Assignment #9 for **Mathematics for Economists**
Economics 362M, Spring 2010

Due date: Tue. April 13.

Readings: CSZ, Ch. 4.7.

Compactness is a crucial idea throughout the mathematics that economists use. One of the crucial results is that a subset of $\mathbb{R}^\ell$ is compact if and only if it is both closed and bounded. There are several equivalent formulations of compactness, and they are useful in different contexts.

Homework 9.1. From Chapter 4.7: *4.7.6.*

Homework 9.2. From Chapter 4.7: *4.7.11.*

Homework 9.3. *For each of the following sets,*

(a) *give a sequence with no convergent sequence,*
(b) *give an open cover with no finite subcover,*
(c) *show that it is either not bounded or not complete.*

1. $\mathbb{R}^2_+ = \{\mathbf{x} \in \mathbb{R}^2 : \mathbf{x} \geq 0\}$.
2. $\mathbb{R}^2_{++} = \{\mathbf{x} \in \mathbb{R}^2 : \mathbf{x} \gg 0\}$.
3. $\{\mathbf{x} \in \mathbb{R}^2 : [x_1 \leq 1] \wedge [x_2 \leq 1] \wedge [\,[x_1 > 0] \vee [x_2 > 0]\,]\}$.
4. $\{1/n : n \in \mathbb{N}\}$.
5. $\mathbb{Q} \cap [0, 1]$.

Homework 9.4. *For each of the following sets, show directly that each of the following holds true:*

(a) *every sequence has a convergent sequence;*
(b) *every open cover has a finite subcover; and*
(c) *it is closed and bounded.*

[*The basic Theorem in* §4.7 *tells you that proving any one of the preceding holds true implies that both of the others hold true. In this problem however, I want you to do things in the more difficult fashion, that is, I want direct proofs for each of the three properties for each of the three sets.*]

1. $\{0\} \cup \{1/n : n \in \mathbb{N}\} \subset \mathbb{R}$.
2. $\{\mathbf{x} \in \mathbb{R}^\ell : 0 \leq x_i \leq i,\ i = 1, \ldots, \ell\}$.
3. $F_N = \text{cl}\,(\{q_n : n \geq N\})$ *where* $\{q_n : n \in \mathbb{N}\} = \mathbb{Q} \cap [0, 1]$.