BUYING TRUTH IN A COMPETITIVE MARKET

HONG XU, MAXWELL B. STINCHCOMBE, AND ANDREW B. WHINSTON

ABSTRACT. Organizations face a competitive certification market for their statements, the statements do not convince third parties unless certified, the organizations are sometimes better served by a lie, and honest mistakes are possible. In our model of such a market: if certifiers are liable for mistakes, certifier contracts must be contingent; when certification is inelastically demanded, increases in certifier liability effectively reduce third party trust; organizational liability for mis-statements has a strong deterrent effect on mis-statements and increases third party trust; and after a strong negative shock to the financial system, loosening certification standards can only make it harder to raise third party trust levels.

The truth is rarely pure and never simple. (Oscar Wilde, The Importance of Being Earnest)
If men were angels, no government would be necessary. If angels were to govern men, neither external nor internal controls on government would be necessary. (James Madison, Federalist Papers No. 51)

1. INTRODUCTION

Certifications affect substantial investment and consumption decisions. Whether or not pension funds can purchase investment offerings depends on rating agencies certifying the offerings as being of “investment grade” or higher. Issuers of debt or securities, be they operating firms raising capital, or financial institutions selling securitized debts are interested in obtaining capital at low cost. Without some kind of certification, potential investors will not trust the statements describing these offerings. Without testing laboratory certifications of the safety and quality of the products of manufacturing firms, many products will not be carried by retailers.

The essential problem in the market for certification is to guarantee, without angelic intervention, that the net result, statements and their certification, convey the information relevant for the third parties who make decisions using the certified statements. Recent events have cast doubt on the credibility of certifiers. Historically, there have been two

Cases of early 21st century accounting scandals that have made the news include Enron (2001), AOL (2002), Bristol-Myers Squibb (2002), Halliburton (2002), WorldCom (2002). More recently, Ernst & Young, as the auditor of the Lehman Brothers, has faced inquiry by the U.K. government after the U.S. bankruckcy
keys to determinants of certifiers’ reliability: who buys the certification, the firms or the investors? and what is the balance of power between the certifiers and the firms?

Focusing on potential investors as the interested third party, we first study the case in which the investors buy the certification, and then the case in which the organizations raising capital buy the certification, both in the context of competition between certifiers that is intense enough to reduce economic profits to zero. We examine the balance of power by analyzing the possibility that certifiers may strategically misreport their findings in the direction favored by the debt/security issuer. We limit this possibility by requiring that the investors be sophisticated, and that the certification be reliable enough that investors are willing to invest in the operating firms that earn certification.

1.1. Background. When the investors are the buyers of the certification, they can directly reward or punish the certifier for accuracies or inaccuracies. A bank’s in-house auditors working on loan approvals are in this position, and for the first three and a half decades of its existence, Underwriters Laboratory (UL) was in a similar position. Supported by a consortium of insurance companies in the late 19th and early 20th century, UL developed expertise in evaluating the safety of electric devices of all sorts. The insurance companies used these ratings and the safety standards behind them in the design and pricing of their policies, making a UL certification extremely valuable. In 1935, UL broke from the insurance companies, reorganizing itself as a non-profit company and beginning to charge manufacturing companies to have goods and processes certified. Perhaps because of UL’s near monopoly, its corporate culture, or perhaps because of enlightened self-interest, manufacturers have never achieved significant power in this market.

In a similar fashion, during the late 19th and early 20th century, the rating agencies, Moody’s, Fitch’s, and Poor’s (later of Standard and Poor’s) specialized in collecting, aggregating and interpreting huge amounts of data on the railroads. The buyers of these analyses were potential investors, and over time, the industries covered by the ratings agencies grow. With the passage of the 1934 Securities Exchange Act, banks had a reserve requirement written in terms of “investment grade” assets, and the ratings agencies were the ones assigning this rating.

In the 1970’s, perhaps as a result of the plunging cost of photocopies, the market structure for the rating agencies shifted, and the asset issuers, rather than the investors, became the buyers of the certification. By itself, this change might not have been problematic, but in the 1990’s, three developments shifted the balance of power in favor of the institutions issuing the assets. First, the assets being issued began to change from corporate bonds to novel court examiner revealed evidence that the Lehman Brothers had prepared misleading financial statement using an accounting gimmick called “repo 105” (Justin Baer and Helen Thomas, “Valukas smooths way to legal action, Financial Times, March 13, 2010). The same gimmick was used by Bank of America and Citigroup to hide billions of dollars of debt (Michael Rapoport, B of A, Citi make ‘Repo’ errors, Wall Street Journal, May 27, 2010). On April 13, 2011, the Senate Permanent Subcommittee on Investigations released their report, “Wall Street and the Financial Crisis: Anatomy of a Financial Collapse,” presenting extensive evidence that Moody’s and Standard & Poor’s skewed their assessments to please their clients.

2Being almost a monopoly, UL sets prices for their services and sets the safety standards. Being a non-profit, the prices have been small enough that the manufacturers have not systematically complained. Being a self-consciously expert organization, the standards are set in consultation with industry and government, and again, there have not been systematic complaints.
structured financial assets, and the novelty of the structured financial assets meant that different agencies were more likely to assign different ratings to the same asset, encouraging ‘ratings shopping.’ Second, in the interest of “transparency,” the ratings agencies made public the criteria by which they assigned ratings. Knowledge of the criteria used by the agencies turned the issuers’ problem of constructing a profit maximizing financial asset into a quickly solved linear programming problem. Third, the issuers began to offer consulting contracts to the ratings agencies, and began to hire individual raters from the agencies at salaries much higher than the agencies were paying them (Partnoy (2009) contains more detail on these developments.)

Our analysis will focus on the possibility that certifier and/or issuer liability might increase the accuracy of the statement/certification process. For ratings agencies, the question of liability has little relevance. The 1934 Securities Exchange Act specifically exempts the raters from liability, and it is settled law that their ratings are “commercial speech,” hence under stringent 1st Amendment protection, and the ratings agencies have never lost a case trying to hold them liable for damages arising from their actions.

Auditors of firms’ financial statements are in a position intermediate between UL and the ratings agencies. The buyers of the certification are the firms whose statements are being certified, and auditors routinely perform consulting services for the firms buying the certification. However, unlike the ratings agencies, the consulting activities are both somewhat limited by regulation, and, since the passage of the Sarbanes-Oxley Act in 2001, consulting fees must be reported. Further, auditing firms in the U.S. must be partnerships rather than limited liability corporations, and the partners can be held liable for damages resulting from errors however they arise. Reputational concerns also play a large role for auditors, both as a marketing tool for their services and as a potent dissuader of lawsuits.

1.2. Description of the Model. Throughout this and subsequent sections, our discussion will be phrased in terms of a competitive market for audits of the SEC-required, quarterly financial statements of publicly traded firms. Mutatis mutandi, the model without liability applies quite directly to the case of ratings agencies certifying the quality of structured financial products.

