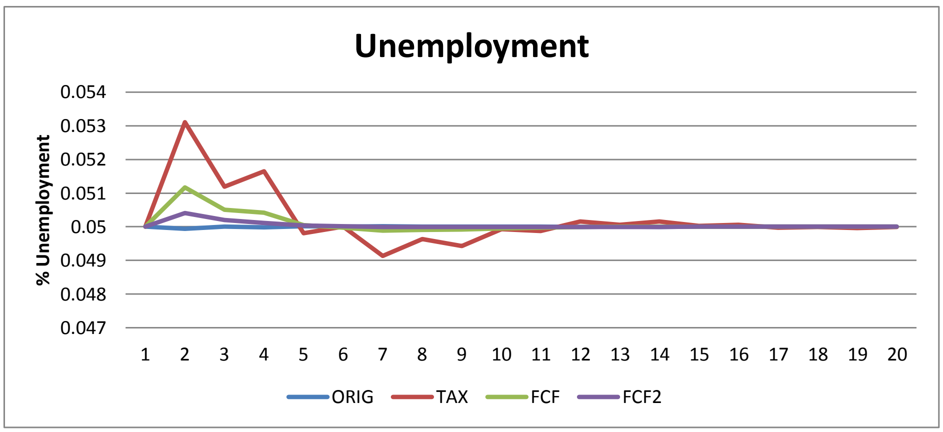

Figure 9: Fluctuations in Unemployment (FCF)

My first concern after the TAX model was the increase in unemployment and its subsequent fluctuations. Employment stability is an extremely important factor in social and political stability, and thus governments are always cautious to make sure unemployment stays low, as seen in Figure 9. The FCF model has a small rise in unemployment during quarters 2 and 3 but quickly returns to the natural rate of unemployment via increased government expenditures (Figure 9). The FCF2 model, however, experiences an even smaller change in unemployment even after increased taxes, signaling that increases in the money supply have a positive effect in increasing investment and maintaining GDP stable (see Figure 8).

In Figure 10, we see the effectiveness of monetary policy in stabilizing prices. Again, a similar pattern is observed: FCF was able to reduce the fluctuations on the price level, but FCF2 was able to do so much more effectively. This can be understood by the extra factor in the monetary policy rule of FCF2, which made the growth coefficient bigger and thus had more impact in price stability.

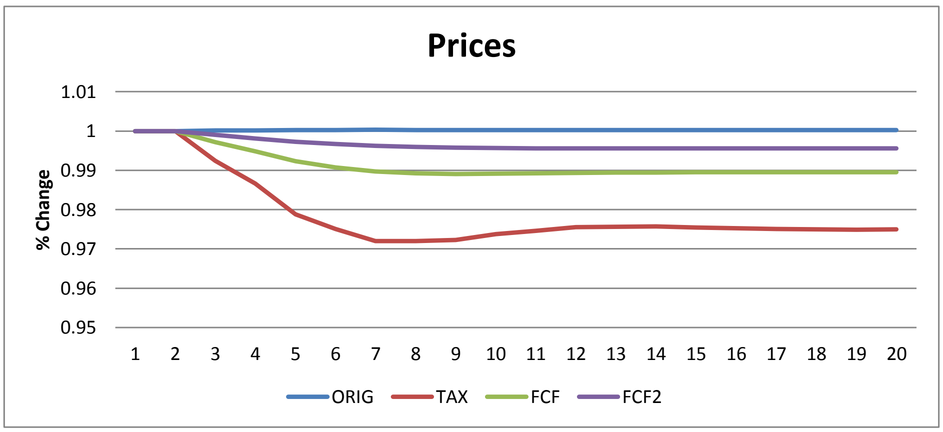

Figure 10: Price Level (FCF)

So far, the FCF2 model appears to be better at attaining the initial goals than the FCF model. However, there must be some tradeoffs between the two models, and now those will be investigated. Now that we have seen how the state variables behaved, we must analyze the policies enacted by both models and judge whether they are realistic.

3.2 policies

Looking at Figure 11, the tax rate in the TAX model seems to converge at 0.20, a 6% increase from the original model. Because of increases in government expenditures, the FCF model necessitates a higher tax rate to

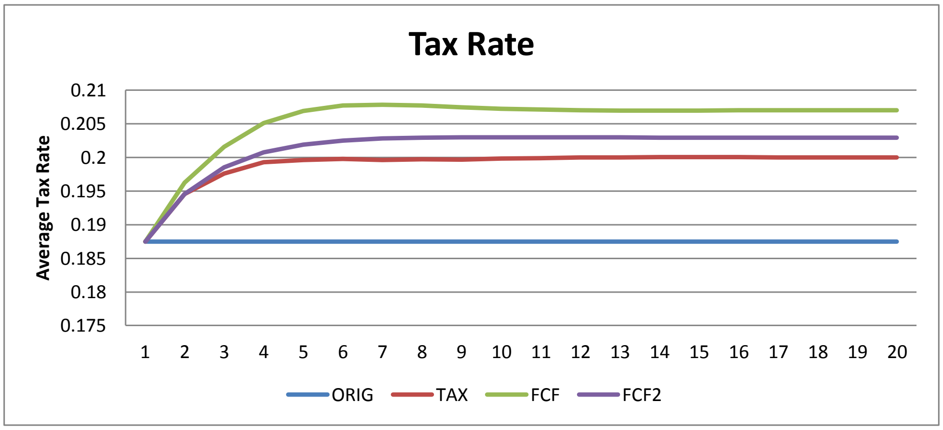

Figure 11: Increase in the Average Tax Rate (FCF)

attain a balanced budget. Its tax rate stabilizes at 0.207, a 10.4% increase. The FCF2 model, however, stabilized at a tax rate of 0.203, only an 8.26% increase. For now, it is a little unclear why this is the case, but it will become apparent after the next graph.

