Two-Stage Budgeting: A difficult problem

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Abstract

Utility maximization with a weakly separable utility function requires a consumer create an optimal budget for each separable subgroup. We show that computational complexity of optimal budgeting is the maximum of a exponential in the number of alternatives and a quadratic in the number of budget increments. From a budget survey of undergraduates we show that an undergraduate procedural consumer can obtain a budget estimate from the experience of previous students and can monitor the flow of funds and can make adjustments at a minuscule fraction of the calculations needed for optimal budgeting.

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1 Introduction

H. Simon (1976) has long advocated that economists study economic procedures. In Norman et al (2000), Norman et al (2000a), and this paper we take the first steps towards creating a sequential procedural model of a consumer. In this paper we examine the difficulties of optimal budgeting using complexity theory. A brief summary of the complexity concepts used in this paper is contained in Appendix A

In Section 2 we analyze the budget problem for a discrete consumer model with a separable utility function that has the two-stage budget property. Given modern packaging most goods are sold in units. Goods like gasoline are discretized in the sense that the consumer purchases gasoline in increments of the smallest coinage, for example cents in the US. Thus, the discrete consumer optimization model is actually more realistic than the continuous calculus model or nondenumerable set model and eliminates the need to consider ϵ approximations to the standard noncomputable consumer optimization models. For this problem, we show that the computational complexity of optimal budgeting is the maximum of a exponential in the number of alternatives and a quadratic in the number of budget increments.

Given the intractability of optimal budgeting, we study how students living in apartments and rented houses actually budget. In Section 3 we analyze a a budget survey of undergraduates who have moved out of the dormitories into rental housing and must budget to allocate funds among the various activities of student life. Students obtain a workable estimate of their expenses from students who have moved into rental housing before them. Also, as procedural consumers they do not consider large numbers of alternative bundles rather they monitor the flow of funds in item-by-item searches and make contingent decisions. Finally, in Section 4, we conclude.

2 Two-Stage Budgeting

Two-stage budgeting dates back to the creation of the separable utility model, Strotz (1957). To see a summary of the literature on separability and budgeting see Deaton and Muelbauer (1982) or Gorman's collected papers edited by Blackorby and Shorrocks (1995). We shall characterize the computational complexity of optimal budgeting for the discrete problem, which has the two stage budget property, described below.

Let us start with the following separable utility function defined over a discrete number of argument values:

$$U((x)_i) = U_1([x_i,1]) + U_2([x_i,2]) + \ldots + U_l([x_i,l]) \tag{1}$$

where

$$\begin{aligned} (x)_i &= (x^1_{i_1}, x^2_{i_2}, \ldots, x^n_{i_n}) \\ x^1_{i_1} \in X^1 &= \{x^1_1, x^1_2, \ldots, x^1_q\} \\ x^2_{i_2} \in X^2 &= \{x^2_1, x^2_2, \ldots, x^2_q\} \\ &\vdots \\ x^n_{i_n} \in X^n &= \{x^n_1, x^n_2, \ldots, x^n_q\} \end{aligned} \tag{2}$$

and

$$[x_i,j] = (x^{\beta(j)}_{i_{\beta(j)}}, x^{\beta(j)+1}_{i_{\beta(j)+1}}, \ldots, x^{\gamma(j)}_{i_{\gamma(j)}}) \in \{X,j\} = X^{\beta(j)} \times \cdots \times X^{\gamma(j)} \tag{3}$$

$\beta(j)$ is the index of the first category of the jth subset, $\gamma(j)$ is the index of the last category, and for simplicity the number of alternatives in each set X^l is equal to q.

Given the discrete nature of modern packaging, consumers mostly make discrete purchases of one or more items from a category of close substitutes, X^i. The category X^j might represent a finite set of different types and amounts of fruit. Those few items that consumers buy in continuous amounts are discretized by rounding off to the nearest cent, and X^k might present different types and amounts of gasoline. The subgroup $\{X, j\}$ might represent food or clothing.