Information is a public good, and markets underprovide public goods. The low cost of reproducing information compounds this problem by making it harder for firms in the business of providing information to profit. To counteract this, publicly traded firms are required, by the SEC, to file quarterly financial statements of their assets and liabilities. ‘Talk’ is often worth no more than the paper it is printed on, and without some kind of verification requirements, one might expect the same for financial statements. Certification by auditors is relatively cheap. Francis (2004a) gives 0.1% of company sales revenue as a reasonable figure for auditing expenses. We assume that alternative credible signals are, by comparison, prohibitively expensive.

Our model assumes that, due to a number of stochastic factors, firms may find themselves in a good or a bad situation at the time the quarterly filing is due. For us, a good situation is one which, if known to investors, would give the firm access to low cost capital, and a bad situation is one which, if known to investors, would not allow the firm this access. The proportion of the time in which firms find themselves in a good position, denoted $\mu$, plays a crucial role throughout.
Firms know their situation, compose a financial report claiming that it is either good or bad, and submit it to their choice of auditor. Auditor $A$ has higher costs and lower error rates than auditor $B$. We assume that competition between the auditors drives their economic profits to zero. This models our assumption that the power is in the hands of the operating firms.\footnote{For us the crucial error rate is the issuance of unmodified opinions offered to problematic clients. Geiger and Rama \cite{Geiger2003} show that opinions from Big 4 auditors, PricewaterhouseCoopers, Deloitte Touche Tohmatsu, KMPG, and Ernst & Young, had lower error rates than the opinions of smaller auditors, though among the Big 4, this crucial type of error was more prevalent for Ernst & Young.}

Auditors perform their audits, and, again stochastically, their audits may or may not mistakenly accept a claim that the company is in a good situation. With this information, the auditor must announce whether or not the operating firm’s financial statement is supported. Financial statements not found to be supported cannot be filed. Therefore, if the auditor chooses to say that the claim of being in a good situation is not supported, then the financial statement is changed to a claim of being in the bad situation. In the ratings agency interpretation of our model, the supported good/bad reports correspond to having earned a higher/lower evaluation.

Investors see which auditor the firm went to, and the final report, either good or bad. Investors, being savvy, use all available information to update their prior, $\mu$ before deciding whether or not to invest. They have outside opportunities available that yield a market rate of return, and only invest if the updated information convinces them that they will make this rate of return (or higher). The essential tension arises because, whatever their true situation, firm profits are higher if there is more investment, i.e. more access to low cost capital.

We study two variants of the model, one with non-strategic auditors who report their internal results no matter what they are, and one with strategic auditors who have the latitude to exercise their judgement in choosing their report. Alternatively, the non-strategic auditors can be thought of auditors with a complete firewall between marketing concerns and the performance of audits. As well as providing a useful benchmark, the non-strategic auditor case is of independent interest — it corresponds to recent policy proposals to replace the requirement that financial statements be audited with the requirement that they be insured for accuracy in the presence of liability for damages caused by the inaccuracy \cite{Ronen2010}. In this case, the presence of insurance and the size of the premium paid would be the investors’ signal.

In both the strategic and the non-strategic auditor analysis, our focus is on equilibria with: active competition between the certifiers; certifications containing useful information; and investors earning a market rate of return on their investments. Active competition is the requirement that, in equilibrium, both auditors have clients. The production of useful information is the requirement that, in equilibrium, investors are willing to invest after the auditors assert that the operating firms claim to be in a good state is supported. The investors earning a market rate of return in equilibrium is the requirement that investors’ prior for a good situation, $\mu$, be in an intermediate range: if $\mu$ is too high, then investors

\footnote{This could arise from Bertrand competition between the two auditors, or from the operating firms being in a position to make take-it-or-leave-it offers to the auditors. Our conclusions are robust to other constant divisions of the surplus between the operating firm and the auditors. To the extent that management deciding to change auditors is interpreted as a bad signal by the market, this assumption understates auditor power.}
would earn higher than a market rate of return; if $\mu$ is too low, then no assurance by an auditor will induce investment, meaning that there would be no investment and that the market for certification does not produce useful information.

1.3. **Description of Results.** The only outcomes satisfying the above three requirements arise from a set of mixed-pooling equilibria in which different proportions of both types of firms patronize the different auditors. No pure strategy equilibrium satisfies our requirements: in a separating equilibrium, firms’ choice of auditors will be a perfect signal of their type, auditors’ reports become irrelevant to the investors, only the lowest cost auditor can stay in business, and firms in a bad situation will get 0 investment and hence have incentive to mimic those in a good situation; pooling equilibria may occur when the proportion of good firms is very high or very low, but they too involve only one type of active auditors.

In working with the indifference conditions that characterize a mixed strategy equilibrium, one determines the likelihood that one person will make a choice between options that seem equally good to them by the condition that someone else be made indifferent between their options. This can be counter-intuitive, and we follow the large population interpretation of equilibria with randomization in [Sandholm (2007)](Sandholm2007). In more detail, [Harsanyi (1973)](Harsanyi1973) showed that randomization can be interpreted as there being a small amount of smoothly distributed, independent, private information about perturbations to player payoffs to different actions. In the game with perturbations, with probability 1, every player has a strict preference for their choice of actions, and when the perturbations are small, the outcome is arbitrarily close to the mixed strategy equilibrium. Sandholm further demonstrated that: there exist versions of Harsanyi’s payoff-perturbed games with larger perturbations that still allow for Harsanyi’s interpretation of mixed equilibria; and the strict equilibria of the perturbed games have a large population interpretations with asymptotically stable dynamics.

From this perspective, investors’ randomization in the investment decision is a population distribution of judgement calls after reading the same firm statement and auditor certification, but bringing to it a different set of experiences and investment needs. In the same way, firms’ randomization over auditors reflects a steady state proportion of a population of firms, and the steady state proportion has stable dynamics. The auditors’ randomized decisions about whether or not to support a firm’s statement of being in a good state is also a population distribution, here it is a proportion of the judgement calls, some of them perhaps quite close and difficult, that go in the firm’s favor.

The main conclusions from our model are as follows.

1. Auditor liability is ineffective at increasing the accuracy of the certification market, but management liability is effective. To put it another way, fines/punishment for failures to detect management deception are less effective than fines/punishment for the deceptive practices. This is because, when audit services are demanded inelastically, fines levied on competitive auditors are entirely passed through, as a fixed cost, to the firms purchasing the audit services. Unless these costs are sufficiently large to close the market, they have no other effect.

2. Differences in auditor quality reduce investment, and in the case of strategic auditors, competition introduces pressures for the better auditors to lower their accuracies and to act more like the weaker auditors.
3. In the presence of liability and/or reputation concerns by auditors, contingent fees, even if only implicit, are necessary to induce quality reporting.

4. When the proportion of firms in a good situation is lowered, e.g. after a cyclical downturn or a significant negative macro shock, loosening audit standards can only make matters worse — lowering audit standards will either further reduce investment, or, if the shock has seriously disrupted investment, lowering audit standards increases the degree of recovery necessary to return investment to regular levels.