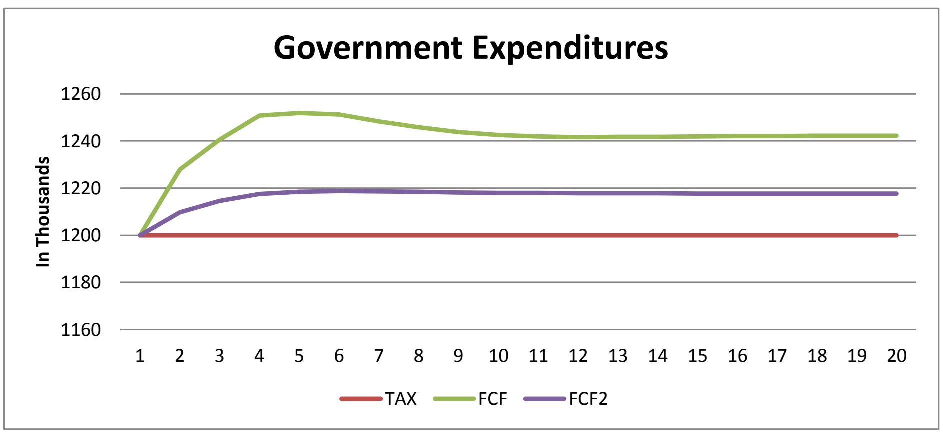

Figure 12: Increase in Government Expenditures (FCF)

Because unemployment stabilizes much quicker in FCF2, there is less reliance on government expenditures to enhance employment. As Figure 12 demonstrates, FCF2 necessitates only a 1.5% increase in government expenditures to sustain employment, while FCF needs a 3.5% increase. Because of this, FCF2 puts less of a burden on tax payers to pay off government deficits and thus requires a smaller tax rate.

So far in our analysis of both models, FCF2 seems to do more with less; more employment and price stability with less taxes and government expenditures. Figure 13 shows the trade-off that marks the major distinction between the two models.

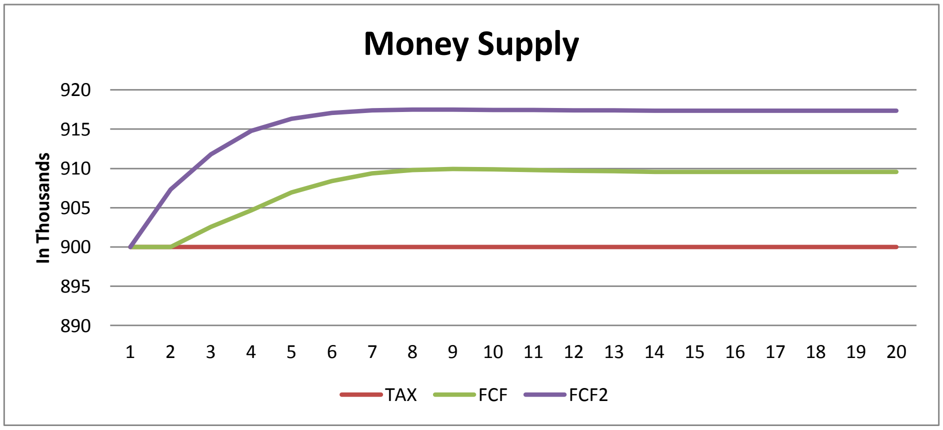

Figure 13: Increse in Money Supply (FCF)

The FCF model stabilizes prices by increasing the money supply by 1.11%. In this case, however, the FCF2 model actually has a more drastic policy than its counterpart; it increases the money supply by 1.93%. In foregoing higher government expenditures to increase employment, the FCF2 model relies more on the money supply to stabilize the state variables. It should be noted, however, that while the change from FCF to FCF2 decreased government expenditures by almost 1.7%, it increased money supply by only 0.8%. Although I cannot make any conclusions with certainty, this fact does however raise the question of whether fiscal policy is more effective than monetary policy.

3.3 investment

Another important factor worth analyzing in these two models is the role of investment. As we have already seen, government expenditures vary with each model, and thus investment plays a big factor in stabilization.

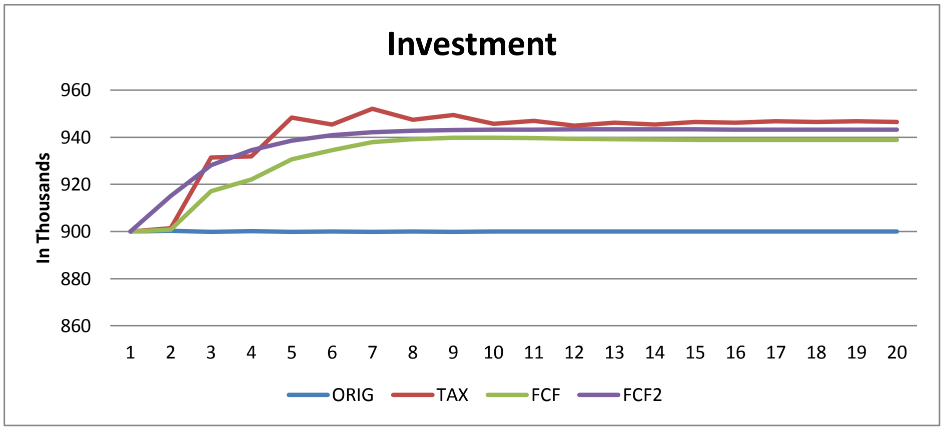

Figure 14: Increase in Investment (FCF)

In the TAX model, the tax increase created a fall in consumption, and given constant government expenditures, stabilization was driven by interest rates going down and investment increasing (see Figure 14 and Figure 15). The decrease in interest rates was in part driven by a contraction of GDP, which caused prices to go down and money demand to go up. In the FCF model, increases in government expenditures play a bigger role in stabilization, discouraging lower interest rates and increased investing; a crowding out effect. As discussed earlier, the FCF2 model did not rely as much on government expenditures, and thus the crowding out effect becomes less apparent. Thus, while the FCF model relies more heavily on government expenditures to stabilize, the FCF2 model has a more balanced approach and indeed promotes higher levels of investment.

