Solutions for Assignment #2 for **Environmental and Resource Economics**
Economics 359M, Spring 2017

Due date: Wednesday, March 1, 2017

A. Kolstad, Ch. 3, problem 1.

Ans. (a) The Pareto criterion fails completeness, it cannot choose between options when there is not and increase/decrease in everyone's preference ordering simultaneous, e.g. different points on the Pareto frontier. (b) Plurality fails IIA, see p. 59 of text for an example. (c) Majority rule fails transitivity as we saw in the Condorcet example in lecture (or the solution to Kolstad, Ch. 3, 9 below). (d) Randomly picking an option fails unanimity.

B. Kolstad, Ch. 3, problem 3.

Ans. Equal taxation would mean \$2 per person, and a vote on this would only have 40% approval, and this also tells us that the proposition is not Pareto improving, 60% would be against it because it makes them worse off.

Using the compensation principle, the 40% in favor could compensate the 60% disapproving it sufficiently to change their minds. The compensation problem, applied to any single problem, is not particularly convincing. However, if there are many proposals all of which satisfy the compensation criterion, and there is also sufficient randomness in peoples' preferences, it becomes more sensible.

C. Kolstad, Ch. 3, problems 8 and 9.

Ans. For 8, follow the suggestions in the text. With '$A \succ_i B$' denoting 'person i prefers A to B, we have

$$X \succ_1 Y \succ_1 Z$$

$$Y \succ_2 Z \succ_2 X$$

$$X \succ_3 Z \succ_3 Y$$

The Borda counts are $5, 6, 7$ for X, Y, Z respectively.

Now add W as follows

$$X \succ_1 Y \succ_1 W \succ_1 Z$$

$$Y \succ_2 Z \succ_2 W \succ_2 X$$

$$X \succ_3 Z \succ_3 Y \succ_3 W$$

The Borda counts are $6, 6, 7, 11$ for X, Y, Z, W respectively. Y now ties for first place with X.

For 9, consider the Condorcet cycle example,

$$X \succ_1 Y \succ_1 Z$$
$$Y \succ_2 Z \succ_2 X$$
$$Z \succ_3 X \succ_3 Y$$

Majority rule has A wins from $\{A, B\}$, B wins from $\{B, C\}$, and C wins from $\{C, A\}$.

Efficiency

D. The aggregate endowment of good 1 is 12, the aggregate endowment of good 2 is 10. There are two consumers, a and b. For each of the following utility function and allocations, determine whether or not it is Pareto optimal. If it is not Pareto optimal, give the set of allocations that Pareto improve on it. If it is Pareto optimal, give prices which, if faced by the two consumers, would leave them no incentive to move away from the allocation.

Ans. For these, we will work through the method given in class. Other methods can also be used so long as you give complete arguments.

1. The utility functions are $u_a(x_1, x_2) = \log(x_1) + 2\log(x_2)$, $u_b(x_1, x_2) = 2\log(x_1) + \log(x_2)$, and the allocation is $(6, 5)$ for both consumers.

   Ans. For any utility level u_2°, consider the problem

   $$\max_{x_1, x_2 \geq 0} u_a(x_1, x_2) \text{ subject to } u_b(12 - x_1, 10 - x_2) \geq u_2^\circ.$$

   The first two FOCs for the Lagrangean are

   $$\frac{\partial u_a(x_1,x_2)}{\partial x_1}_{|(x_1,x_2)=(6,5)} = \lambda \frac{\partial u_b(x_1,x_2)}{\partial x_1}_{|(x_1,x_2)=(6,5)}$$
   $$\frac{\partial u_a(x_1,x_2)}{\partial x_2}_{|(x_1,x_2)=(6,5)} = \lambda \frac{\partial u_b(x_1,x_2)}{\partial x_2}_{|(x_1,x_2)=(6,5)}$$

   The first equation asks that $\frac{1}{6} = \lambda \frac{2}{6}$, the second asks that $\frac{2}{5} = \lambda \frac{1}{5}$, and λ cannot simultaneously be $\frac{1}{2}$ and 2 as these require.
   To find the set of allocations, we need the set of x_1, x_2 such that $u_a(x_1, x_2) > u_a(6, 5)$ and $u_b(12 - x_1, 10 - x_2) > u_b(6, 5)$. This is a lens to the NW of the initial allocation in the Edgeworth box diagrams we used in lecture.
2. The utility functions are $u_a(x_1, x_2) = \log(x_1) + 2\log(x_2)$, $u_b(x_1, x_2) = 2\log(x_1) + \log(x_2)$, and the allocation is $(12, 10)$ for consumer a and $(0, 0)$ for consumer b.