We shall divide n into l equal subsets so that the computational complexity of each subproblem is the same; otherwise the computational complexity is dominated by the computational complexity of the largest subgroup. We shall consider values of the growth parameter n such that $\frac{n}{l}$ is integer. Also we shall divide I equally into m integer quantities, $a_1, a_2, \ldots, a_m$ with value $V(a_i) \geq 1$ cent. For example, $V(a_i)$ could equal 1 cent, 1 dollar, or 100 dollars. These m quantities are to be optimally allocated among the elements of $(I)_l$. The discrete consumer problem is:

$$\max_{(I)_l} \sum_{j=1}^{l} (\max_{[x_i, j]} U([x_i, j]) \text{ subject to } \sum_{k=\beta(j)}^{\gamma(j)} p^k_{i_k} x^k_{i_k} \leq I_j) \text{ and } \sum I_j = I \tag{4}$$

where $p^k_{i_k} > 0$, $I > 0$, and $I_j \geq 0$.

Now let consider the relationship to this problem and the two stage budgeting problem for the continuous weakly separable utility function. For the latter problem, Strotz (1957) asked under what condition can we optimally allocate

I into $(I)^*_l = I^*_1, I^*_2, \ldots, I^*_l$ where $\sum I^*_j = I$ and solve l smaller optimization problems instead of one large optimization problem. Gorman resolved this issue with a set of Slutsky conditions, see Chapter 2 of Blackorby and Shorrocks (1995).

For the discrete case above, the second stage problem is

$$\max_{[x_i,j]} U([x_i, j]) \text{ subject to } \sum_{k=\beta(j)}^{\gamma(j)} p^k_{i_k} x^k_{i_k} \leq I^*_j \text{ for } j = 1, 2, \ldots, l \tag{5}$$

where the I^*_js are determined by (4) above. It is straightforward to demonstrate by contradiction that the discrete model has the two-stage budget property, which is that an optimal solution to (4) is and optimal solution to (5) and an optimal solution to (5) is and optimal solution to (4).

We make the following assumptions:

1. **Binary Comparison**: We will consider the set of algorithms that solve (4) by comparing two bundles, $[x_i, j]$ and $[x_k, j]$ using a binary ranking operator, $B([x_i, j], [x_k, j]$ where $\rightarrow [x_i, j] \succeq [x_k, j]$ if $U_j([x_i, j]) \geq U_j([x_k, j])$ else $[x_i, j] \prec [x_k, j]$. We treat this binary comparison operator as a computational primitive.

2. **Interactions Affects**: In order to determine the utility of a subgroup bundle, $[x_i, j]$ an algorithm must evaluate $U_j([x_i, j])$. This eliminates additive utility functions.

3. **Bundle Organization**: In order to create an algorithm we have to specify how the bundles are organized that the consumer is comparing. For now, we shall assume that the bundles in each category arranged in ascending cost with index $k = 1, 2, \ldots, q^{\frac{n}{l}}$. The cost of the kth bundle is represented by $c(k)$ and the

preferred bundle in separable group s for the rth level of income is represented by $[x_*, s, r]$. The rows for the various subgroups are adjacent to each other.

Now, let us consider the following algorithm:

Block Bundle Search Algorithm, Block-Search:

Step 1: Set $s = 0$, Repeat steps 2-5 l times

Step 2: Increment s by 1. Set $k = 1$. $max_{s,r} = 1$ and $r = 1$

Step 3: While $c(k) \leq I$ and $k < q^{\frac{n}{l}}$ increment k by 1 and perform Steps 4 and 5

Step 4: If $c(k) \leq rV(a_1)$

If $B([x_{max_{s,r}}, s], [x_k, s]) \neq [x_{max_{s,r}}, s] \succeq [x_k, s]$, then $max_s = k$

Step 5: If $c(k) > rV(a_1)$

Record $[x_*, s, r]$ and $U([x_*, s, r])$

Increment r by 1 and $max_{s,r} = k$

END

When the algorithm terminates, a table of $[x_*, s, r]$ and $U([x_*, s, r])$, the preferred bundle and associated utility for each income level for each separable group. This table will be designated the *Opt* table. In the worst case we shall again assume that all bundles are feasible and in the expected case that $\alpha q^{\frac{n}{l}}$ are feasible where $0 < \alpha < 1$.