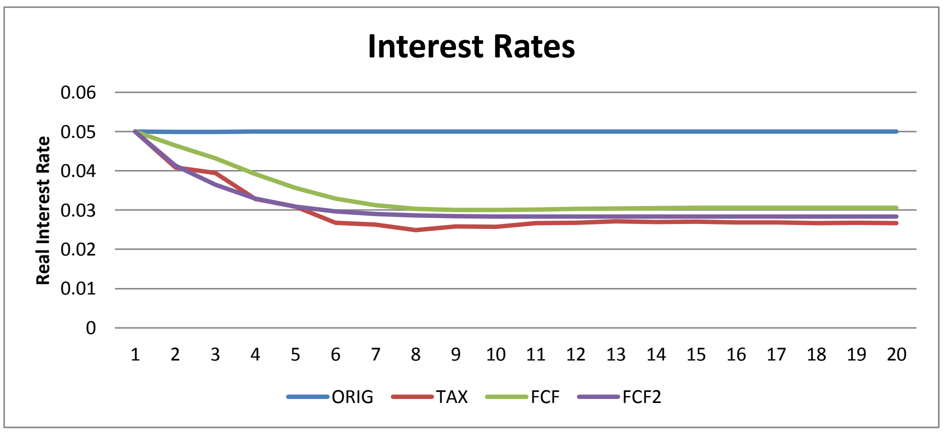

Figure 15: Fluctuations in Real Interest Rate (FCF)

Part II

TIME VARYING RULES

4

GENERAL NONLINEAR FEEDBACK MODEL

In the previous chapters I used nonlinear feedback rules to obtain desired paths. These rules were considered desired insofar as no interference was made to set priorities and weights on different policy tools. In order to expand the framework of nonlinear feedback rules, however, I will incorporate time varying weights to generalize policy options.

4.1 A GENERAL APPROACH

To recapitulate, the general nonlinear feedback rules used in Chapter 1 yield an equation for the evolution of the control variable which is of the form

$$u_t = \alpha_t u_{t-1} \tag{18}$$

where u_t is a control variable and α_t is a nonlinear function consisting of state variables that by construction converge to one.

Having shown that these time varying geometric sequences converge only when the product of α_t's converges, I now move onto developing a framework for time varying and weighted nonlinear feedback rules. The idea is to add a weight to the prescribed growth coefficient by extracting its growth rate. The following substitution is made without loss of generality

$$\alpha_t = 1 + \beta_t \tag{19}$$

where β_t is defined as the growth rate. Because we want to assign a weight to this growth rate, we want a geometric ratio of the form

$$\alpha'_t = 1 + \omega_t \beta_t \tag{20}$$

Substituting $\beta_t = \alpha_t - 1$ into the previous equation, we obtain

$$\alpha'_t = 1 + \omega_t(\alpha_t - 1) \tag{21}$$

Using this geometric ratio, we arrive at the following recurrence equation for the control variable

$$x_t = \alpha'_t x_{t-1} = (1 + \omega_t(\alpha_t - 1))x_{t-1} \tag{22}$$

4.2 TIME VARYING NONLINEAR FEEDBACK RULES

We can now take the rules presented in Chapter 2 and apply our general approach.

The two fiscal policy rules are

$$\tau_t = \left(1 + \omega_t^T \left(\frac{G_t}{\tau_t Y_t} - 1\right)\right) \tau_{t-1} \tag{23}$$

$$G_t = \left(1 + \omega_t^G \left(\frac{U_t}{U_N} - 1\right)\right) G_{t-1} \tag{24}$$

where ω_t^T is the weight assigned to the tax rule and ω_t^G is the weight assigned to the government expenditures rule at time t.

The monetary policy rule is

$$M_t = \left(1 + \omega_t^M \left(\frac{P_{t-1}}{P_t} - 1\right)\right) \left(1 + \omega_t^U \left(\frac{U_t}{U_N} - 1\right)\right) M_{t-1} \tag{25}$$

where ω_t^M is the weight assigned to the inflation correction part of the rule and ω_t^U is the weight assigned to unemployment correction at time t.

5

EXPERIMENTS ON DEFICIT REDUCTION AND MACROSTABILIZATION

We can now use the framework established in Chapter 4 to develop experiments with different weights on policy tools.

An important argument made by prominent liberal economists, such as Paul Krugman, has been that to solve the long term debt problem, we must first focus on raising employment and recovering the economy. In this spirit, the following experiment will be based on the idea of focusing on unemployment in the short run but on the deficit in the long run.

5.1 A SIMULATED RECESSION

To obtain some insights from this framework, we must first simulate a recession in our dynamic system.

5.1.1 *Autonomous Economy*

The recession of 2008 generated deep changes in the economy. Private investment collapsed, and seemed largely unresponsive to stimulation from the FED targeting lower interest rates.

To simulate this effect within the Hall/Taylor Model, I add an additive noise ν_t to the Investment equation and reparametrize the interest elasticity of investment d as d' to obtain the equation

$$I_t = e - d'R_t + \nu_t \tag{26}$$

We must also redefine initial GDP to reflect a large unemployment rate that will take some time to recover. The change in initial GDP is of a similar magnitude to ν_0. As explained in Part i, a basic assumption of this model is that the economy is self correcting, and therefore the **Autonomous** dynamics of this model will tend towards equilibrium.

5.1.2 *The "Krugman" Approach*

Nobel prize winning economist Paul Krugman has been a fierce opponent of austerity measures. According to him, the United States must first focus on getting the economy back on track and lowering unemployment. He proposes a 2 stage solution to the current crisis:

1. focusing on unemployment and recovery in the short run
2. dealing with the debt in the long run

Using the framework established in the previous chapter, the following table summarizes the **"Krugman"** approach.

The first row defines quarters $1-8$ where the tax rate remains constant but the other rules are in full effect. This ensures a dynamics of macrostabilization and fast recovery. The second row defines quarters $9-20$ where the dynamics of macrostabilization are ignored and the tax rate increases until the deficit is nullified.

TIME	ω_t^τ	ω_t^G	ω_t^M	ω_t^U
1-8	0	1	1	1
9-20	1	0	0	0

Table 3: The "Krugman" Approach

5.1.3 *The Feedback Control Approach*

Although the "Krugman" approach provides a sensible mechanism for economic recovery and deficit reduction, it doesn't take full advantage of the tools afforded by the general framework established in Chapter 4. By the results of Part i, we can assume that only focusing on the deficit during the second stage will inject instability to the model. We can thus compare the "Krugman" approach to one that makes a moderate use of **Feedback Control** rules in the second stage.

TIME	ω_t^τ	ω_t^G	ω_t^M	ω_t^U
1-8	0	1	1	1
9-20	1	0.5	0.7	0.4

Table 4: The Feedback Control Approach

The first row remains unchanged. However, the second row contains constants instead of zeros. The choice of these is largely arbitrary with illustrative purposes. The prescribed growth rate of government expenditures is scaled by 0.5 while the one for money supply to keep up with inflation is scaled by 0.7. Since we're already partially controlling for unemployment with G_t, the weight on the growth rate prescribed by U_t is 0.4.

