

U.T. Economics Summer 2011 Math Camp
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Topics: Relations, optimal choice for finite sets, the weak axiom of revealed preference

Readings: CSZ, Ch. 2.4-5

A. Let X be the set containing the cities Austin, Tokyo, and Mumbai. Represent each of the following relations by filling out the following boxes.

1. xRy if city x strictly precedes city y alphabetically.

Mumbai			
Tokyo			
Austin			
	Austin	Tokyo	Mumbai

2. xSy if the second letter of city x 's name is strictly earlier than the second letter of city y 's name.

Mumbai			
Tokyo			
Austin			
	Austin	Tokyo	Mumbai

3. xEy if city x and city y are spelled with the same number of letters.

Mumbai			
Tokyo			
Austin			
	Austin	Tokyo	Mumbai

B. The observation about majority voting that appears in this problem is originally due to Nicolas de Caritat, the marquis de Condorcet (1743-1794). Suppose that X is a three point set, specifically $X = \{a, b, c\}$, and that persons $i = 1, 2, 3$ have complete transitive preferences orderings \succeq_i over X that satisfy $a \succ_1 b \succ_1 c$ for person 1, $c \succ_2 a \succ_2 b$ for person 2, and $b \succ_3 c \succ_3 a$ for person 3.

1. Fill in the tables below that describe \succeq_i , $i = 1, 2, 3$.

c			
b			
a			
	a	b	c
	\succeq_1		

c			
b			
a			
	a	b	c
	\succeq_2		

c			
b			
a			
	a	b	c
	\succeq_3		

2. For non-empty $B \in \mathcal{P}(X)$, define

$$C_V(B) = \{x \in B : (\forall y \neq x \in B)[\#\{i : x \succ_i y\} \geq 2]\},$$

i.e. $C_V(B)$ is the set of options in the set B that win a head-to-head vote against everything else in B . [Mnemonically, $C_V(B)$ is the “chosen by pairwise voting on options in B ” set.] Give $C_V(B)$ for each subset of X containing either 2 or 3 points.

3. Define $\succeq_V^* \subset X \times X$ by $x \succeq_V^* y$ if $(\exists B \in \mathcal{P}(X))[x, y \in B] \wedge [x \in C_V(B)]$. Fill in the following table for \succeq_V^* .

c			
b			
a			
	a	b	c

4. Prove that \succeq_V^* cannot be represented by any utility function.

- C. Problem 2.5.3 in CSZ.
- D. Problem 2.5.5 in CSZ.
- E. Problem 2.5.7 in CSZ.
- F. Problem 2.5.8 in CSZ.