Lemma 1: If the n categories are divided into l subsets with an equal number of members, the worst and expected computational complexity of determining the *Opt* table is $lq^{\frac{n}{l}}$.

Proof:

a. $O(lq^{\frac{n}{l}})$: The consumer needs to perform no more than $l[q^{\frac{n}{l}} - 1]$ binary comparisons in the worst case and $\alpha l[q^{\frac{n}{l}} - 1]$ in the expected case using the Block-Search algorithm.

b. $\Omega(q^{\frac{n}{l}})$: Each set contains $q^{\frac{n}{l}}$ bundles. Given interactions affects, the consumer must perform at least $q^{\frac{n}{l}} - 1$ binary comparisons in the worse case and $\alpha q^{\frac{n}{l}} - 1$ binary comparisons in the expected case. By definitions D1-D3 of the appendix the computational complexity of determining the *Opt* table is $lq^{\frac{n}{l}}$.

END

Now we need to consider how we can create an efficient algorithm using this table to determine the optimal budget allocation among the l disaggregated consumer optimization problems. Let:

(1)$[x_*, j, r]$ be the preferred bundle in $\{X, j\}$ for $1 \leq r \leq m$ units of income.

(2)$[x_*, (1, j), r]$ be the preferred bundle in the concatenation of separable subgroups $\{X,1\}$ though $\{X, j\}$ for r units of income.

(3) $U(i, j, r) = U([x_*, (1, i-1), j]) + U([x_*, i, r-j])$.

(4) $U^*(i, r) = \max_j U(i, j, r)$ where $j = 0, 1, 2, \ldots, r$

The optimal budget allocation algorithm:

Step 1: Set i=2

Step 2: While $i \leq l$ repeat steps 3-5

Step 3: Use binary comparison, $>$ to determine and record $U^*(i, m)$ and $[x_*, (1, i), m]$ from $U(i, j, m)$

Step 4: If $i < l$ use binary comparison, $>$ to determine and record:

(1) $U^*(i, m-1)$ and $[x_*, (1,i), m-1]$ from $U(i, j, m-1)$

(2) $U^*(i, m-2)$ and $[x_*, (1,i), m-2]$ from $U(i, j, m-2)$

$\vdots$

(m-1) $U^*(i, 1)$ and $[x_*, (1,i), 1]$ from $U(i, j, 1)$

Step 5: Increment i by 1

END.

Note that this algorithm determines the optimal budget by concatenating the subgroups together one by one. Step 4 determines the optimal current concatenation for each budget level so that the next subgroup can be added. The operation that characterizes this algorithm is the number of binary comparisons, $>$, we must perform to determine the various $U^*(i, r)$s and $[x_*, (1,i), r]$s. Let us treat this operation, which involves lookup and comparison, as a primitive with cost $c_>$.

Lemma 2: If the n categories are divided into l subsets with an equal number of members, then the real and expected computational complexity of determining the optimal budget given the table of $[x_*, j, k]$s and $U([x_*, j, k])$ is lm^2s.

Proof:

a. $O((l)m^2)$: To perform steps 4 and 5 in the Optimal Budget Allocation Algorithm requires $O(m^2) >$ operations. Steps 4 and 5 are repeated $(l-2)$ times.

b. $\Omega((l)m^2)$: By induction: Prop(1) is the case i=3. Given the compensatory definition of the Utility function now applied to each block the number of $>$ operations is $\Omega(m^2)$. Prop(t) $\rightarrow$ Prop(t+1). Same number of steps as in the case of Prop(1). END

Theorem 3: The real and expected computational complexity of the optimal budgeting is $\max(lq^{\frac{n}{l}}, lm^2)$.

Proof: By the two lemmas the computational complexity of budgeting $\max(lq^{\frac{n}{l}}, lm^2)$ and is minimized when l is such that $q^{\frac{n}{l}} = m^2$. END

In determining the optimal budget there is a tradeoff between the number of bundles, which decreases with increasing number of subsets, and the number of possible allocations, which increases with increasing number of subsets. For the sake of discussion we shall ask if l were arbitrary what l would minimize the computational complexity of determining an optimal budget allocation.