5.2 RESULTS

First, we analyze the result that motivates this project, deficit reduction. As Figure 16 shows, the **Autonomous** model has a decreasing deficit via a self correcting economy. However, when the economy reaches an equilibrium at around quarter 9, so does deficit at a positive value. The feedback control models, on the other hand, appear to increase the deficit during those same quarters but successfully reach an equilibrium deficit of zero by quarters $13 - 15$. It is important to note that the **Feedback Control** model has a lingering deficit slightly above the **"Krugman"** model. These observations will be explored later.

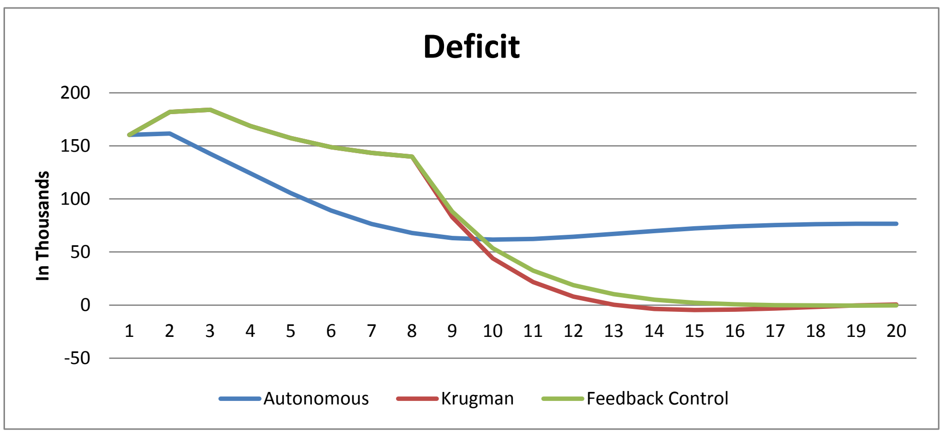

Figure 16: The Path of Deficit (TVFCF)

Seeing that we have achieved our initial goal of deficit reduction, we must now check the stability of the economy. Figure 17 shows the **Autonomous** model struggling to get back to equilibrium, doing so after quarter 7. The correcting models, having the same weights in the short run, recover much more quickly, after quarter 3. The major distinction comes after quarter 8. While the **"Krugman"** model suffers a relatively larger dip in GDP, the **Feedback Control** model is minimally affected and gets back to equilibrium more quickly. This is because the second model takes advantage of feedback rules in the long run.

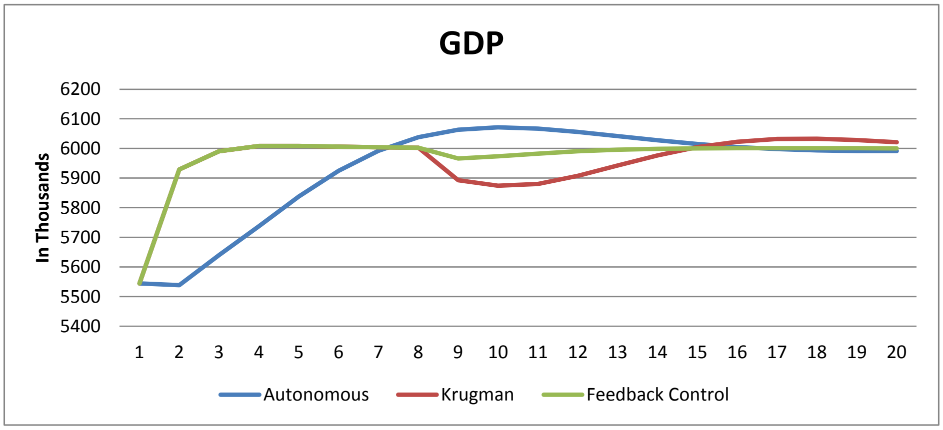

Figure 17: Fluctuations in GDP (TVFCF)

We now compare the path of the relevant control variables, namely unemployment and price level. Figure 18 shows the unemployment rate for the models, which appears as the horizontal mirror image of GDP. Again, the **"Krugman"** model suffers an increase in unemployment in the quarters after 8 due to the nonlinear rules being turned *off*.

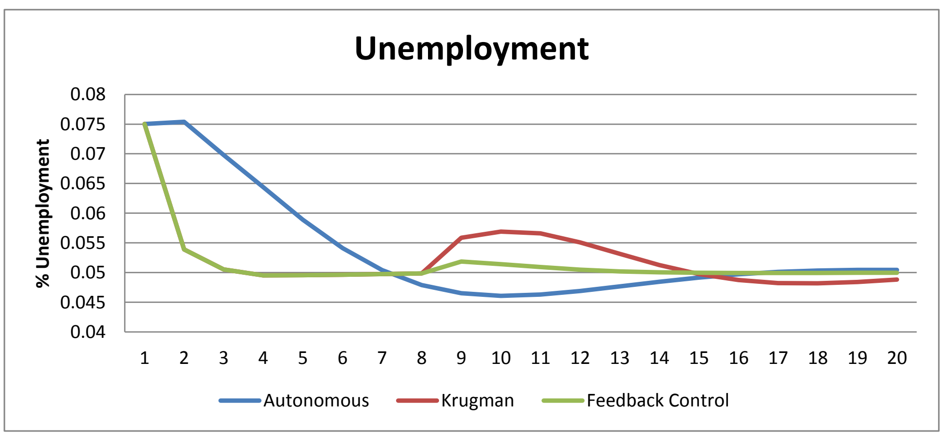

Figure 18: Fluctuations in Unemployment (TVFCF)

Similarly, price level does relatively better under the **Feedback Control** model, as shown by Figure 19.