1.4. Related Theoretical Literature. There is an extensive literature on information intermediaries. Topics covered include adverse selection, signaling, coordination, the role of competition in the incentives to provide quality information, reputation concerns, implicit contracts, and the roles of litigation and liability.

In the context of the middlemen, Biglaiser (1993) demonstrates that, in the presence of adverse selection, expert intermediaries can improve the social welfare of the trading partners. Lizzeri (1999) mostly focuses on a monopoly certifier who can choose how much to reveal to the third parties, finding that the monopoly certifier can absorb a large share of the surplus by giving very little information — a binary signal that is positive at very low levels, and also finding the striking result that firms have incentive to be certified even if the certification is uninformative. Boot et al. (2006) show that intermediaries, specifically a monopoly credit ratings agency with no statistical knowledge about the quality of a firm, can coordinate firms and investors on equilibrium outcomes desirable for the firms when there are multiple equilibria.

Our model also has binary signals, but they are provided by competing certifiers with different degrees of statistical expertise. Further, in our model, all firms require certification to do business, so adverse selection is not an issue. When we study strategic auditors, we select from the multiple possible equilibria those that maximize the welfare of the purchasers of certification. This involves the more accurate certifier increasing their average rating, i.e. acting as if they have adopted a less accurate set of audit practices. However, competition nullifies this effect at the market level, and aggregate certifier market accuracy is unchanged across the equilibria.

The relations between the intermediaries and their clients have also been studied in this class of markets, and in particular the mixed incentives these relations provide for the intermediaries to provide quality information. For credit rating agencies, when consulting and other non-rating services generate a major share of the income for a rating agency, Mathis et al. (2009) argues that they produce overly optimistic ratings to attract clients and that reputational concerns are not strong enough to overcome this effect. Becker and Milbourn (2011) showed that from 1998 to 2006, the material entry of a third ratings agency, Fitch, coincided with higher ratings levels, lower correlations between ratings and market-implied yields, and lower ability of ratings to predict default.

In an experimental setting, Russo et al. (2000) discovered that auditors tend to distort information to support their clients’ preferred opinions. Heron and Lie (2009) also reported

\[\text{5By contrast, the analysis of competition in Lizzeri (1999) and the study of entry in the insurer ratings industry of Doherty et al. (2012) shows that competition can lead to better information. In cases when the investors and not the firms are buying the certification, entrants may attract experienced investors by supplying more precise information Jin et al. (2010).}\]
that financial manipulation, in the form of back-dated stock options for high-level management, is more prevalent among firms that work with smaller auditors. Competition for business could also take the form of ‘ratings shopping.’ If this is possible, and if investors take ratings at face value, certifiers have high incentive to inflate ratings, leading to lower information quality as compared to the monopoly case (Bolton et al., 2012). The incentive for rating shopping is even higher when the underlying asset is complex in nature, which leaves room for very different ratings from certifiers (Skreta and Veldkamp, 2009). Similarly, when investors cannot distinguish between a firm with no rating and a firm chooses not to disclose the rating, the equilibrium outcome may involve firms hiding their ratings, and competition among certifiers will lead to lower information disclosure (Faure-Grimaud et al., 2009). While we do select the equilibria most profitable for the firms, and our equilibria often involve supporting a good report when the certifier’s internal evidence is against it, we do not model or investigate the ratings shopping aspect of the problem because changing auditors is a costly undertaking, often regarded as a sign that the firm has something to hide.

In our model, when certifiers can be held liable for not detecting false claims made by their clients, supporting a good report becomes risky, and some kind of risk premium becomes necessary if good reports are to be supported. In the auditing industry, explicit tying of the outcome of the auditor’s report to payment is illegal. For us, it is an implicit contract for services above and beyond certification, e.g. consulting and other management advisory services, that provides this contingent payment. Of particular interest in this context is the study of implicit and incomplete contracts in Bernheim and Whinston (1998). They argue that when second stage actions by two parties are strategic complements, for example management effort by the firm and management advisory services by an auditor, implicit contracts can dominate complete and explicit contracts. To the extent that some part of the payments for management advisory services are risk premia paid for support of a positive rating, one should expect, as found in Banker et al. (2005), that auditors more heavily invested in such services demonstrate higher measured productivity growth.

Information can be dispersed across markets, and the credibility of ratings is also influenced by the accessibility of rating information across different markets. In the model of a single rater of Damiano et al. (2008), ratings are often inflated in equilibrium. To increase the credibility of the ratings, clients with correlated qualities should be rated in a centralized fashion, and the ratings should be made available to all relevant investors. The possibility of cross-referencing increases the credibility of ratings and improves the payoffs to the monopoly rating agency. By contrast, in our model, all investors have the same information, and competition for business between the two certifiers drives their economic profits to zero.

Competition is sometimes understood as a free entry condition. Hvide and Heifetz (2001) has a certification market with a number of certifiers, free entry, and perfect discovery capability. Rather than competing for the same clients, the results show that each certifier targets a segment of clients. Entry of a third firm in the ratings industry has happened in the last two decades, but only exit has happened among the large auditing firms. We model certifier competition for business, but we do not model entry as part of the competition. For us, segmentation or separation can never emerge as an equilibrium satisfying our requirement, rather, both certifiers cater to all types of clients.

7
Litigation and liability has the possibility of changing certifier incentives to put in effort, and consequently certification quality: Partnoy (2001) proposed a new legal regime to regulate the credit rating industry through a contract of liability sharing with their clients; White (2010) recommended more regulations on financial institutions and fewer for the ratings agencies, as well as a decrease of investors’ reliance on ratings by providing multiple new sources of information. In a study of IPO’s, Venkataraman et al. (2008) show that a higher litigation regime is associated with higher audit quality, indicating more effort, and higher audit fees, perhaps also indicating a higher risk premium. However, the net effect of certifier liability may be ambiguous — as argued in Pae and Yoo (2001), anticipating the high pressure auditors bear under high litigation, clients have less incentive to invest on internal control and rely more heavily on the auditors. Besides affecting audit quality and audit fees, evidence in Simunic and Stein (1996) suggests that increased litigation induces clients to switch to low-quality auditors. Lowered reputation is often thought of a form of liability that may influence intermediaries’ reporting. Corona and Randhawa (2010) showed that reputation concerns can actually lead auditors to misreport, e.g. to protect their own reputation, an effect also discussed in Mariano (2012). Our model focuses on the impact of the litigation cost, be it the cost of legal services or the cost of a lower reputation, as it acts through auditors’ strategic reporting decisions, rather than through their effort choice.