Theorem 4: The computational complexity of solving (4) is minimized when l is defined by $q^{\frac{n}{l}} = m^2$

Proof: Obvious. END

Now let us reconsider that arrangement of bundles in adjacent ascending price rows. To obtain some perspective of the numbers involved with $\alpha = 0.5$, let us assume that $n = 30$ and that $q = 10$. These numbers are conservative even for a modern grocery store. In this case $q^n = 10^{30}$. In experiments with pens, we found that a subject could make a binary comparison between two pens in 3.2 seconds, Norman et al (2000). If it only took 10 seconds to make a binary comparison between two bundles of 30 items, then it would take only

1.59×10^{23} years to find the preferred bundle. If each bundle were placed in a shopping cart with each cart taking 3 feet then the consumer would have to travel 5.68×10^{27} miles just to view all the bundles.

The consumer shops item-by-item rather than as a bundle because the number of alternatives increase linearly rather than multiplicatively with each category. Sellers organize their merchandise item-by-item and not bundles to vastly reduce their display space. This means that it would be more difficult than specified for a consumer to consider bundles in the marketplace and our results are a lower bound in a market setting.

3 Actual Budgeting

The optimal budgeting algorithm is intractable for humans even aided by a computer. We performed a budget survey of The University of Texas at Austin, UT undergraduates who live in rented apartments to investigate how they solved this difficult problem. We focused on four issues:

1. How did the subjects obtain their initial budget estimate?
2. What procedures did subjects use in budgeting?
3. How flexible were their budgeting procedures?
4. How did they make budget adjustments?

We chose UT undergraduates who had lived in a rented apartment or house for at least part of a semester. For such students life in rented apartments is their first experience with budgeting because they have to deal with the apartment expenses, food, social expenses, and the entertainment expenses. We performed two budget surveys. The first involved over 100 students from which we obtained 85 good results. We later ran a second budget survey with 25 UT undergraduates. We shall display only those questions useful for this paper.

3.1 Initial Estimate

If students were able to obtain a good initial estimate to their budget problem, they may be able to obtain reasonable performance even though the problem is intractable. We asked two questions aimed at determining how students made their initial budget estimate.

1. Before moving into an apartment or other rental, how did you determine how much it would cost you to live in an apartment?

   a. Created a budget on paper or using a computer

   b. Mentally created a budget

   c. Simply guessed what your expenses would be

The results are displayed in the table below:

Answer	Frequency	Percent	Cum Frequency	Cum Percent
a	39	45.9	39	45.9
b	31	36.5	70	82.4
c	15	17.6	85	100.0

This table indicates that slightly less than 1/2 created a budget on paper or using a computer.

2. In determining how much it would cost, did you discuss the issues with:

   a. Family members

b. Friends or acquaintances in apartments

c. No one

The results are displayed in the table below:

Answer	Frequency	Percent	Cum Frequency	Cum Percent
Blank	1	1.2	1	1.2
a	28	32.9	29	34.1
b	51	60.0	80	94.1
c	5	5.9	85	100.0

Sixty percent of student planing to move into apartments queried students who were already living in apartments. Another 32.9% queried family members who reputedly had some experience in budgeting. Only five students talked with no one, but of these 3 created a budget on paper. Ten of the 15 students who simply guessed their expenses talked with friends or acquaintances in apartments.

Students obtained a imperfect estimate for their expenditures because 88% said that they underestimates expenses in one or more categories and 44% said that they overestimated expenses in one or more categories.

3.2 Budget Procedures

We focused on three aspects of budgeting. Since subjects spend money through time, they need to monitor the flow of funds because bouncing a check costs a student $25+. We investigate how active students were in monitoring the flow of funds. In addition, we investigated students planning procedures and to what extent they kept records.