   Ans. This is as unfair as possible, but it is Pareto optimal, it is not possible to make b better off without taking something away from a.

To find prices, consider the consumer demand problem

$$\max_{x_1,x_2} u_a(x_1,x_2) \text{ subject to } p_1x_1 + p_2x_2 \le p_1 \cdot 12 + p_2 \cdot 10.$$

The first two FOCs for the Lagrangean are

$$\frac{\partial u_a(x_1,x_2)}{\partial x_1}_{|(x_1,x_2)=(12,10)} = \lambda p_1 \frac{\partial u_a(x_1,x_2)}{\partial x_2}_{|(x_1,x_2)=(12,10)} = \lambda p_2.$$

Evaluating the left-hand sides, we find that

$$Du_a\left(\begin{smallmatrix}12\\10\end{smallmatrix}\right) = \begin{pmatrix}1/12\\2/10\end{pmatrix}.$$

Any vector of prices p that is proportional to Du_a will work in the sense that you can then find a positive λ satisfying all of the FOCs. For example, $p = \left(\begin{smallmatrix}5\\12\end{smallmatrix}\right)$ works, (this comes from multiplying Du_a by 60, and I only chose it because I like integer prices).

3. The utility functions are $u_a(x_1,x_2) = \log(x_1)+x_2$, $u_b(x_1,x_2) = \log(x_1)+\log(x_2)$, and the allocation is $(6,9)$ for consumer a and $(6,1)$ for consumer b.

   Ans. For any utility level u_2°, consider the problem

$$\max_{x_1,x_2\ge 0} u_a(x_1,x_2) \text{ subject to } u_b(12-x_1, 10-x_2) \ge u_2^\circ.$$

   The first two FOCs for the Lagrangean are

$$\frac{\partial u_a(x_1,x_2)}{\partial x_1}_{|(x_1,x_2)=(6,9)} = \lambda \frac{\partial u_b(x_1,x_2)}{\partial x_1}_{|(x_1,x_2)=(6,1)}$$
$$\frac{\partial u_a(x_1,x_2)}{\partial x_2}_{|(x_1,x_2)=(6,9)} = \lambda \frac{\partial u_b(x_1,x_2)}{\partial x_2}_{|(x_1,x_2)=(6,1)}$$

   At the given allocations, the gradients are $Du_a = \left(\begin{smallmatrix}1/6\\1\end{smallmatrix}\right)$ and $Du_b = \left(\begin{smallmatrix}1/6\\1\end{smallmatrix}\right)$, which can be solved by setting $\lambda = 1$. Therefore, this allocation is Pareto optimal, and any vector of prices proportion to Du_a (or Du_b) will work, for example, $p = \left(\begin{smallmatrix}1\\6\end{smallmatrix}\right)$.

4. The utility functions are $u_a(x_1,x_2) = \log(x_1)+x_2$, $u_b(x_1,x_2) = \log(x_1)+\log(x_2)$, and the allocation is $(3,7)$ for consumer a and $(9,3)$ for consumer b.

   Ans. Repeating the analysis in the previous problem (and not writing out the Lagrangean or the FOCs), at the given allocation, the gradients are $Du_a = \left(\begin{smallmatrix}1/3\\1\end{smallmatrix}\right)$ and $Du_b = \begin{pmatrix}1/9\\1/3\end{pmatrix}$, and since these are proportional (pick $\lambda = 3$), we are at a Pareto optimal point.

E. Kolstad, Ch. 4, problem 1.

Ans. (a) In a competitive market equilibrium, price is equal to marginal cost. Marginal cost is 10, and price is equal to 10 at the quantity $q_{comp} = 40$ (solve $Q = 50 - 10$ for Q). (b) For a monopolist, price is equal to marginal revenue, revenue is $Q(50-Q)$ so MR is $50-2Q$, and this is equal to 10 and $q_{Mon} = 20$, which yields a price of $p_{Mon} = 30$. (c) In the competitive case, the producer surplus is 0 and the consumer surplus is the area in the triangle bounded by the marginal cost curve (flat in this example) and the demand curve, so the total surplus is 800. Since the damage is 600, the net surplus is 200. In the monopoly case, the producer surplus is 400, the consumer surplus is 200, the damage is 300, and the net surplus is $(400 + 200) - 300 = 300$.