The Sarbanes-Oxley Act established the Public Company Accounting Oversight Board (PCAOB) in 2001. The PCAOB has recently called for auditor rotation, because, according to member of the accounting oversight board, Steven B. Harris, the evidence showed that there are still “strong incentives that lead some auditors to serve the interests of the company paying the bills rather than those of investors.” (New York Times, “Accounting Board to Seek Comments on Rotating Auditors,” August 17, 2011, B3). Ghosh and Pawlewicz (2009) compared audit fees before and after Sarbanes-Oxley, and documented an economically large increase in audit fees following the enactment of Sarbanes-Oxley. Other evidences include Choi et al. (2009), which concluded that auditors charge higher fees for firms that are cross-listed in countries with stronger legal regimes and the premium increases with the difference in the strength of legal regimes in the two countries. On the one hand, auditors can reduce the risk of litigation with more working hours spent auditing and higher levels of caution interpreting what they find, and on the other, they shift the perceived legal costs to their client firms. Bell et al. (2001) and Simunic and Stein (1996) present evidence that auditors bill more hours and charge higher audit fees for clients with higher business and litigation risk. One of our key interests is the contrast between auditor liability and firm liability, two different penalty structures contained in Sarbanes-Oxley.

There are many areas in which tensions similar to the ones we examine here play out. When a doctor prescribes, or takes part in a study evaluating a medicine, they act as an expert that certifies the quality and appropriateness of the medicine, but pharmaceutical company compensations to doctors provide the same kind of conflict of interest studied here. Lerner and Tirole (2006) study what we call the ‘ratings shopping’ part of the puzzle in a model of forum shopping. One of the intended purposes of a board of directors is the

---

6From a different perspective, Laux (2010) demonstrated that litigation possibilities against outside directors, also imposed in Sarbanes-Oxley, has an ambiguous impact on board behaviors and accounting information, especially when oversight is costly and difficult.
monitoring of a CEO, but, as pointed out and analyzed by Hermalin and Weisbach (1998), positions on the board are often chosen through processes highly influenced by the person to be monitored.

1.5. Related Empirical Literature. As just discussed, many of our modeling choices are made with an eye to auditors and ratings agencies. An extensive literature on the empirical regularities of the auditing industry further informs our choices. Perhaps the two most important assumptions in the model are that certifiers cannot catch all false claims, and that certifiers sometimes inflate their reports in a direction favored by the clients. We discuss evidence for these two assumptions in turn.

Most audits are successful, enough so that it is only audit failures that are newsworthy, and there are many well-documented determinants of the likelihood of a successful or an unsuccessful audit. Francis (2004) is a survey of the empirical research on this. However, due to the numerous earnings management strategies in common use, research on this topic has low statistical power, it can only find egregious cases, e.g. Carcello and Palmrose (1994) find that only 30% of bankruptcies are preceded by auditor issuance of ‘going concern’ report.

‘Earnings management’ strategies can take many forms, and it is well-known that the managers of publicly traded firms may have incentive to hide information from the investors, especially if their compensation is tied to short-term earnings reports or stock price, and Guttman et al. (2006) provide a rational expectations model of this kind of mis-reporting. Hammersley (2006) shows that it often takes an auditor with specialized, industry specific experience to spot inconsistencies or odd patterns in the financial statements: core expenses can be shifted into other categories as special items, overstating core earning (McVay, 2006); firms can engage in less transparent reporting formats, such as comprehensive income items (Hunton et al., 2006).

A different, and more dynamic perspective on this issue is taken by Geiger and North (2006), which showed that hiring a new CFO leads to lower discretionary accruals in the subsequent financial statements. ‘Cozy’ auditor-client relationships also contribute to audit failures. Empirical evidence suggested that longer audit partner tenure leads to lower audit quality, such as larger abnormal accruals (Carey and Sinnett, 2006), and, at least after evidence of accountant troubles has come to light, switching to a new auditor leads to less abnormal accruals in the reported earnings (Cahan and Zhang, 2006). The “revolving door” phenomenon, i.e. hiring former partners of the current auditor, has a similar impact (Menon and Williams, 2004).

Perhaps a better source of information are the ‘natural experiments’ that have arisen. For example, since 2001 but not before, it has been a legal requirement in the U.S. that both audit and nonaudit fees be disclosed. This regime shift was studied in Francis and Ke (2005), which finds that, post-disclosure, in stark contrast to the pre-disclosure era, conditioning on quarterly earnings surprises, the earnings of firms with higher nonaudit fees are assigned significantly lower market value, at several different lags, than earnings of firms with lower nonaudit fees. They further find that the change is almost entirely driven by firms that have both high nonaudit fees and large accruals. As they note, accruals are “subject to greater

---

7There is an extensive literature on this topic, see e.g. the survey by Francis (2006).
managerial discretion than cash flows and therefore would be viewed (by the market) with more suspicion if their quality were not verified by a credible independent auditor.”

In a related natural experiment, [Carey and Simnett (2006)] study the effect of a regime shift in the auditing rules in Australia. Pre-shift, there was no limit to the tenure of an auditor-client relation, post-shift there was. In the post-shift world, there were several indications that the distribution of judgement calls had shifted against the firms.

1.6. Outline of the Paper. The next section describes the multi-stage game model in more detail. The subsequent section analyzes the equilibria of our model when certifiers are non-strategic. This represents the ideal certifier, who supplies the most reliable information possible without additional incentives. The focus is on the determinants of equilibrium accuracy, the effects of negative macroeconomic shocks to the system, and on the relative effectiveness of the two different penalty structures contained in the Sarbanes-Oxley act. Throughout this and the subsequent section, the only equilibria with multiple auditors useful and active in the market are mixed-pooling equilibria. Throughout, we offer interpretations of these equilibria as the dynamically stable points of perturbed versions of the games.

The penultimate section reprises the analysis for the full model, the one in which auditors have the discretion to alter their reports. In this model, there are multiple equilibria, each corresponding to different levels of accuracy by the auditors. To sharpen the analysis, we focus on the equilibria having the highest industry profits. The last section contains an overview of our conclusions and indications of some directions for future research.

2. Model

We analyze a game with three kinds of players, firms, certifiers, and potential investors. Firms can be in one of two states, denoted \( H \) and \( L \), and only the firm knows which state obtains. There are two certifiers, \( A \) and \( B \), that differ in accuracy and cost, \( A \) being more accurate and more costly. Firms seek to raise capital from the investors by releasing statements about their performance. Certifiers are capable of providing opinions about the statements. Before choosing whether or not to invest, the investors observe the firms’ choice of certifier, the certifiers’ opinion, and the payments made by the firm to the certifier. Based on all of this information, the investors decide whether to make an investment.

2.1. Game Description. The game has the following stages.

1. Firms learn which state they are in, \( H \)igh or \( L \)ow. This is private information known only to the firm. Investors would like to learn the state because it determines/can be identified with, the rate of return to investing in the firm. The proportion of \( H \)-firms is \( \mu \), the proportion of \( L \)-firms is \( 1 - \mu \). \( \mu \) is publicly known, and we think of it as varying across industries and across time.

