Assignment #2 for **Mathematics for Economists**
Economics 392M.8, Fall 2013

Due date: Mon. Sept. 16.

Readings: CSZ, Ch. 3.6-9.

Problems

A. **From Chapter 3.6**: 3.6.4 (p. 91), 3.6.6, parts 1 and 2 (p. 92).

B. **From Chapter 3.7**: 3.7.12 (p. 96), 3.7.16 and 17 (p. 98).

C. Suppose that the growth curve in the fishery model of §3.7 is twice continuously differentiable. Your job is to compare the optimal growth paths and optimal steady states for the three utility functions $u(x_t) = \log(x_t)$, $v(x_t) = 2\sqrt{x_t}$, and $w(x_t) = x_t$.
   1. Give the three corresponding Euler equations.
   2. Give the optimal steady states as a function of the discount factor.
   3. Starting from the same x_0 below the steady states you just found, which utility function involves faster growth of the fish stock? [For the utility function $w(\cdot)$, you need to remember that the Euler equations were derived under the assumption that the solutions were strictly positive, and that may not be true.]
   4. Suppose now that x_0 is above the steady states and repeat the previous.

D. Write down the infima and suprema of the following sets. In each case say whether the infima and suprema are minimum and maximum elements.
   1. $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$.
   2. $\{\frac{3n}{4n+1} : n \in \mathbb{N}\}$.
   3. $\mathbb{Q} \cap (0, 1)$.
   4. $\{q \in \mathbb{Q} : q^2 < 7\}$.

E. Verify carefully from the definition of supremum that
   1. $\sup\{\frac{3n-1}{4n} : n \in \mathbb{N}\} = \frac{3}{4}$.
   2. $\sup\{\cos(\frac{1}{n}) + (-1)^n : n \in \mathbb{N}\} = 2$.

F. Let $X, Y \subset \mathbb{R}$ be non-empty sets. Prove the following.
   1. $\sup(X \cup Y) = \max\{\sup X, \sup Y\}$.
   2. $\sup(X + Y) = \sup X + \sup Y$ where $X + Y := \{x + y : x \in X, y \in Y\}$.
   3. $\sup(XY) \neq \sup X \sup Y$ where $XY := \{xy : x \in X, y \in Y\}$.

G. Show the following.
   1. $\lim_{n\to\infty} \frac{2n^2+19}{3n^2+10^6} = \frac{2}{3}$.
   2. $\lim_{n\to\infty} \frac{2n^2+\sin n}{n+19} = 2$.
   3. $\lim_{n\to\infty}(\sqrt{n+1} - \sqrt{n}) = 0$.

H. Suppose that $x_n \to x$, show that $(\frac{1}{n}\sum_{t\leq n} x_t) \to x$.