In this example, the monopolist's reduction of quantity lowers the damages enough that the total surplus is higher when the monopolist is in control. This is a general observation — if there are external damages to an action, reducing that action can improve life. However, the general Pigouvian solution is to charge people for the damages they cause and then to let the market work. In this case, if we made the generating firms pay for the damages that their production causes, the total marginal cost to producing would be $25 = 10 + 15$, so in the competitive case, we would have $Q = 50 - 25$, for the (maximal possible) total surplus of 312.5.

F. Kolstad, Ch. 4, problems 4 and 5.

Ans. (4) The production possibility frontier should, at a solution, be tangent to Brewster's indifference curve, and the slope will be $-p_w/p_G$ at the solution. (5) For the marginal rates of substitution given, we expect brewster would be willing to retain more garbage in exchange for the wine he gave up.

G. A society consisting of individuals a and b has 100 units of a consumption good. If it sacrifices s of the consumption good, it can produce $y = 10\sqrt{s}$ of a public good. The utility functions are $u_a(x, y) = \log(x) + \log(y)$ and $u_b(x, y) = \log(x) + 2\log(y)$.

1. Solve the problem $\max_{x,y} u_a(x, y)$ subject to $y \leq 10 \cdot \sqrt{100 - x}$. This gives person a's choice if they are the only person in the economy.

   Ans. Solve the problem

$$\max_{0 \leq s \leq 100} \log(100 - s) + \log(10s^{\frac{1}{2}}).$$

   The FOCs are $\frac{1}{100-s} + \frac{1}{2s} = 0$, or $100 - s = 2s$, so $s = 33\frac{1}{3}$, which gives $x_a = 66\frac{2}{3}$. Alternatively, solve for x_a directly,

$$\max_{0 \leq x_a \leq 100} \log(x_a) + \log(10(100 - x_a)^{\frac{1}{2}}),$$

   which has FOCs $\frac{1}{x_a} - \frac{1}{2(100-x_a)}$, or $3x_a = 200$, yielding again $x_a = 66\frac{2}{3}$.

2. Solve the problem $\max_{x,y} u_b(x, y)$ subject to $y \leq \sqrt{100 - x}$. This gives person b's choice if they are the only person in the economy.

   Ans. Solving

$$\max_{s \geq 0} \log(100 - s) + 2\log(10s^{\frac{1}{2}}),$$

   the FOCs are $\frac{1}{100-s} + \frac{1}{s} = 0$, or $100 - s = s$, so $s = 50$, giving $x_b = 50$.

3. For $\theta > 0$ being the weight on person a solve for $x_a^*(\theta)$, $x_b^*(\theta)$, and $y^*(\theta)$ in the problem

$$V(\theta) = \max_{x_a, x_b} [\theta u_a(x_a, y) + u_b(x_b, y)] \text{ s.t. } y \leq 10\sqrt{100 - (x_a + x_b)}.$$

Ans. To solve

$$\max_{x_a,x_b} M(x_a, x_b; \theta) = [\theta \log(x_a) + \log(10 \cdot (100 - (x_a + x_b)^{\frac{1}{2}})] + \log(x_b) + 2\log(10 \cdot (100 - (x_a + x_b)^{\frac{1}{2}}),$$

where $M(x_a, x_b; \theta)$ is the objective function, the FOCs are

$$\frac{\partial M}{\partial x_a} = \frac{\theta}{x_a} - \frac{\theta}{2} \frac{1}{(100 - (x_a + x_b))} - \frac{1}{(100 - (x_a + x_b))} = 0,$$
$$\frac{\partial M}{\partial x_b} = -\frac{\theta}{2} \frac{1}{(100 - (x_a + x_b))} + \frac{1}{x_b} - \frac{1}{(100 - (x_a + x_b))} = 0.$$

From the first equation, we arrive at

$$\frac{\theta}{x_a} = \left(1 + \frac{\theta}{2}\right) \left(\frac{1}{(100 - (x_a + x_b))}\right),$$

from which we find

$$x_a = \tfrac{2\theta}{2+\theta}(100 + (x_a + x_b)).$$

From the second equation we find (after similar re-arrangements)

$$x_b = \tfrac{2}{2+\theta}(100 + (x_a + x_b)).$$

One implication of the two equations is $x_a = \theta x_b$. Substituting this into the equation for x_b leads to

$$x_a^*(\theta) = \tfrac{200\theta}{4+3\theta}, \; x_b^*(\theta) = \tfrac{200}{4+3\theta}.$$