2. Firms prepare a report about the state they find themselves in, choose a certifier, \( A \) or \( B \), and offer a take-it-or-leave it contract for certification services[

3. The certifiers evaluate the truthfulness of the firms’ reports, a process that is private to the certifiers and which has the potential for errors. The process leads to evidence of the form \( h_A \) or \( l_A \) for \( A \), respectively \( h_B \) or \( l_B \) for \( B \). We interpret \( h_A \) as ‘auditor \( A \)’s evidence

[8]Alternatively, the certifiers could Bertrand compete for business.
supports the firm’s report of $H$,’ and we interpret $l_A$ as ‘auditor $A$’s evidence supports the firm’s report of $L$ or better,’ with similar interpretations for $h_B$ and $l_B$. We assume that $Pr(h_A|H) = Pr(h_B|H) = 1$, that is $H$-firms can generate completely convincing evidence of their true state. We assume that $L$-firms who do not declare “$L$” have some ability to mislead auditors, but that auditor $A$ is better able than $B$ to detect such purposeful misstatements. In terms of error rates, this is $0 < \varepsilon_A := Pr(h_A|L) < \varepsilon_B := Pr(h_B|L) < 1$, and the equivalent formulation in terms of sensitivity/accuracy is $1 > t_A := Pr(l_A|L) > t_B := Pr(l_B|L) > 0$.

4. Certifiers issue their reports, $h_A$ or $l_A$ for $A$, $h_B$ or $l_B$ for $B$. In our analysis of non-strategic certifiers, their report is, by assumption, equal to their evidence. In our analysis of strategic certifiers, they may issue certifications that differ from the internal evidence.

5. The investors observe the firm’s statement, which certifier the firm hired, the payments made to the certifier by the firm, and the certifier’s report. On the basis of this information, investors decide whether to invest in the firm or to take their outside option. The value of the outside option is defined as the market rate of return.

6. After investments and realization of returns, fines, litigation and/or regulatory actions, if any, occur.

2.2. Payoffs. Management payoffs are highest when they are in state $H$ and the investors decide to invest, and lowest when they are in state $L$ and investors decide not to invest. We

---

9We have investigated the model with the other two error rates, $Pr(l_A|H)$ and $Pr(l_B|H)$ strictly positive. These make the algebra significantly messier, and require some extra qualifications for some of the results, but do not change the basic conclusions.
normalize the highest and the lowest payoffs to 1 and 0, and define $a$ as their payoff when they are in state $L$ and receive investment, and $b$ as their payoff when they are in state $H$ and receives no investment. When there are no fines or litigation to be considered, reporting $L$ is weakly dominated for the firm. Deleting this strategy yields the game tree (partially given) in Figure 1.

Certifier payoffs are given by the difference between their equilibrium level of revenue and costs. Certifier $A$’s cost of evaluating a firm’s statement is given by $c_A$, certifier $B$’s cost is $c_B$, the difference is $\Delta_c := c_A - c_B$, and by assumption, $\Delta_c > 0$. Given the small size of auditing costs, less than $1/10$ of one percent of revenues on average, the assumption that management’s gain from access to low cost capital is always larger than $\Delta_c$ is innocuous.

**Assumption 1.** $\min\{a - 0, 1 - b\} \gg \Delta_c$.

We denote the market rate of return by $r$. If investors invest in a firm in state $H$, their rate of return is $r + r_h$, if they invest in a firm in state $L$, their rate of return is $r - r_l$, and both $r_h$ and $r_l$ are strictly positive. Without any information from the certification process and without a credible way for the firms in state $H$ to signal to investors that they are not in state $L$, investors’ expected rate of return from investment would be $\bar{r} := \mu(r + r_h) + (1 - \mu)(r - r_l) = r + (\mu r_h + (1 - \mu)(-r_l))$. Define $\kappa = \frac{\mu r_h}{1 - \mu r_l}$, the ratio of expected gains to expected losses (relative to the market rate of return $r$) from investing without extra information about the firm, and define $\bar{\mu}$ by $\frac{\bar{r}}{1 - \bar{\mu} r} = 1$. We assume throughout that information is necessary for investment.

**Assumption 2.** $r > \bar{r}$, equivalently, $\kappa < 1$ or $\mu < \bar{\mu}$.

Note that, if firm type is observable, investors will only invest in $H$-firms. Without such observability, investors make conjectures over how likely a firm is $H$ based on the certification. In equilibrium, investors may invest in both types of firms with positive probability, and we denote this probability the investment rate.

The net benefit to the $L$ firms to receiving access to low cost capital is $a - 0$. If an $L$ firm hires the accurate certifier and submits a report claiming $H$, their expected utility is therefore $a \cdot \varepsilon_A$, which is large if either $a$ is large, or if the more accurate auditor has a high error rate when companies are trying to mis-lead them. The $H$ firm’s net benefit to receiving access to low cost capital is $1 - b$. This is small if $H$ firms can easily finance projects from internal funds. As we will see in the proof of Theorem 1, the following implies that investor decisions can motivate an $L$ firms to imitate an $H$ firm’s choice of auditor.

**Assumption 3.** $a \varepsilon_A > (1 - b)$.

2.3. Market Equilibria with Certifier Competition. We are interested in equilibria involving competition between certifiers who are accurate enough that investors are willing to act on their approval of positive reports.

**Definition 1.** A market equilibrium with certifier competition is an equilibrium in which

(a) both certifiers have clients, and
(b) both have sufficiently reliable recommendations that investors earn the market rate of return, \( r \), after investing in a firm with a positive report from either certifier.

Receiving the same expected return after seeing a positive report from either auditor does not imply that the investors react to both reports in the same way. Our first result below, Theorem 1(a), shows that investors are more likely to invest after certifier \( A \)’s positive report, and this result carries through the entire analysis of this and the subsequent section with strategic auditors.

Crucial to characterizing the equilibria will be the proportions of auditor \( A \)’s and auditor \( B \)’s clientele that are \( H \) firms. Define

\[
\theta_{h_A} = \frac{\varepsilon_A r_l}{r_h + \varepsilon_A r_l} \quad \text{and} \quad \theta_{h_B} = \frac{\varepsilon_B r_l}{r_h + \varepsilon_B r_l}.
\]

(1)

If more (less) than \( \theta_{h_A} \) of \( A \)’s clientele is from \( H \) firms, then investors earn more (less) than the market rate of return if they invest after learning \( h_A \), with the parallel definition for auditor \( B \). Since certifier \( A \) is more accurate than certifier \( B \), certifier \( B \) is more accurate than no information at all, and information is needed for investment, we have \( 0 < \theta_{h_A} < \theta_{h_B} < \mu \). These provide the boundaries between different types of equilibria.

i. \( \mu < \theta_{h_A} \) — this corresponds to an industry in such poor shape that there is no investment in any equilibrium of the model.

ii. \( \theta_{h_A} < \mu < \theta_{h_B} \) — this corresponds to an industry in which only market equilibria with certifier competition are possible.

iii. \( \theta_{h_B} < \mu < \mu \) — this corresponds to an industry in such good shape that investors will, in all equilibria of the model, make more than the market rate of return, and there is only one auditor with any clients.

