Assignment #3 for **Mathematics for Economists**
Economics 362M, Spring 2010

Due date: Tue. Feb. 9.

Readings: CSZ, Ch. 2.6, 2.7, and 2.8

Chapter 2.6 reviews the definitions of functions and correspondences, their inverses, their compositions, and their properties, one-to-one, onto (surjectivity), into (injectivity). Chapter 2.7 introduces a list of properties that a relation may or may not have. Most important for our purposes are the weakly ordered sets, the partial ordered sets (POSETs), and lattices. Chapter 2.8 covers the basics of what are called "monotone comparative statics." The results here give sufficient conditions for movement in something that affects a decision maker to move the decision makers choice in a monotonic/unidirectional fashion.

Problems to do

From Chapter 2.6: There are 10 problems on pp. 34-38. Your job is to know how to do all of the problems, but you do not have to hand in any of them.

On p. 34, in Theorem 2.6.4(5), there is a typographical error. The claimed result is that $f(E\Delta F) \subset f(E)\Delta f(F)$. The true result is the reverse, $f(E)\Delta f(F) \subset f(E\Delta F)$. The easy way to go about proving this is to start by showing that $f(E) \setminus f(F) \subset f(E \setminus F)$. Taking $A = [0, 1]$, $f(x) = x(1 - x)$, $E = [0, 1]$ and $F = [\frac{1}{2}, 1]$ gives you a lovely example with the subset relation being strict.

From Chapter 2.7: 2.7.6, & 10 (p. 41), 2.7.15 (p. 42).

From Chapter 2.8: 2.8.2 (p. 43), 2.8.3 (p. 44).

From Chapter 2.8: 2.8.11 (p. 47), 2.8.21 (p. 48).

On the next page, there is one more problem.

Homework 3.A. *A software designer, s, and a marketer, m, form a partnership to which they contribute their efforts, respectively $x \geq 0$ and $y \geq 0$. Both have utility functions, $u_s = \$_s - x^2$ and $u_m = \$_m - y^2$, where $\$_s$ and $\$_m$ are monies received by s and m from the profits of the firm. The profit function is $\pi(x, y) = 4(x + y + cxy)$ where $0 < c < \frac{1}{4}$, and the profits are shared evenly between the two partners.*

(1) *Find the best response curves for the two players, i.e. find $x^*(y)$ that solves $\max_{x \geq 0} u_s(x, y)$ and $y^*(x)$ that solves $\max_{y \geq 0} u_m(x, y)$. Explain how the best response curves depend on c and why this is reasonable.*

(2) *A* ***Nash equilibrium*** *is a pair (x^*, y^*) that satisfies*

$$x^* \text{ solves } \max_{x \geq 0} u_s(x, y^*) \text{ and}$$

$$y^* \text{ solves } \max_{y \geq 0} u_m(x^*, y).$$

Find the Nash equilibrium when the two must simultaneously choose their effort levels. Explain how the equilibrium depends on c and why this is reasonable.

(3) *Now suppose, as you might think reasonable, that the software designer chooses her effort level, x, first. After x is observed by the marketer, she chooses her effort level, y. Find the equilibrium for this game. Explain how the equilibrium depends on c and why this is reasonable.*

(4) *Find the (x, y) pair that maximizes $u_s(x, y) + u_m(x, y)$. Explain why the answer to this problem is different than either of the equilibrium levels you found above. Also, explain how this efficient point depends on c and why this is reasonable.*