4. For θ very large, show that $x_a^*(\theta)$ and $y^*(\theta)$ is very close to the a's choice if they are the only person in the economy.
   Ans. As $\theta \uparrow \infty$, using l'Hôpital's rule (from calculus), we have $x_a^* \to \frac{200}{3} = 66\frac{2}{3}$ as above, and $x_b^* \to 0$.
5. For θ very small, show that $x_b^*(\theta)$ and $y^*(\theta)$ is very close to the b's choice if they are the only person in the economy.
   Ans. When $\theta = 0$, we have $x_a^* = 0$ and $x_b^* = 50$ as above.

Public Goods/Bads

H. Kolstad Ch. 5, problem 1.

Ans. For (a), the inverse aggregate demand curve is

$$P_{ag}(Q_{ag}) = \begin{cases} 2 - 1\frac{1}{2}Q_{ag} & 0 \leq Q_{ag} < 1 \\ 1 - \frac{1}{2}Q_{ag} & 1 \leq Q_{ag} \leq 2. \end{cases}$$

For (b), the efficient quantity is $Q_{ag} = 0.8$ where the supply curve (marginal cost curve) crosses the demand curve.

I. Kolstad Ch. 5, problem 6.

Ans. For (a), the Total Marginal Willingness to Pay curve is

$$MWP_T(Q_T) = \begin{cases} MWP_R(Q_T) + MWP_O(Q_T) = 25 - 3Q_T & 0 \leq Q_T < 6 \\ MWP_O(Q_T) = 13 - 2Q_T & 6 \leq Q_T \leq 13. \end{cases}$$

The efficient level is $Q_T = 5$ where $MWP_T(Q_T)$ is equal to the marginal cost, 10.

For (b), producers in each region are paying the cost of their damages, and would choose the efficient level from the previous problem, 5. The amounts received is the area under the $MWP_T(\cdot)$ curve for all units of pollution that are <u>not</u> abated, given 32 to O and 1 to R. As the producers do not care where the money goes (insofar as their profit maximizing decisions are concerned), there is no difference if the money goes to the UN.

For (c), the negotiations in O would end up with $13 - Q_0 = 10$ for $Q_O^* = 3$ and in R, $12 - 2Q_R = 10$ for $Q_R^* = 1$, a total reduction of 4 rather than 5. Here the producers are only paying for part of the damages they cause.

J. There are two routes into the business district, a Bridge and a Tunnel. There are 400,000 people who make the daily commute, there is no car-pooling. The time it takes to commute by the Bridge is $30 + \frac{n_B}{20{,}000}$ minutes if n_B people use the Bridge, the corresponding figure for the Tunnel is $40 + \frac{n_T}{5{,}000}$.

1. Suppose that each of the 400,000 people chooses so as to minimize their time commuting. This means that, in equilibrium, the commute times are equal. What are the equilibrium n_B and n_T? And what is the equilibrium total commute time in people-hours?

   Ans. Solve $30 + \frac{400{,}000 - n_T}{20{,}000} = 40 + \frac{n_T}{5{,}000}$ for the tunnel traffic, $n_T = 40,000$, and bridge traffic $n_B = 360,000$. $400,000$ people spend 48 minutes per trip, or $320,000$ person-hours per commute. (To put this in a bit more perspective, suppose that you valued a person hour at only \$6, with two commutes per day, the daily cost is \$3,840,000/work day, that is, approximately one billion dollars per year.)

2. Give the n_B and n_T that minimize the total commute time. Valuing a person-hour at \$6 (that is, \$1 for every 10 minutes), what is the daily value of the equilibrium inefficiency?

Ans. Solve the problem

$$\min_{n_T}(400,000 - n_T) \cdot (30 + \frac{400,000 - n_T}{20,000}) + n_T \cdot (40 + \frac{n_T}{5,000})$$

for $n_T = 60,000$ and $n_B = 340,000$. To reach the socially optimal total commute time, one needs to reduce the bridge traffic from 360,000 to 340,000, a bridge toll of 50 cents achieves this. This involves a total of (roughly) $316,300$ person hours per commute, about a 1% overall improvement. At $6 per hour, the savings of $3,700$ hours is worth $44,400 per day.