3. Nonstrategic Certifiers

Nonstrategic certifiers issue certifications that are consistent with their assessment, simplifying the game tree to that in Figure 2. As noted above, this corresponds to the proposal in Ronen (2010) that firms purchase “financial statement insurance” because this would put the auditors in a position like that of a bank’s in-house auditors. After discussing some of the essential aspects of mixed strategy equilibria and their interpretation, we give some of the basic properties of our strictly mixed equilibrium: the determinants of equilibrium accuracy; and equilibrium investment levels. We also address two policy design questions.

i. Response to negative macro shocks: If there is a strong and negative macro shock, which we interpret as a large decrease in the proportion of firms that are in the \( H \) state, what would need to happen to certification standards, such as auditing standards, or CRA regulations, in order to restore investor confidence and re-start investment? We find that, loosening standards, which we understand as reductions in certifiers’ ability to find mis-statements by the operating firms, increases the distance from the situations with positive investment.

---

10 If \( \theta \) of certifier \( A \)’s business is from \( H \) firms, then \( \beta := \frac{\theta}{\theta + \varepsilon_A (1 - \theta)} \) is the proportion of firms receiving \( h_A \) that are actually in state \( H \). If \( \beta \) satisfies \( \beta r_h + (1 - \beta) (-r_l) = 0 \), then investors are exactly indifferent between investing and not after \( h_A \). Solving yields \( \theta_{h_A} \) with parallel calculations for \( \theta_{h_B} \).
ii. **Certifier versus firm fines:** In case of misreporting by the certifiers, should fines be levied on the certifiers, as in many recent policy proposals, or on firm management? We find that, at least as currently done, levying fines on auditors has no effect on firm behavior and equilibrium accuracy, and reduces investment levels. By contrast, fines on management for mis-statements increase equilibrium accuracy, and often increases investment levels.

3.1. **Mixed-pooling Equilibria.** Our model will not have any interesting pure strategy equilibria. Here we provide some intuition for this result as well as our interpretations of the indifference conditions that characterize the mixed equilibria.

If, for example, all of the $H$ firms went to auditor $A$ and all the $L$ firms to auditor $B$, then savvy investors would understand the choice of auditor $A$ a perfect indicator of a higher-than-market rate of return. This would lead the $L$ firms to incur the small additional cost, $\Delta_c$, on the chance that they too would receive a positive report from $A$, and this process could continue until auditor $B$ has no clientelle. However, at some point before that, $H$ firms would wish to switch auditors so as to save $\Delta_c$ while simultaneously separating themselves from the $L$ firms.

Market equilibrium with certifier competition happens when these two tendencies just balance: the $H$ firms must be indifferent between incurring $\Delta_c$ and not incurring it; and the $L$ firms must be indifferent between incurring $\Delta_c$ while having a small chance of their misleading statements being accepted and saving $\Delta_c$ while having a larger chance. These can only be accommodated if there is investment forthcoming after a positive report from $B$, but more investment forthcoming from a positive report from $A$. This in turn requires that investors’ tendencies to invest and not invest in the two firms just balance.
3.2. Determinants of Investment Levels and Accuracy. An equilibrium for this game is a 4-tuple, $(\sigma^*_H, \sigma^*_L, p^*_A, p^*_B)$, giving the proportion, $\sigma^*_H$, of $H$ firms that choose certifier $A$, the proportion, $\sigma^*_L$, of $L$ firms that choose certifier $A$, the investment rate, $p^*_A$, after a positive report by certifier $A$, and the investment rate, $p^*_B$, after a positive report by certifier $B$. Recall that $\kappa = \frac{\mu_s - \mu_l}{1 - \mu_l}$ is the ratio of expected gains to expected losses relative to the market rate of return $r$, and that $\kappa < 1$ is equivalent to information being necessary for investors to choose to invest, and smaller values of $\kappa$ correspond to investors needing more accurate positive signals in order to be willing to invest.

**Theorem 1.** The market equilibrium with certifier competition between non-strategic certifiers is strictly mixed, can only happen if $\mu$ satisfies $\theta_h A < \mu < \theta_h B$, and has the following properties:

(a) investors treat certifier $A$ as a more reliable guide to investment, $p^*_A > p^*_B$;

(b) the investment rates, $p^*_A$, $p^*_B$, as well as $(p^*_A - p^*_B)$, are all higher for higher values of $\Delta_c$;

(c) investment rates are higher for higher values of $\varepsilon_A$ and for lower values of $\varepsilon_B$; and

(d) equilibrium sensitivity to false filings, $[\sigma^*_L + (1 - \sigma^*_L)t_B]$, is equal to $(1 - \kappa)$, independent of the certifier accuracy rates.

Proofs can be found in the appendix, here we sketch some of the intuitions for the results, and Lemma 1 (in §4.3) characterizes the range of parameters for which a market equilibrium with certifier competition exists.

As noted above, we understand investment rates as the proportion of investors that read the firm’s report and auditor certification as an indicator that investment in the firm is the correct decision. In this context, 1(a) and (b) tell us that the investors read the willingness of a firm to submit to a more costly audit procedure as a signal that they are a better investment. On the other hand, 1(c) implies that the differential strength of the statistical test that the firms submit to, $(\varepsilon_B - \varepsilon_A)$, has the opposite effect. When $(\varepsilon_B - \varepsilon_A)$ is larger, there is more incentive/temptation for the $L$ firms to go to auditor $B$, and the only way to balance this force is for a larger proportion of the investors to treat auditor $B$’s recommendation, $h_B$, as not quite being enough to induce investment. Balancing this requires that more of the investors also treat $h_A$ as not quite being enough to induce investment.

An important measure of the functioning of the certification market is the equilibrium accuracy rate, $[\sigma^*_L + (1 - \sigma^*_L)t_B]$, the probability of the audited statements truthfully revealing an $L$ firm’s type. One might believe that encouraging the weaker auditor to adopt the known standards and practices of the stronger auditor would increase overall accuracy in the certification market. However, 1(d) is a strong neutrality result telling us that this effect is completely washed out by a countervailing effect — as the weaker auditor becomes more accurate, it becomes more similar to the strong auditor and more business flows to it. It is the distribution of profits and losses and equilibrium market structure that determine equilibrium accuracy, not certifier accuracy.

3.3. Negative Macro Shocks. The policy response to the latest downturn in investor confidence in the accuracy of balance sheet statements has been a loosening of auditing
standards.\textsuperscript{11} To the extent that the value of other investor options, captured by \( r \), did not fall as much as the value of investing in the operating firms, captured by \( r + r_h, r - r_l \) and \( \mu \), our model suggests that this policy, decreasing standards so that more firms have a positive rating, has exactly the opposite effect.

We model a negative macro shock as being either a decrease in \( \mu \), or as a decrease in \( \frac{r_h}{r_l} \), which increases both \( \theta_{hA} \) and \( \theta_{hB} \). It is a severe shock if it is sufficiently large to move to a situation with \( \mu' < \theta_{hA}' < \theta_{hB}' \). However such a change happens, it is disastrous for investment levels, as is the policy response of loosening auditing standards, understood as increasing either \( \varepsilon_A \) or \( \varepsilon_B \) or both so that more firms receive positive ratings.

