U.T. Economics Summer 2013 Math Camp

Date: Wednesday, August 7, (A)

Catch-22 logic: Let: $\mathcal{P}$ denote the set of pilots; $E(p)$ is the statement "pilot p is Excused from flying because they are unfit" so that $E \subset \mathcal{P}$ is the set of pilots Excused from flying because they are unfit; $I(p)$ is the statement "pilot p is Insane" so that $I \subset \mathcal{P}$ is the set of Insane pilots; $R(p)$ is the statement "pilot p Requests an evaluation so that $R \subset \mathcal{P}$ is the set of pilots who Request an evaluation.

Lemma 1 (Joseph Heller)**.** *If* $[E \Rightarrow [I \wedge R]]$ *and* $I \Rightarrow \neg R$*, then* $\neg E$*.*

There are two premises in this lemma. The first is that a pilot can only be excused if he is insane and requests an evaluation. The second is that insane pilots do not request evaluations — only the sane pilots want to be excused from flying missions.

There are different ways of writing the statement in the lemma. Here is one, using the quantifier "$\forall$," that is, "for all."

$$(\forall p \in \mathcal{P})[\,[E(p) \Rightarrow [I(p) \wedge R(p)]] \wedge [I(p) \Rightarrow \neg R(p)]\,] \Rightarrow (\forall p \in \mathcal{P})[\neg E(p)]. \tag{1}$$

In terms of sets, with R^c bein the complement of R, we could also write the lemma as

$$[\,[E \subset (I \cap R)]\&[I \subset R^c]\,] \Rightarrow [E = \varnothing]. \tag{2}$$

Here is the proof using logical symbols.

Proof. By the definition of implication, $[I \Rightarrow \neg R] \Leftrightarrow [\neg I \vee \neg R]$. By DeMorgan, $[\neg I \vee \neg R] \Leftrightarrow \neg[I \wedge R]$. But $[E \Rightarrow [I \wedge R]]$ and $\neg[I \wedge R]$ imply $\neg E$. □

The last step comes from what is, in fancier places, called "*modus tollens*." This is the observation that in the truth table definition of implication, "True $\Rightarrow$ False is a False statement."

Here is the proof using sets.

Proof. $I \subset R^c$ iff $I \cap R = \varnothing$. But $E \subset (I \cap R)$ and $(I \cap R) = \varnothing$ imply $E = \varnothing$. □

Here is Joseph Heller's proof. The first question is posed by John Yossarian, a bomber pilot in WWII, to the doctor after learning that being crazy meant that you didn't need to fly bombing runs, and that you could request an evaluation of your sanity. The Doctor told him not to be quite so excited at the prospect.

> "You mean there's a catch?"
>
> "Sure there's a catch," Doc Daneeka replied. "Catch-22. Anyone who wants to get out of combat duty isn't really crazy."