U.T. Economics Summer 2011 Math Camp
M. B. Stinchcombe

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Topics: Proofs, efficiency via prices (FFTWE), products, correspondences, functions,
Readings: CSZ, Ch. 1, esp. 1.5

Notation for exchange economies, $\mathcal{E}$, is taken from CSZ 1.5.

A. [The immense distance between efficiency and fairness] In both parts of this problem, there is one cherry pie to be divided between two people. Person 1 receives $\alpha \geq 0$ of the pie, person 2 receives $\beta \geq 0$, and $\alpha + \beta \leq 1$.
   1. Person 1's utility from an allocation (α, β) is given by the strictly increasing function $u_1(\alpha)$, person 2's by the strictly increasing $u_2(\beta)$. Prove that no efficient allocation has $\alpha + \beta < 1$. [This is also an exercise in writing proofs.]
   2. With the same assumptions as in the previous problem, prove that for **any** $r \in [0, 1]$, $(\alpha, \beta) = (r, 1 - r)$ is an efficient allocation.
   3. Suppose now that person 1's utility from an allocation (α, β) is $u_1(\alpha, \beta) = 7\sqrt{\alpha} + \sqrt{\beta}$ while person 2's is a strictly increasing function $u_2(\beta)$. Give the set of efficient allocations.

B. [Exchange efficiency via prices] Find the Pareto efficient allocations and the equilibrium for the following exchange economies.
   1. Suppose that $I = \{1, 2\}$, $\ell = 2$, $\mathbf{y}_1 = (5, 2)'$, $\mathbf{y}_2 = (2, 5)'$, $\succ_1$ is given by the utility function $u_1(x_{1,1}, x_{1,2}) = x_{1,1} + x_{1,2}$, and $\succ_2$ is given by the utility function $u_2(x_{2,1}, x_{2,2}) = x_{2,1} \cdot x_{2,2}$.
   2. Suppose that $I = \{1, 2\}$, $\ell = 2$, $\mathbf{y}_1 = (9, 1)'$, $\mathbf{y}_2 = (3, 8)'$, $\succ_1$ is given by the utility function $u_1(x_{1,1}, x_{1,2}) = \min\{x_{1,1}, x_{1,2}\}$, and $\succ_2$ is given by the utility function $u_2(x_{2,1}, x_{2,2}) = x_{2,1} + x_{2,2}$.
   3. Suppose that $I = \{1, 2\}$, $\ell = 2$, $\mathbf{y}_1 = (2, 1)'$, $\mathbf{y}_2 = (6, 8)'$, $\succ_1$ is given by the utility function $u_1(x_{1,1}, x_{1,2}) = 2\log(x_{1,1}) + \log(x_{1,2})$, and $\succ_2$ is given by the utility function $u_2(x_{2,1}, x_{2,2}) = \log(x_{2,1}) + 3\log(x_{2,2})$.

C. [Productive efficiency via prices, version I, from Milgrom and Roberts *Economics, Organization, and Management*] You are in charge of the Department of Highway Safety. You have 3,000 crew hours to allocate to projects, your job is to reduce the expected numbers of lives lost. The data on last year's possible projects is given in the following table.

Project	Crew hours	Lives saved
1	800	4
2	900	3
3	800	2
4	500	1
5	1,300	2
6	700	1

1. Last year, projects 2, 3, and 5 were carried out. Show that this was inefficient and give the efficient choice of projects. (Hint: start by calculating the lives saved per 1,000 crew hours.)
2. Suppose that the same set of projects was available but that you had had only 2,500 hours of crew time available. What are the efficient set of projects?
3. Suppose now that x thousand crew hours are available.
   a. Give the maximal number of lives saved as a function of x on the assumption that partial projects yield partial rewards, i.e. that half of project 5 costs 650 crew hours and saves 1 life (in expectation).
   b. Repeat the previous on the assumption supposing that projects deliver no benefits unless completed.
4. The projects for this coming year are not yet known, they will not all be available for review at the same time, Further, because the projects are complex, staff members in several regional offices will be responsible for making the estimates of the crew hours and lives saved. Show that there is some number p with the property that if only projects saving p or more lives per 1,000 crew hours are chosen, then an efficient coordinated choice will be made.