3. Except for congestion, there is a 0 marginal cost to having commuters on either route. Suppose that 10 minutes of commute time is worth $1 to a commuter. Find tolls t_B and t_T for the Bridge and Tunnel, one of them equal to 0, that have the property that equilibrium for the commuters is the one that minimizes total commute time. What revenues are generated?

   Ans. Most of this was done in the previous problem, 50 cents for each of $340,000$ bridge commuters, twice a day, yields $340,000 per work day. [At 300 work days per year, this is 102 million per year, which pays for a chuck of road maintenance.]

4. Suppose that 1/4 of the commuters car pool with 2 people in each car (so that there are 350,000 cars per day). Recalculate the minimal possible total commute time and the value of the potential savings.

   Ans. Do the obvious things, include putting the car pools in the quicker routes.

K. (A common pool resource) There are I different organizations, countries or firms, that can put out fishing fleets to catch from schools of fish. Use the number $a_i \geq 0$ to represent the number of fishing boats in the fleet of organization i, $i = 1, \ldots, I$ and let $a = \sum_i a_i$ denote the total size of the fishing fleet. The marginal cost of a boat is constant, and equal to c, the **per boat** return is $R(a) = 100,000 - \frac{1}{10}a^2$.

1. Verify that $R'(a) < 0$ and $R''(a) < 0$ for $a > 0$.

   Ans. $R'(a) = -\frac{a}{5}$, and $R'' = -\frac{1}{5}$.

2. Let $V(c) = \max_a [aR(a) - c \cdot a]$. Before you do any work, why should you expect that $a^*(c)$ is decreasing in c? Give the FOCs, $a^*(c)$ and $V(c)$.

   Ans. Looking at the objective function, it is submodular in c and a, hence we expect that $a^*(\cdot)$ decreases as $c \uparrow$.

   The FOCs are

   $$R(a) + aR'(a) = c.$$

   Putting in $R(a) = 100,000 - \frac{1}{10}a^2$ yields

   $$(100,000 - \tfrac{1}{10}a^2) - \tfrac{a^2}{5} = c,$$

yields $a^*(c) = \sqrt{\frac{10}{3}(100,000 - c)}$. Putting this into the objective function, $aR(a) - c \cdot a$ yields

$$V(c) = \left(100,000 - \frac{1}{3}(100,000 - c) - c\right) \sqrt{\frac{10}{3}(100,000 - c)}.$$

Simplifying a bit, this is

$$V(c) = \frac{4}{3}(100,000 - c) \sqrt{\frac{10}{3}(100,000 - c)}.$$

3. Now suppose that $I = 2$ and that the two countries choose a_1 and a_2 to solve

$$\max_{a_1} \; a_1 R(a_1 + a_2) - c \cdot a_1 \text{ and}$$
$$\max_{a_2} \; a_2 R(a_1 + a_2) - c \cdot a_2.$$

Give the FOCs for the equilibrium, $a_1^e(c)$ and $a_2^e(c)$. Show that $a^e(c) = a_1^e(c) + a_2^e(c)$ cannot solve the FOCs from the previous problem. Give, as a percentage, the efficiency losses when $c = 10,000$.

Ans. The FOCs are

$$R(a_1 + a_2) + a_1 R'(a_1 + a_2) = c$$
$$R(a_1 + a_2) + a_2 R'(a_1 + a_2) = c.$$

Summing, dividing both sides by 2, and setting $a^e = a_1^e + a_2^e$ yields

$$R(a) + \tfrac{1}{2} a R'(a) = c.$$

Since $R'(a) < 0$ and $R''(a) < 0$, the same a cannot solve these FOCs and the previous ones, the fact $\frac{1}{2}$ precludes this.

Being more explicit, the total equilibrium number of ships is $a^e = \sqrt{5(100,000 - c)}$, and both countries put out $\frac{1}{2}$ of this, $a_1^e = a_2^e = \frac{1}{2}\sqrt{5(100,000 - c)}$. To calculate the loss, calculate $V(10,000)$, then subtract from this the revenues minus the costs at the higher a^e.

4. Repeat the previous for I being any integer ≥ 2.

Ans. The FOCs sum to

$$R(a) + \tfrac{1}{I} a R'(a) = c.$$

The logic is just as above, the losses become larger as I increases because more countries are ignoring more damages.

L. Referring to the previous problem, Oström *et al.* describe a solution when the fish in question are north Pacific halibut. The solution involved changing the open access rules into another one of the four property-rights systems used to regulate CPRs. Which one? Explain.