**Corollary 1.1.** If \( \theta_{hA} < \mu < \theta_{hB} \) changes to \( \mu' < \theta_{hA}' < \theta_{hB}' \), investment drops to 0. If \( \mu' < \theta_{hA}' < \theta_{hB}' \), then increasing audit standards so that \( \varepsilon_A \) decreases enough restores investment, and simultaneously decreasing \( \varepsilon_B \) restores more investment. If \( \varepsilon_A \) and/or \( \varepsilon_B \) are increased, the distance between \( \mu' \) and \( (\theta_{hA}', \theta_{hB}') \) increases.

Essentially, looser standards lead to more errors in certification, giving low quality balance sheets higher “official” rating. Savvy investors understand this, they know that the official rating is now less trustworthy. This means that firms that are in good shape have difficulty accessing capital at appropriate prices, which further worsens the macroeconomic situation.\textsuperscript{12}

3.4. Sarbanes-Oxley Fines. The Sarbanes-Oxley Act established the PCAOB (Public Company Accounting Oversight Board) to encourage better auditor behavior, and it also established penalties for management mis-statements, knowing or unknowing, on financial filing with the SEC. Our conclusion is that penalties levied on auditors, at least in the present form, have no effect on the equilibrium accuracy achieved by the certification market and decrease investment rates. By contrast, penalties for management mis-statements have positive effects on equilibrium accuracy and increases investment rates if there is a significant effect on investor returns.

3.4.1. Fines on Auditors. At present, if the PCAOB finds that an auditor has problems with its audits, there are provisions in Sarbanes-Oxley to protect the image and reputation of the auditor: the identity of the auditor and the existence of problems is to be kept confidential unless the auditor fails to fix the problems during the next year. If not fixed during the year, the identity is to be kept confidential for the period of an appeal to the PCAOB if the auditor chooses to appeal. If an auditor’s appeal to the PCAOB fails, the identity is to

\textsuperscript{11} Section 132 of the *Emergency Economic Stabilization Act of 2008* was titled “Authority to Suspend Mark-to-Market Accounting.” It restated the SEC’s authority to suspend the application of mark-to-market accounting if the SEC determines that it is in the public interest and protects investors. As Bloomberg reported on March 29, 2009, “Four days after U.S. lawmakers berated Financial Accounting Standards Board Chairman Robert Herz and threatened to take rulemaking out of his hands, FASB proposed an overhaul of fair-value (mark-to-market) accounting that may improve profits at banks such as Citigroup Inc. by more than 20 percent.” By early April 2009, the FASB implemented this overhaul.

\textsuperscript{12} To put it another way, when trust is short, investors put their trust in cash. As Arthur Levitt, former chair of the SEC asserted in a September 1997 meeting of the Inter-American Development Bank, “high quality accounting standards ... improve liquidity (and) reduce capital costs.” In a different context (the Economic Club of Washington, April 2000), he observed that “quality information is the lifeblood of strong, vibrant markets. Without it, liquidity dries up. Fair and efficient markets simply cease to exist.”
be kept confidential for the period of the auditor’s appeal to the SEC. If that appeal fails, the auditor’s identity and some summary statistics about their audits is revealed. So far, 41 months is the length of the delay between the PCAOB noting problems and release of this information to investors, and the release has not included the identity of the firms whose mis-statements were mis-audited. We think of the opportunity cost of auditor management time and lawyers as part of the fines that are imposed on an auditor, but since auditor reputation is a potent dissuader of litigation, this is only part of the fines.

In terms of the model, if an expected fine of $F_{aud}$ is imposed on auditor $A$ when they give an $h$ report to an $L$ firm, then the expected liability is $F_A = (1 - \theta_A)\varepsilon_A F_{aud}$ because $(1 - \theta_A)$ is the equilibrium proportion of their clientele that is mis-stating and $\varepsilon_A$ is their error rate. By the same logic, auditor $B$’s expected fine is $F_B = (1 - \theta_B)\varepsilon_B F_{aud}$.

We shall see below that $F_B > F_A$, that is, the expected fines are higher for the less accurate auditor. If $(F_B - F_A) > \Delta_c$, then fines are so high that they reverse the cost ranking between the two auditors, leaving auditor $B$ less accurate, more costly, and out of business. The equilibrium with auditor fines is a 4-tuple, $(\sigma_{aud}^A, \sigma_{aud}^L, p_{aud}^A, p_{aud}^B)$.

**Corollary 1.2.** If $\Delta_c > (F_B - F_A)$ and $(\sigma_{aud}^A, \sigma_{aud}^L, p_{aud}^A, p_{aud}^B)$ is a market equilibrium with certifier competition, then

(a) auditor fines are more expensive for the weaker auditor,
(b) fines on auditors have no effect on $\sigma_{aud}^A$ and $\sigma_{aud}^L$, the behavior of the operating firms, and
(c) investment levels, $p_{aud}^A$ and $p_{aud}^B$, are lower for higher auditor fines.

The essential intuition is seen directly in the proof. It shows that, since fines are more costly for the lower quality auditors, they lower the cost difference between the auditors. This reduces the signaling value of the willingness to submit one’s financial statement to the more accurate auditor.

Auditing services are demanded inelastically as they are a requirement for publicly traded companies. There is a 100% pass-through of (expected) certifier fines to their clients as a fixed cost, and increases in these fixed costs have no effect on firm behavior. Corollary 1.2(c) shows that the investors act as if they are aware that some of their money is being used to pay fines. Corollary 1.2(a) tells us that fines act as a tax on the least accurate auditor, and one might expect that this would lead to incentives for the less accurate to become more accurate, hence to increase the overall accuracy of the certification market. However, as we saw in Theorem 1(d), there is a strong countervailing effect — as the weaker auditor becomes more accurate, it becomes more similar to the strong auditor and more business flows to it.

### 3.4.2. Fines on Management

Under the part of Sarbanes-Oxley act that deals with management behavior, there are penalties for management mis-statements on financial filing with the SEC, significantly stiffer if the mis-statements are knowing rather than unknowing. Unlike the treatment of auditor errors, such fines allow investors to identify the firms. This has a profoundly different effect on the equilibrium.

Recall that without fines to mis-leading financial statements, for $L$ firms, telling the truth is a weakly dominated strategy. So long as mis-statement exists as an equilibrium phenomenon, the fines are not large enough to overturn the equilibrium that we study. We conduct this analysis under two assumptions about the incidence of the fines, $F_{mgt}$. 
1. We assume that management suffers when fines are imposed. This would not hold if the fines and other legal expenses are paid from company, i.e. shareholder, funds. Specifically, we are assuming that the payoff to the \(H\) firms are unchanged, while the payoff to the \(L\) firm going to auditor \(A\) is \(\varepsilon_A p_A a' - \Delta_c\), while going to auditor \(B\) gives them \(\varepsilon_B p_B a'\) where \(a' := (a - F_{\text{mgt}})\).

2. We also assume that investors payoffs are lower if there are fines, reflecting the identifiability of the firm in which mis-statements happen. Specifically, we assume that the amount the investors lose increases with \(F_{\text{mgt}}\) — if the investors have invested in a \(L\) firm, they receive \(r - r'_l\) where \(r'_l := (r_l + \gamma \cdot F_{\text{mgt}}) > r_l, \gamma > 0\). The penalty term has two parts, the \(\gamma\) and the \(F_{\text{mgt}}\). We think of \(\gamma\) as the multiplier determining the hit to the stock price when fines of size \(F_{\text{mgt}}\) are imposed.