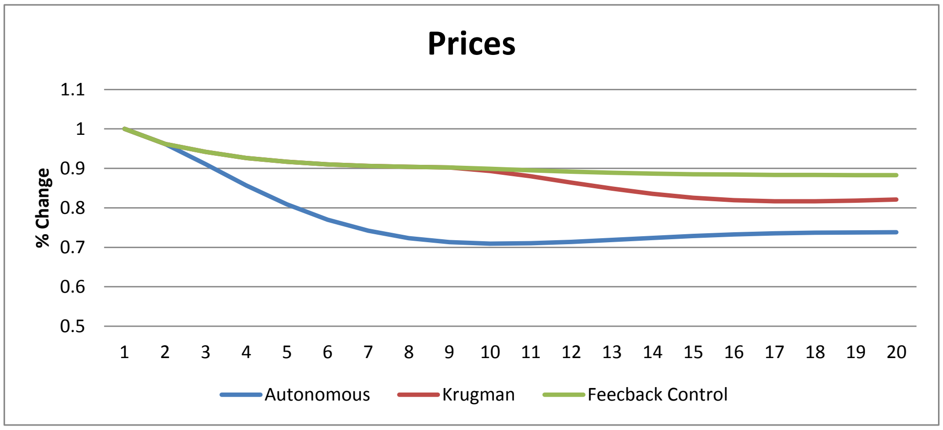

Figure 19: Price Level (TVFCF)

Figure 20 helps partially explain the better performance of unemployment stabilization in the **Feedback Control** Model. The weight ω_t^G is the same during the first stage in the two correcting models, and we can see that the prescribed policy is the same. However, in the **"Krugman"** model the weight is *turned off*, while the **Feedback Control** model prescribes a moderate priority on unemployment control. In this sense, the two models diverge to different equilibriums. To attenuate unemployment fluctuations, the **Feedback Control** model makes use of a 10% increase in government expenditures from the autonomous baseline of 1200 thousands of dollars, while the **"Krugman"** model uses a 5.4% increase. The question then becomes whether this increase is worth the welfare added by unemployment control.

Differences in government spending under these models also affects the average tax rate prescribed by the nonlinear rules (see Figure 20). With a stable level of government expenditures in the second stage, the **"Krugman"** model necessitates an increase in taxes from 18.75% to 21% to nullify the deficit. As the **Feedback Control** model prescribes government expenditure increases to control for unemployment, so is the aver-

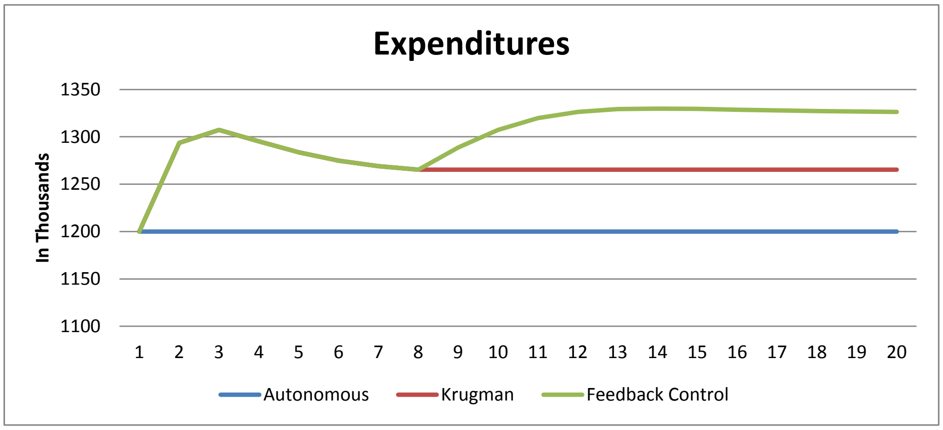

Figure 20: Increase in Government Expenditures (TVFCF)

age tax rate to decrease the deficit affected. In this model, the average tax rate increases to 22.1%, a 1.1% higher than **"Krugman"** model.

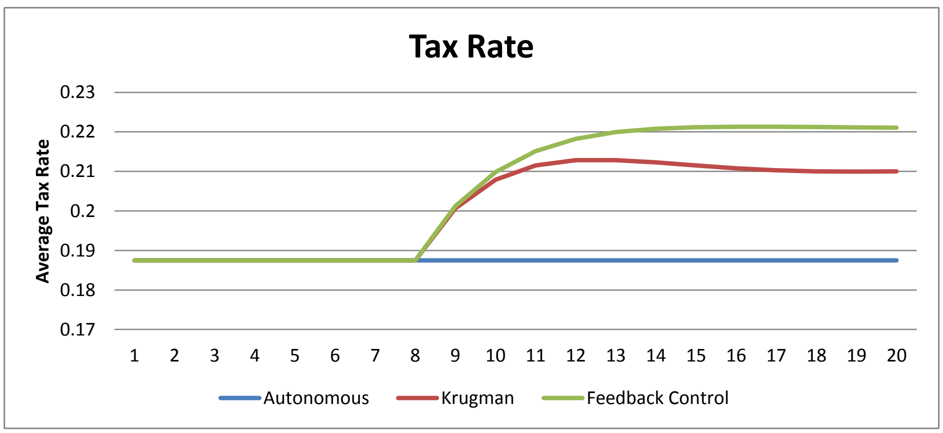

Figure 21: Increase in Average Tax Rate (TVFCF)

One last point to make is in the money supply (Figure 22), where similar story occurs. The marginal use of a monetary policy rule aimed at unemployment helps ease the need of higher increases in government expenditures, and the weight on inflation helps keep prices more stable (see Figure 19).

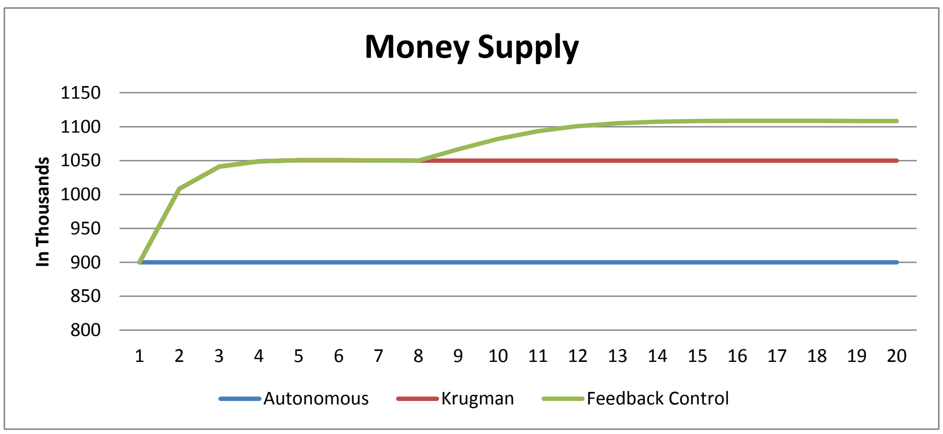

Figure 22: Increase in Money Supply (TVFCF)

6

CONCLUSIONS

The TAX model was developed to solve the problem of budget deficits. In doing so, however, the model disrupted indicators that are vital for social and political stability, namely employment and prices. Thus I developed two correcting models based on feedback control in order to ameliorate these conditions and argue for the use of such a framework.