If the fish in question are near the bottom of the food chain, in what way are the efficiency calculations in the previous problem mis-leading? Which part of the arguments in Oström *et al.* does this relate to? Does this necessarily depend on where the fish are in the food chain? Explain.

Ans. The transferable quotas for halibut turned this into an individual property situation where those owning the quotas excluded anyone else from commercially fishing for halibut. If the fish are near the bottom of the food chain, increasing their number increases the catch of everything that feeds on them. This is the simplest of what Oström *et al.* call the *complications of interlinked CPRs* (p. 281). Externalities will exist no matter where the fish are in the food chain, they will either compete with other fish for food, or else eat other fish.

Discounting and Decisions

M. Kolstad Ch. 6, problem 1.

Ans. For (a), the NPV is approximately $26.4 \cdot 10^6$ dollars. For (b), the NPV is approximately negative $50.5 \cdot 10^6$ dollars. For (c), one needs to numerically solve (e.g. using a spread sheet), and the answer is approximately 4.35%.

N. Kolstad Ch. 6, problem 2.

Ans. Here, we solve assuming that costs and benefits accrue at the end of the periods and that growth occurs after the first period. Other assumptions will change the answers slightly, and if clearly given, will also lead to full credit.

For (a), the NPV is negative 174,410 dollars, don't buy. For (b), the maximum is the NPV of benefits, approximately $1.8 \cdot 10^6$ dollars. For (c), with benefits growing at 3%, the NPV is positive, approximately $1.1 \cdot 10^6$, and the maximal willingness to pay is $3.1 \cdot 10^6$.

O. The production function turning today's investment, which is foregone consumption today, s_0, into consumption tomorrow is $c_1 = 10\sqrt{s_0}$. You have x_0 available to consume today, $c_0 \leq x_0$. The remainder, $s_0 = x_0 - c_0$, is invested and turned into consumption tomorrow. The utility function is $u(c_0) + \beta u(c_1)$ where $u(c) = 3\log(c+1)$. Let $V(x_0) = \max_{0 \leq s_0 \leq x_0} [u(c_0) + \beta u(c_1)]$ subject to $c_1 \leq 10\sqrt{s_0}$.

1. Verify that the production function and the utility function are increasing (have positive first derivative) and concave (have negative second derivative).

   Ans. $u'(x) = \frac{3}{x+1} > 0$, $u''(x) = -\frac{3}{(x+1)^2} < 0$. The production function is $f(x) = 10\sqrt{x}$, $f'(x) = \frac{10}{\sqrt{x}} > 0$ and $f''(x) = -\frac{10}{2x^{3/2}} < 0$.

2. Without doing any FOC calculations, how do savings/investment, s_0, depend on β? Explain.

   Ans. The objective function, $[u(x_0 - s_0) + \beta u(f(s_0))]$, is supermodular in s_0 and β, hence we expect $s_0^*(\cdot)$ to be increasing in β.

3. Without doing any FOC calculations, how do savings/investment, s_0, depend on x_0? Explain.

   Ans. The objective function, $[u(x_0 - s_0) + \beta u(f(s_0))]$, is supermodular in x_0 and s_0 (as done in lecture).

4. Without doing any FOC calculations, how does $V(\cdot)$ depend on x_0? Explain.

   Ans. Here is a pattern of argument that will be useful many times: if x_0 increases to x_0' and we do not change s_0, then utility goes up strictly; therefore, the optimal changes to s_0 after the change to x_0' must make utility go up even more. Therefore $V(\cdot)$ is strictly increasing in x_0.

5. Give the FOCs for $V(x_0)$ and solve them for $s_0^*(\beta, x_0)$.

   Ans. The FOCs are

$$\frac{3}{x_0 - s_0 + 1} = \frac{3\beta}{10\sqrt{s_0} + 1} \frac{1}{\sqrt{s_0}}.$$

   Rearranging yields a quadratic in $m = \sqrt{s_0}$,

$$(1 + \tfrac{1}{\beta})m^2 + \tfrac{1}{\beta}m - (1 + x_0) = 0.$$

   The only positive root of this equation has

$$m = \frac{1}{2(1 + (10/\beta))} \cdot \left((1/\beta) + \sqrt{(1/\beta)^2 + 4(1 + 10/\beta)(1 + x_0)}\right),$$

   which is an absolute mess, so figuring out comparative statics using this involves taking some complicated derivatives.