In order to determine how frequently subjects monitored the flow of funds, we asked subjects the following question :

11. Checking account balance:

How often do you check your checking account balance? _____

Note: If you know your balance at all times, put 0. If you check your balance every couple of days, put 2 or 3. Once a week, put 7. When you receive your statement, put 30

Frequency of monitoring bank account

Every __ Days	Frequency	Percent	Cum Frequency	Cum Percent
0	19	24.4	19	24.4
1	6	7.7	25	32.1
2	10	12.8	35	44.9
3	11	14.1	46	59.0
4	1	1.3	47	60.3
5	2	2.6	49	62.8
7	15	19.2	64	82.1
14	4	5.1	68	87.2
20	1	1.3	69	88.5
30	9	11.5	78	100.0

What this table clearly indicates is that most students actively monitor the flow of funds. While slightly over 10% waits until they receive their bank statements, 80% check their balance at least once a week. Students use more than one approach to monitoring the flow of funds as shown in the table below:

Methods for Monitoring Bank Account

Balance Checkbook	Phone Bank	ATM	Use Internet
47%	42%	65%	33%

For example, 65% of the subjects monitor there bank account through an ATM machine. The totals add to more that 100% because students use more than one method.

While most students actively monitor the flow of funds, less than 60% of students subjects budget on a regular basis, the most common period being a month. Students who budget on an irregular basis vary from two weeks to two months. What do students do when budgeting. Over 90% check their bank balances and consider future bills that must be paid. Over 75% check to see what checks have cleared, and 60% consider future events such as trips. It is important to note that less than 30% keep records on paper or in a computer. Over 70% simple consider categories intuitively.

There are wide variations in how students budget. A few students take pride is saying they do not budget at all. At the other extreme are students who keep detailed records in spreadsheets and carefully plan future expenditures. One student, who appeared to have a successful procedure, budgeted once a week. First he brought up his account on the internet and checked his balances and what checks had cleared. Then he considered what bills he had to pay that week. Then he intuitively thought about food and entertainment expenses. He keep no written records.

3.3 Flexibility

Student budgeting is characterized by great flexibility. The response to the question of budget flexibility was:

Click on the one button from responses a,b,c,d,e, below
that best describes your flexibility

Choice	Action	No.
a	Once I determine the amount in a particular category, I never go beyond that amount	1
b	All categories vary a little from month to month	7
c	Some categories may vary considerably from month to month	1
d	All categories may vary considerably from month to month	3
e	Rent is fixed, utilities except perhaps phone are beyond my control, food varies somewhat, but I am flexible in my eating out, entertainment expenses and car expenses (if applicable)	13

3.4 Adjustment Process

The next question to consider is how do students make adjustments to their budgeting process. The issue is do students first adjust the amounts in their spending categories or do they, without directly considering their budget amounts, use simple rules to adjust expenses that result in changing the amount in various categories.

Almost 90% of students asked said they underestimated some expenses in their initial budget. Amusingly enough over half had to learn how to cook, at least to the extent of using a microwave, to reduce the cost of eating out. They adjusted by using some combination of getting a job, increasing the hours they worked, obtaining more money from parents or loans, or cutting expenses. Only 1/3 said they sat down and recalculated their budget. In contrast almost 80% said that they "Cut expenses without recalculation of my budget. For example, less entertainment, less eating out and when eating out less expensive restaurants." Over 2/3s agreed with the statement "Do you think your cutting expenses could be characterized by a simple rule such as I cut back on luxuries on several areas." Later in the survey we asked students how they would cut expenses \$100/month. Over 90% agreed with this statement "Cut back on luxury goods mostly in entertainment and eating out, but some in other categories too." This suggests that simple rules adjust the amount in various categories as students make each expenditure decision and students do not adjust the amounts in budget categories beforehand. One student who had to reduced her expenditures \$200 a month used a decision rule to 'cut out frivolous expenses' such as eating ice cream, something she liked. For many students entertainment expense is a residual after they taken care of their other expenses.

3.5 Cognitive Decision Theory

Thaler (1991) asserts that the substantive consumer model implies that money should be fungible that is money should not have labels attached to it. Thaler's fungible money is inconsistent with weakly separable utility theory wherein consumers optimally allocate funds to divide up the consumer problem into many smaller utility problems and necessarily place labels on the components of their income.