The FCF and FCF2 models base their fiscal policy on unemployment reduction while they vary on monetary policy. The FCF model determines monetary policy from a feedback on price level changes, while the FCF2 model determines monetary policy from a feedback on both price level changes and unemployment changes. The results provide two different paths that attain a balanced budget, price stability, and a natural rate of unemployment. They differ, however, in the policy variables they use to reach these goals.

The FCF model results in a higher reliance on government expenditures and tax rate increases, while the FCF2 model removes some of the burden on the taxpayers and relies more on money supply and investment. In particular, FCF2's lower tax rate and government expenditures seem more appealing and feasible in a political system where fiscal policy is highly static and heavily regulated by Congress. The model allows for more liberty with monetary policy, which the FED already has. Therefore, it would seem to me that policies such as the ones derived from FCF2 would have less friction in passing through Congress, whereas FCF's higher government expenditures would draw criticism from Congress conservatives. Practically, FCF2 also provides earlier convergence in all control variables, and thus, I would argue, it would be more desirable than FCF.

A further effort is made to generalize the framework of nonlinear feedback rules by adding time varying weights. This allows flexibility to the model, permitting any user to input their priorities and weights on specific rules and observing the effects that their personalized policies have in the economy. The general framework also allowed me to focus on an experiment consisting of two regime stages. The first stage, from quarters 1-8, focuses on unemployment reduction and macrostabilization. The second stage, from quarters 9-20, focuses on the long run problem of deficit.

The results are quite satisfactory, and provide insights into policies that would alleviate our economic condition. While the use of nonlinear feedback rules at first puts a higher weight on government expenditures to salvage unemployment, the model suggests a quick convergence to a balanced budget and resurgence in the economy. The alternative is an autonomous economy that struggles to get back to equilibrium, and exposed to further austerity, would generate grave fluctuations in unemployment.

This paper thus provides an argument for a feedback control framework with nonlinear "handcrafted" rules and suggests policy approaches to reducing the deficit. Given the conditions of the U.S. economy, austerity measures and tax reductions would only make the problem worse by launching Americans into a deeper recession. With corporate taxes at an all-time low and unemployment still lingering above 7%, a reform to a more progressive tax code and an expansionary policy could provide the economy with the strength to come out of the slump.

Part III

APPENDIX

A

SOURCE CODE

Throughout this paper, the **General Algebraic Modeling System (GAMS)** was used to construct models and obtain data. What follows is a detailed explanation of the various models referred to in the paper as constructed in GAMS.

A.1 ORIG.GMS

The `ORIG.gms` file specifies the autonomous model used in Part i of this paper. This model contains no feedback rules and is based on the assumption that the economy is self correcting. This model is used to show the equilibrium of the economy and to serve as a base for comparison of other models.

First, we specify `SCALARS` to be used in the model.

Listing 1: `SCALARS`

```
SCALARS

tau       tax  rate                           / 0.1875 /
a         consumption equation constant       / 220 /
b         marg prop to consume                / 0.7754 /
e         investment equation constant        / 1000 /
d         interest elast  of invest.          / 2000 /
k         income elast of money dem.          / 0.1583 /
h         interest elast of mon  dem.         / 1000 /
alpha     coef on single-lagged inflation     / 0.4 /
beta      coef on double-lagged inflation     / 0.2 /
f         coef on excess aggregate demand     / 0.8 /
q         exchange rate equation constant     / 0.75 /
v         interest effect on exchange rate    / 5 /
```

```
g          net exports equation constant       / 600 /
m          income elast of net exp             / 0.10 /
n          real ex rate elast of net exp       / 100 /
mu         elast of empl wrt GDP               / 0.33 /
SSMlev     steady-state money level            / 900 /
SSGovlev   steady-state gov't expenditures     / 1200 /
SSYNlev    steady-state potential GDP          / 6000 /
gamma      growth investment                   / 0.3 / ;
```

We then move on to defining SETS over which the model will run.

Listing 2: SETS

```
1 SETS  T     TIME HORIZON     / 0*20 /
2       T0(T) PERIOD ZERO
3       T1(T) PERIOD ONE ;
4
5       T0(T) = YES$(ORD(T) EQ 1);
6       T1(T) = YES$(ORD(T) EQ 2);
7       DISPLAY T0, T1;
```

We define the set T over 20 quarters and in lines 2 and 3 specify names for periods 0 and 1, respectively. This is done by assigning the first ordinal value of the set T to T0(T) in line 5 and the second ordinal value to T1(T) in line 6.

Now, we specify PARAMETERS that might change in the model according to user input.

Listing 3: PARAMETERS

```
PARAMETERS

M(T)        money stock  (% change)
Gov(T)      Govt expenditure (% change)
YN(T)       potential GDP (% change)
PW(T)       foreign prices (% change)

Mlev(T)     money stock (level)
Govlev(T)   Govt expenditure (level)
YNlev(T)    potential  GDP (level)
plevw(T)    foreign prices (level)  ;

M(T) = 0 ;   Gov(T) = 0 ;  YN(T) = 0 ; PW(T) = 0 ;
```

```
15 Mlev(T) = SSMlev ;
16 Govlev(T) = SSGovlev ;
17 YNlev(T) = SSYNlev ;
18 plevw(T) = 1 ;
```

Lines 3 − 6 above represent percentage changes to be made to respective control variables, initialized at zero. An example is presented in the listing below, which allows for changes in Money Supply. We define a time period `TS1a(T)` from quarters `/4*10/` and set a percentage change by changing the zeros in `M(TS1a)`. The following line carries out the change into the model, setting money level to the steady state plus the percentage change only during those periods. Government expenditures, potential GDP, and world price levels have the same mechanism for defining percentage changes.

Listing 4: `Money Supply Shock`

```
SETS
TS1a(T) periods for shock 1a   / 4*10  /
TS1b(T) periods for shock 1b   / 15*20 / ;

M(TS1a) = 0.0 ;
M(TS1b) = 0.0 ;

Mlev(TS1a) =  SSMlev * (1 + M(TS1a)) ;
Mlev(TS1b) =  SSMlev * (1 + M(TS1b)) ;
```

Now we can move on to defining the actual model. First, we define `VARIABLES` and `EQUATIONS` and assign them names.

