Assignment #8 for **Mathematics for Economists**
Economics 392M.8, Fall 2013

Due date: Mon. November 4

Readings: Chapters 4.11, 4.12

Problems

The first three problems are about Markov Chains, the last three about Markovian dynamic programming.

A. [Random walk around a circle] The state space is $S = \{1, \ldots, N\}$, the transition matrix is $p_{i,i} = \rho$, $p_{i,i+1} = (1 - \rho)$ for $i = 1, \ldots, N - 1$, $p_{N,1} = (1 - \rho)$.
   1. Show that the unique ergodic distribution is $\pi = (1/N, \ldots, 1/N)$.
   2. A doubly stochastic matrix is one having all row sums and all column sums equal to 1. Show that the previous result about the unique ergodic distribution being the uniform distribution holds for all doubly stochastic matrixes.

B. [A simple diffusion model] r black balls and r white balls are distributed between 2 boxes subject to the constraint that each box contains r balls. We let the state space be $S = \{0, 1, \ldots, r\}$, interpreted as the possible numbers of white balls in the first box. At each point in time, one ball is chosen at random from each box, and the balls are interchanged. Give the transition matrix and the unique ergodic distribution when $r = 3$. [In general, the ergodic distribution is hypergeometric.]

C. [Gambler's ruin, aka a random walk with absorbing barriers] The state space is $S = \{0, 1, \ldots, N\}$, the transition matrix is $p_{0,0} = p_{N,N} = 1$, $p_{i,i-1} = \rho$ and $p_{i,i+1} = (1 - \rho)$ for $i \neq 0, N$. We are interested in the probability that, starting at n, the system ends up at 0 or ends up at N. Let $v_{i,0}$ and $v_{i,N}$ denote the probabilities that, starting at i, the system ends up in 0 or N. We know that $v_{0,0} = v_{N,N} = 1$.
   1. Give the entire set of ergodic distributions for this matrix.
   2. Show that $v_{i,0} + v_{i,N} = 1$ for all i.
   3. Show, using the Markov property, that $v_{1,0} = \rho + (1 - \rho)v_{2,0}$.
   4. More generally, show that $v_{i,0} = \rho v_{i-1,0} + (1 - \rho)v_{i+1,0}$.
   5. Solve the the $v_{i,0}$ as a function of ρ and N.

D. **From Chapter 4.11**: 4.11.15.

E. **From Chapter 4.11**: 4.11.19.

F. **From Chapter 4.11**: 4.11.20.