Given the difficulty of budgeting we would expect individuals to make errors and to be constantly searching for better allocations to their various separable categories. Even optimally allocating a marginal increase in available funds is very difficult. Suppose a student obtains an an unexpected additional \$100 from his or her grandparents. If the student is using three budget categories, there are 5151 possible allocations in the three categories to consider if the student allocates in \$1 increments. And, this assumes he or she knows the marginal utility of each allocation. The fact that even marginal budgeting is difficult we should not be surprised that many individuals treat bonuses differently from regular income.

Some psychologists such as Heath and Soll (1996) have focused on a questionable assumption that subjects in their mental accounting have designated fixed amounts for such categories as food, entertainment, rent. In the 1930s a form of budgeting known as tin can or envelope budgeting was common. Budgeters put cash into containers for each category of expenditure. Then they only spend what had been allocated for than category and did not move cash from one category to another. In order to claim that humans have had fixed budget amounts for decades, Heath and Soll reference tin can accounting', (page 155, Rainwater, Coleman, and Handel, 1959) . We quote from the above reference a working class housewife's explanation of so called tin can accounting,

> "I have a silly little system. Whenever my husband gets paid I take away so much for my grocery money and put it in my kitchen drawer. Then I take all the rest and I put it into a tin can. If we can pay a bill in person we take the cash out of the can–otherwise we may write a check on our bank account. I try to pay as many bills in person as I can though, because I don't like spending the time it takes to write a check. Now, whatever is left over in the tin can by the time the next payday comes we transfer into the bank account to pay our future bills. If my husband doesn't have enough money for gas out of his allowance, or if we go out for some entertainment we just take the money out of the tin can. Sometimes there is only a little left in the tin can at the end of the period, and sometimes there is a lot–it just all depends on the weeks. I've tried to budget with envelopes labeling them for this and that, but then we always took money out of the wrong envelop whenever we ran low, so it didn't really work. Now I've found the checking account together with the tin can the best system."

This passage clearly indicates that 'tin can' budgeting was unworkable and that her budget accounts are flexible and this agrees with our findings among undergraduates. Webley (1995) also suggests that the current evidence for mental accounting is weak. We claim that the issues of how flexible individuals are with categories in current income is an empirical issue. With the 20th century movement to a credit economy, even Thaler's stipulated barrier between current and future income has been decreasing.

4 Conclusion

Practical budgeting is currently a topic in consumer finance such as Quinn (1997). Also, numerous computer programs such as Quicken have been created to help individuals and households budget. These resources offer consumers very little good advice on the intractable problem of budget allocation.

Readers of this journal write code, therefore they tend to think in terms of procedures. Creating code that would improve consumers' budget allocation would be an interesting research question and the basis for a startup. For the continuous case interested readers should consider Gorman's optimal budget adjustment equations [eqn 5.1, page 28, Blackorby and Shorrocks (1995)]. They also should consider Carpentier and Guyomards' (2001) approximation for the first stage budget allocation and Velupillai's (1999) demonstration that solving the Leijonhufvud-Marshall consumer model is tractable.

For the discrete case, researchers must consider how to drastically reduce the relevant number of bundles. For example, if a household buys a particular computer and operating system, all consideration of software that does not run on that system can be excluded.

References

1. Heath, C., and J. Soll, (1996), Mental Budgeting and Consumer Decisions *Journal of Consumer Research* vol 23, pp 40-52

2. Carpentier, A. and H. Guyomard, (2001) Unconditional Elasticities in the Two-Stage Demand Systems: An Approximate Solution, *American Journal of Agricultural Economics* v83, n1, 222-229

3. Gorman, W. M., (1995), *Separability and Aggregation: Volume 1, Collected Works of W. M. Gorman* Blackorby, C. and Shorrocks, A. (eds), (Clarendon Press: Oxford)