Listing 5: `VARIABLES` and `EQUATIONS`

```
VARIABLES

Y(T)         gdp
Yd(T)        disposable income
C(T)         consumption
I(T)         investment
R(T)         interest rate
plev(T)      price level
Pi(T)        inflation rate
```

```
Pie(T)       expected inflation rate
Ex(T)        nominal exchange rate
X(T)         net exports
GD(T)        government deficit
U(T)         unemployment rate
SUP          supplemental variable


EQUATIONS

gdp(T)           gdp identity
dispinc(T)       disposable income
consump(T)       consumption
invest(T)        investment
mdemand(T)       money demand
expinfl(T)       expected inflation
inflation(T)     inflation rate
pricelev(T)      price level
exrate(T)        real exchange rate
netex(T)         net exports
govdef(T)        government deficit
uer(T)           unemployment rate
supeq            supplemental equation;
```

An important part of this code is the `supplemental equation`. Because we are not optimizing anything in this model, the supplemental equation serves as a dummy for the model. Without this, the code will not compile.

Having defined all parts of the model, we can finally assert the model's equations.

Listing 6: `MODEL`

```
supeq..           SUP =E= 0 ;
gdp(T+2)..        Y(T+2)  =E=  C(T+2) + I(T+2) + Govlev(T+2) + X(T+2)
     ;
dispinc(T+2)..    YD(T+2)  =E= (1 - tau) * Y(T+2) ;
consump(T+2)..    C(T+2) =E= a + b * YD(T+2) ;
invest(T+2)..     I(T+2) =E=  e - d * R(T+2) + gamma * (Y(T+2)-Y(T+1)
    ) ;
mdemand(T+2)..    Mlev(T+2) / plev(T+2) =E= k * Y(T+2) - h * R(T+2);
expinfl(T+2)..    Pie(T+2) =E= alpha * Pi(T+1) + beta * Pi(T) ;
inflation(T+2)..  Pi(T+2) =E= Pie(T+2)
                  + f * (Y(T+1) - YNlev(T+2))  /  YNlev(T+2) ;
pricelev(T+2)..   plev(T+2) =E= plev(T+1) * (1 + Pi(T+2)) ;
exrate(T+2)..     Ex(T+2) * plev(T+2) / plevw(T+2) =E= q +  v * R(T
    +2) ;
```

```
netex(T+2)..      X(T+2) =E= g - m * Y(T+2)
                  - n * ( Ex(T+2) * plev(T+2) / plevw(T+2)) ;
govdef(T+2)..     GD(T+2) =E= Govlev(T+2) - tau * Y(T+2) ;
uer(T+2)..        U(T+2) =E= UN(T+2)
                  - mu * (Y(T+2) - YNlev(T+2)) / YNlev(T+2) ;
```

We guess initial values to give **GAMS** a starting point. If we do not do so, **GAMS** may find a solution to this problem that is unrealistic or nonsensical. Because dividing by zero is bad, we also set a lower limit for P_w.

Listing 7: GUESS

```
 R.L(T+2) = 0.09 ;
 Y.L(T+2) = 6000 ;
 Ex.L(T+2) = 1.2 ;
 C.L(T+2) = 4500 ;
 I.L(T+2) = 900 ;
 X.L(T+2) = -100 ;
 GD.L(T+2) = 75 ;
 U.L(T+2) = 0.07 ;
 YD.L(T+2) = 4875 ;
 Pi.L(T+2) = 0.1 ;
 Pie.L(T+2) = 0.2 ;
 plev.L(T+2) = 1.1 ;


* To divide by zero is bad, so we set a lower bound for plev *

 plev.LO(T) = 0.0001 ;
```

Next, we must fix the initial values for variables used in lags.

Listing 8: INIT VAL

```
 plev.FX(T1) = 1 ;
 Pi.FX(T0) = 0 ;
 Pi.FX(T1) = 0 ;
 Y.FX(T1) = 6000 ;
```

We now specify the equations that conform the model, in this case /ALL/. Then we declare how to solve the model. Because this model is inherently nonlinear, we must use a nonlinear solver. The nonlinear solver

NLP maximizes the supplemental equation, a dummy, and in doing so solves the model.

Listing 9: SOLVE

```
MODEL NonLinDyn /ALL/ ;

SOLVE NonLinDyn MAXIMIZING SUP USING NLP;
```

To display the values in an organized manner, we create the PARAMETER REPORTS.

Listing 10: REPORTS

```
PARAMETER REPORTS Solution Values ;
  REPORTS(T,"Pot. GDP") = YNlev(T);
  REPORTS(T,"GDP") = Y.L(T) ;
  REPORTS(T,"DispInc") = YD.L(T) ;
  REPORTS(T,"Consump") = C.L(T) ;
  REPORTS(T,"Invest") = I.L(T) ;
  REPORTS(T,"Govt Exp") = Govlev(T);
  REPORTS(T,"Govt Def") = GD.L(T) ;
  REPORTS(T,"NetExp") = X.L(T) ;
  REPORTS(T,"Money") = Mlev(T);
  REPORTS(T,"IntRate") = R.L(T) ;
  REPORTS(T,"Price") = plev.L(T) ;
  REPORTS(T,"Infl") = ROUND(Pi.L(T),3) ;
  REPORTS(T,"ExpInfl") = ROUND(Pie.L(T),3) ;
  REPORTS(T,"UER") = U.L(T) ;
  REPORTS(T,"Fgn Prc") = plevw(T);
  REPORTS(T,"Ex Rate") = Ex.L(T) ;
```

One of the most important aspects of this model is the addition of an interface with **EXCEL** that helps facilitate data analysis.

Listing 11: INTERFACE

```
execute_unload "fcfdata.gdx" REPORTS

execute 'gdxxrw.exe fcfdata.gdx par=REPORTS rng=ORIG!'
```

These lines of code use a **GAMS** package to write and save the data onto an **EXCEL** file called `fcfdata.xlsx` in the GAMS project directory.

The parameter `par=REPORTS` tells the model what to save, and the parameter `rng=ORIG!` specifies the range in which the data will be saved, namely, an EXCEL sheet called ORIG. This is helpful in saving the data from various models into the same EXCEL file, specifying different ranges for different experiments.

A.2 TAX.GMS

The `TAX.gms` file contains the model used in Section 2.2. Unless otherwise noted, the parts of `ORIG.gms` also apply to this file.

The major change is turning the tax level within the model into an exogenous variable defined by the nonlinear feedback rule.

