Assignment #7 for **Mathematics for Economists**
Economics 392M.8, Fall 2013

Due date: Mon. October 28

Readings: Chapters 4.10, 4.11

Problems

A correspondence from X to Y can be viewed two ways: as a point to set mapping, for each $x \in X$, $\Psi(x) \subset Y$; or as a subset of $X \times Y$, $gr(\Psi) = \{(x, y) : y \in \Psi(x)\}$. Here is a short list of places you will see correspondences: the Walrasian budget set $(\boldsymbol{p}, w) \mapsto B(\boldsymbol{p}, w) := \{\boldsymbol{x} \in \mathbb{R}^\ell_+ : \boldsymbol{pp} \leq w\}$; the demand set is the associated argmax correspondence, $x^*(\boldsymbol{p}, w) = \{\boldsymbol{x} \in B(\boldsymbol{p}, w) : (\forall \boldsymbol{y} \in B(\boldsymbol{p}, w))[u(\boldsymbol{x}) \geq u(\boldsymbol{y})]$; in supermodularity, $\theta \mapsto x^*(\theta)$ is another example of the argmax correspondence; for a game $\Gamma = (A_i, u_i)_{i \in I}$, $(u_i)_{i \in I} \mapsto Eq((A_i, u_i)_{i \in I})$ is the equilibrium correspondence, this is a product of argmax correspondences; in statistics, $X \mapsto \widehat{\theta}(X)$ is the mapping from data, X, to an estimator, again, this is often an argmax correspondence; in partially identified models, $X \mapsto \widehat{S}(X) \subset \Theta$ is a point to set mapping giving the estimated set of parameters consistent with, or best fitting, the data.

We now turn to the study of the basic properties of correspondences, upper and lower hemicontinuity, and the basic result, Berge's Theorem of the Maximum.

A. **From Chapter 4.10**: 4.10.3.
B. **From Chapter 4.10**: 4.10.6.
C. **From Chapter 4.10**: 4.10.14.
D. **From Chapter 4.10**: 4.10.21.
E. A sequence of continuous functions, u_n, on $\mathbb{R}^\ell_+$ converges uniformly on compacts to u if for all compact K, $d_{K,\infty}(u_n, u) := \max_{\boldsymbol{x} \in K} |u_n(\boldsymbol{x}) - u(\boldsymbol{x})| \to 0$.
   1. u_n converges to u on compacts iff $\rho(u_n, u) := \sum_{n \in \mathbb{N}} \frac{1}{2^n} \min\{1, d_{K_n,\infty}(u_n, u)\} \to 0$ where $K_n := \{\boldsymbol{x} \in \mathbb{R}^\ell : \|\boldsymbol{x}\|_\infty \leq n\}$.
   2. For a continuous $u : \mathbb{R}^\ell_+ \to \mathbb{R}$, and fixed $\boldsymbol{p}$ and w, let $V(u) = \max\{u(\boldsymbol{x}) : \boldsymbol{x} \in B(\boldsymbol{p}, w)\}$ and $x^*(u) = \text{argmax}\{u(\boldsymbol{x}) : \boldsymbol{x} \in B(\boldsymbol{p}, w)\}$. Show that $u \mapsto V(u)$ is continuous and $u \mapsto x^*(u)$ is upper hemicontinuous.
   3. For a continuous $u : \mathbb{R}^\ell_+ \to \mathbb{R}$, define $e(u; \boldsymbol{p}, r) = \min\{\boldsymbol{px} : u(\boldsymbol{x}) \geq r\}$ and $h(u; \boldsymbol{p}, r) = \text{argmin}\{\boldsymbol{px} : u(\boldsymbol{x}) \geq r\}$. Show that for fixed $\boldsymbol{p}$ and r, $u \mapsto e(u; \boldsymbol{p}, r)$ is continuous and $u \mapsto h(u; \boldsymbol{p}, r)$ is upper hemicontinuous.
   4. Extend one of the previous two problems either to continuity and upper hemicontinuity in $(u, \boldsymbol{p}, w)$ or $(u, \boldsymbol{p}, r)$. Part of the problem is figuring out which domains for the correspondences make the result true.

F. If Y_n is a sequence of compact technologies and $d_H(Y_n, Y) \to 0$ for some compact technology Y, then for all $\boldsymbol{p}_n \to \boldsymbol{p}$ in $\mathbb{R}^\ell_+$, show that $\Pi(\boldsymbol{p}_n; Y_n) := \max\{\boldsymbol{p}_n\boldsymbol{y} : \boldsymbol{y} \in Y_n\}$ converges to $\Pi(\boldsymbol{p}, Y)$ and the optimal netput set converges upper hemicontinuously. Formulate a continuity result for unbounded sequences of technologies (using the ideas used in the convergence of utility functions on compact sets).
G. **From Chapter 4.11**: 4.11.8 and 4.11.10.
H. **From Chapter 4.11**: 4.11.12.