4. Knuth, D. E., (1973) *The Art of Computer Programming* (Addison-Wesley: Reading)

5. Norman, A., A. Ahmed, J. Chou, K. Fortson, C. Kurz, H. Lee, L. Linden, K. Meythaler, R. Rando, K. Sheppard, N. Tantzen, I. White and M. Ziegler, (forthcoming), An Ordering Experiment, Journal of Economic Behavior and Organization

Available at http://www.eco.utexas.edu/Homepages/Faculty/Norman/ab.pdf

6. Norman A., A. Ahmed, J. Chou, A. Dalal, K. Fortson, M. Jindal C. Kurz, H. Lee, K. Payne, R. Rando, K. Sheppard, and I. White, (2000), Selection by Aspects, Norman and Associates Working Paper http://www.eco.utexas.edu/Homepages/Faculty/Norman/aa.pdf

7. Pagan, F. G., (1981) *Formal specification of programming languages* (Prentice-Hall: Englewood Cliffs)

8. Payne, J., Bettman, J., and Johnson, (1993) *The Adaptive Decision Maker* (Cambridge University Press-New York)

9. Quinn, J. (1997) Making the Most of Your Money, (New York: Simon and Schuster)

10. Simon, H. A., (1976), From substantive to procedural rationality, S. Latsis (ed), *Method and Appraisal in Economics*, (Cambridge University Press, Cambridge)

11. Strotz, R. H., (1957), The Empirical Implications of a Utility Tree, *Econometrica*, 25, 269-280

12. Thaler, R. H., (1991), *Quasi Rational Economics* (New York: Russell Sage Foundation)

13. Velupillai, K., (1999) *Computable Economics*, (Oxford, Oxford University Press)

Algorithms and complexity analysis

In this paper we shall follow the example of Knuth (1973) and describe in English the algorithms that will be sequences of operators. This is an acceptable compromise between rigor and the need to communicate with as wide an audience as possible.

Consider an algorithm $\varphi \in \Gamma$, the algorithm set, that using the operators $\Upsilon = \{\upsilon_1, \upsilon_2, \ldots \upsilon_m\}$ solves a problem element $s \in S$, the problem set, which consists of feasible combinations of prices, income, and utility functions. The cost function, C is: $C(\varphi[s]) = \sum c_{\upsilon_i} \cdot$ (numberof υ_i operations) where c_l is the unit cost of executing υ_l. Similarly, we can also define a time function, T: $T(\varphi[s]) = \sum t_{\upsilon_i} \cdot$(numberof υ_i operations) where t_l is the unit time of executing υ_l. For the rest of the discussion we shall just consider the cost function. The development of the time function would be the same.

For this discussion, we shall only consider the worst case analysis, for which the consumer must consider all possible bundles or items, by defining the cost function for an algorithm φ which will solve all $s \in S$ as: $C(\varphi[S])) = \sup_{s \in S} C(\varphi[s])$. The development of the expected case, for which the consumer considers only the budget feasible fraction, αq^n, of the bundles or items where $0 < \alpha < 1$, is similar. To solve S the consumer uses an efficient search, that is an algorithm φ^*; $\varphi^*(S) = \inf_{\varphi \in \Phi} C(S)$

where Φ is the set of all algorithms which solve S. To define the computational complexity of S let $Y = Y(n)$ be a nonnegative function which we wish to compare with the cost function, $C = C_\varphi(n)$. Frequently Y is n, n^2 etc. Consider the following definitions:

D1. C is $O(Y)$ if there exist $i, j > 0$ such that $C(n) \leq jY(n)$ for all $n > i$.

D2. C is $\Omega(Y)$ if there exist $i, j > 0$ such that $C(n) \geq jY(n)$ for all $n > i$.

D3. S has information-based complexity Y if there exists an algorithm $\varphi_i \in \Gamma$ such that C_{φ_i} is $O(Y)$ and for all algorithms $\varphi_j \in \Gamma$, C_{φ_j} is $\Omega(Y)$.

The concept of computational complexity divides problems into equivalence classes to facilitate comparison of the "difficulty" in problem solving. With these definitions, problems can be identified as easy, (for example, members of the n equivalence class) or hard, (for example, members of the exponential equivalence class or members of the NP complete class.).