Listing 12: `TAX MODIFICATIONS`

```
VARIABLES

TX(T)       tax rate
...

EQUATIONS

tax(T)      tax rate
...

tax(T+2)..  TX(T+2) =E= TX(T+1) * Govlev(T+2) / ( TX(T+2) * Y(T+2)) ;
...

TX.L(T+2) = 0.1875;
...

TX.FX(T1) = 0.1875;
...

PARAMETER REPORTS Solution Values ;
  REPORTS(T,"Tax Rate") = TX.L(T) ;
  ...
```

An important change in the code occurs in the specification of the range for the data to be saved in. The data will be saved in the same `EXCEL` file as the data from `ORIG.gms`, but the range is changed to `TAX!` to allow this data to be saved in a different sheet. This facilitates the creation of comparative graphs within the same file.

Listing 13: `TAX INTERFACE`

```
execute_unload "fcfdata.gdx" REPORTS

execute 'gdxxrw.exe fcfdata.gdx par=REPORTS rng=TAX!'
```

A.3 FCF.GMS AND FCF2.GMS

`FCF.gms` builds on `TAX.gms` to implement the nonlinear rules used in Section 2.3. First, government spending and money supply must be changed from exogenous parameters to endogenous variables. The following additions are made to the code in their respective parts. **Note:** The FCF model also includes the tax rule and the code shown before.

Listing 14: `FCF MODIFICATIONS`

```
VARIABLES

Govlev(T)     government expenditures
Mlev(T)       money supply
...

EQUATIONS

gov(T)        government expenditures
m(T)          money supply
...

gov(T+2)..    Govlev(T+2) =E= Govlev(T+1) * (U(T+2)/UN(T+2)) ;
m(T+2)..      Mlev(T+2) =E= Mlev(T+1) * plev(T+1) / plev(T+2) ;
...

```

```
Govlev.L(T+2) = 1200 ;
Mlev.L(T+2) = 900 ;
...

Govlev.FX(T1) = 1200;
Mlev.FX(T1) = 900 ;
...

PARAMETER REPORTS Solution Values ;
  REPORTS(T,"Govt Exp") = Govlev.L(T);
  REPORTS(T,"Money") = Mlev.L(T);
  ...

execute 'gdxxrw.exe fcfdata.gdx par=REPORTS rng=FCF!'
...
```

`FCF2.gms` uses a monetary policy rule that also controls for unemployment, so the equation `m` is modified.

Listing 15: FCF2 MODIFICATIONS

```
m(T+2)..          Mlev(T+2) =E= Mlev(T+1) * plev(T+1) / plev(T+2) *
    (U(T+2)/UN(T+2))  ;
...

execute 'gdxxrw.exe fcfdata.gdx par=REPORTS rng=FCF2!'
...
```

We can now recreate the experiments analyzed in Part i by running these models. The end result is an EXCEL file with 3 sheets of data. We now use the usual EXCEL procedures to generate graphs for all desired variables. It's important to note that the way the interface is structured allows for running the models again with different parameters. Running a new model and saving over the same range as a previous model will automatically update the graphs already created. This facilitates the task of running various experiments with the models.

A.4 TVFCF.GMS

Lastly, we developed a general model for nonlinear feedback rules in Part ii, and the modifications to **ORIG.gms** are presented below.

Listing 16: `WEIGHTS`

```
PARAMETERS

WG(T)       weight on expenditures
WT(T)       weight on taxes
WM(T)       weight on monetary policy - inflation
WU(T)       weight on monetary policy - unemployment
...

WG(T) = 0 ; WT(T) = 0 ; WM(T) = 0 ; WU(T) = 0 ;
...
```

We instantiate the weights as parameters which will have values between zero and one, initialized at a value of zero. The mechanism to change the weights is similar to the mechanism used for changes in controls established in `ORIG.gms`.

Listing 17: `WEIGHT PRIORITIES`

```
*WEIGHT IN GOVERNMENT EXPENDITURES
SETS
TSG1(T) period for shock 1 in G / 1*8 /
TSG2(T) period for shock 2 in G / 9*20 /  ;

WG(TSG1) = 0;
WG(TSG2) = 0;

*WEIGHT IN TAXES
SETS
TST1(T) period for shock 1 in taxes  / 1*8 /
TST2(T) period for shock 2 in taxes  / 9*20 / ;

WT(TST1) = 0;
WT(TST2) = 0;

*WEIGHT IN  MONEY SUPPLY - INFLATION
SETS
```

```
TSM1(T) period for shock 1 in money supply inflation / 1*8 /
TSM2(T) period for shock 2 in money supply inflation / 9*20 / ;

WM(TSM1) = 0;
WM(TSM2) = 0;

*WEIGHT IN MONETARY POLICY - UNEMPLOYMENT
SETS
TSU1(T) period for shock 1 in money supply unemployment / 1*8 /
TSU2(T) period for shock 2 in money supply unemployment / 9*20 / ;

WU(TSU1) = 0;
WU(TSU2) = 0;
```

Although the code presented only shows two stages for different weights, time sets can be similarly added to implement different policies at different periods of time.

Listing 18: TVFCF EQUATIONS

```
gov(T+2)..        Govlev(T+2) =E= Govlev(T+1) * (1+WG(T+2)*((U(T+2)/
    UN(T+2))-1)) ;
tax(T+2)..        TX(T+2) =E= TX(T+1) * (1 + WT(T+2)*( ( Govlev(T+2)
    / (TX(T+2)*Y(T+2)) ) -1 ) );
m(T+2)..          Mlev(T+2) =E= Mlev(T+1) * ( 1 + WM(T+2) * ( ( plev(
    T+1) / plev(T+2) ) - 1 ) ) * ( 1 + WU(T+2) *  ( ( U(T+2)/UN(T+2)
    )  -1 ) )  ;
```

The equations now include weights, which allows for flexibility and variability in policy implementation. The example used in Part ii uses unemployment control and economic recovery in stage one and deficit reduction in stage two.

Listing 19: TVFCF INTERFACE

```
execute_unload "fcfdata.gdx" REPORTS

execute 'gdxxrw.exe fcfdata.gdx par=REPORTS rng=TVFCF#!'
```

The interface works just as in the previous models; the only change is the range used. For an experiment with all zero weights, the range is defined as `TVFCF0!` which reflects the effects in the autonomous economy.

Other experiments with different weights are numbered in increasing order, and comparative graphs can then be developed from the data file. Setting up the graphs before running experiments also allows users to see updated graphs every time different weights are used.

B

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