Up to a very high point, equilibrium accuracy is increasing in fines of this kind. If \(F_{\text{mgt}}\) is sufficiently high that it wipes out the extra profits of access to low cost capital, then mis-representation becomes a dominated strategy. We think it is unlikely that fines levied on firm management will ever be that high, and study the remaining range. An equilibrium with management fines is a 4-tuple, \((\sigma_{HA}^{\text{mgt}}, \sigma_{LA}^{\text{mgt}}, p_{mgt}^A, p_{mgt}^B)\).

**Corollary 1.3.** If \((\sigma_{HA}^{\text{mgt}}, \sigma_{LA}^{\text{mgt}}, p_{mgt}^A, p_{mgt}^B)\) is a market equilibrium with certifier competition, then

(a) the stronger auditor receives more business and the weaker auditor receives less when fines are higher,

(b) investment levels, \(p_{mgt}^A\) and \(p_{mgt}^B\), are lower for higher fines,

(c) certifier market accuracy is higher when fines are higher, and

(d) for higher values of the multiplier, \(\gamma\), the investment level for the stronger auditor’s clients is higher for higher fines.

In terms of our population interpretations of these mixed equilibria, the \(L\) firms right at the boundary in their preferences between their auditors begin to slightly favor \(A\) because it saves them from the worst results of their own temptation to try to pass off a mis-leading statement. This in turn makes the certifier market more accurate, leading more of the investors right at the investment boundary to take the plunge.

### 4. Auditors and Judgement Calls

In this section, we investigate our full model. Auditors, after having done due diligence, see their own signal indicating that the firms have supported their position, or that they have not. At this point, the auditor must decide on what to allow the firms to report as having been supported — that high claims have been supported, or that low claims have been supported.

#### 4.1. Overview of the Issues

The investors are fully aware that auditors may be compromised. This provides limits on the range of auditor discretion that is compatible with reports being reliable enough to induce investment. We study only equilibria with this baseline level of reliability, believing that it is a precondition for auditors to stay in business.

Explicit dependence of auditor payments on the report is illegal. An implicit contract between the auditor and the firm, perhaps in the form of extra consulting work and the
associated revenues in the form of nonaudit fees, when the report is positive, is not illegal, is widespread, and has two advantages. First, it brings the experience of professional auditing firms to bear on firm problems, potentially leading to improvements in firm performance. Second, positive reports contain risk for the accounting firm that negative/uninformative reports do not, either in terms of fines or reputational costs. Without reward for a good report, auditors’ motivation is to produce uninformative statements about the firm’s claims.

Implicit contracts also have disadvantages. Since 2001, it has been a legal requirement that both audit and nonaudit fees be disclosed. [Francis and Ke (2006)] study the effect of this regime shift and find that post-disclosure, in stark contrast to the pre-disclosure era, conditioning on quarterly earnings surprises, the earnings of firms with higher nonaudit fees are assigned lower market value, at several different lags, than earnings of firms with lower nonaudit fees. They further find that the change is almost entirely driven by firms that have both high nonaudit fees and large accruals. As they note, accruals are “subject to greater managerial discretion than cash flows and therefore would be viewed (by the market) with more suspicion if their quality were not verified by a credible independent auditor.” Further, as discussed above, these kinds of disadvantages can grow as the tenure grows.

4.2. Implicit Contracts and Our Model. An implicit contract specifies what the auditor will be paid in each of the contingencies that arise. For each auditor there are three possibilities, internal signals of \( l \) coupled with support of either \( l \) or \( h \) as a filed statement, and internal signals of \( h \) coupled with support of \( h \) as a filed statement. We suppose here, as we have throughout, that investors are informed and savvy, that any payments, including extra consulting fees/business, are observed by the investors, and that they draw the appropriate conclusions from what they observe.

The first case we analyze has no liability or reputation concern for mistaken reports, essentially giving us a model of credit rating agencies — it is settled law that their ratings have stringent 1’st Amendment protection as “commercial speech.” In this case, there is no cost difference between any of the three contingencies and competition drives the implicit contracts to be constant at the costs of the auditors.

When there are either liability or reputation considerations for the mistaken support of an \( h \) filing, the certifiers must be paid more, for support of an \( h \) than for support of an \( l \). If not, certifiers’ best response is “defensive,” by degrading the informational content of the certification market to 0. Also, since investors will observe if there is any difference between the “see \( h \), support \( h \)” and “see \( l \), support \( h \)” payments, the only credible way to support \( h \) filings (that are perhaps on the boundary) is to have the same contingent fees. Revelation of auditor internal signals is not part of any equilibrium in which both auditors are in business. Therefore, the implicit payments are of the form \((w_{hA}, w_{lA})\) for auditor \( A \) and of the form \((w_{hB}, w_{lB})\) for auditor \( B \), they depend only on the type of the filing that is supported.

The addition of implicit contracts and additional choices by the auditors complicates the analysis of market equilibria with certifier competition, but much of the qualitative content of the results in the previous section goes through. The main difference is that there exists an \( \varepsilon_0 > \kappa \), and we have different kinds of mis-reporting depending on whether \( \varepsilon_B < \varepsilon_0 \) or \( \varepsilon_B > \varepsilon_0 \). If \( \varepsilon_B > \varepsilon_0 \), then only the stronger auditor ever mis-reports (the close cases), if \( \varepsilon_B < \varepsilon_0 \), then both mis-report. In both regions, there are multiple mixed strategy equilibria, and competition drives the the auditors to be indifferent between them. We focus on those
equilibria most preferred by the firms buying the certification. In our interpretation of mixed equilibria, a certifier that is randomizing is paying attention to minor differences in the cases, they are making a close and difficult judgement call about whether a case is above or below the boundary of acceptability.

The next subsection gives the basics of the equilibrium results, including the determinants of investment levels and the equilibrium accuracy of auditor reports. The last two subsections turn to the two policy design questions considered above with non-strategic auditors: should auditor standards be increased or decreased after a strong macro shock has lowered investment? and should fines/penalties for mis-statements on audited statements be levied on auditors or firms? The answers are mostly the same as in the non-strategic case considered above, with one important difference.

i. Lowering auditor standards after a strong macro shock is counter-productive, only an increase in standards can restore investor confidence enough to induce investment. The difference is that now there is some slack in the system — if the auditors toughen up their judgement calls, they can increase standards.

ii. Penalties for auditor mistakes are passed through to the operating firms as a fixed cost, they do not affect firm behavior nor equilibrium accuracy, and they lower investment rates. Management penalties for firm mis-statements change firm behavior and equilibrium accuracy for the better. The difference is that now, in the range of equilibria most preferred by the firms buying the certification, management penalties do not increase investment rates despite the higher equilibrium accuracy.

4.3. Equilibrium, Investment, and Accuracy. Strategic reporting is equivalent to a stochastic transformation (a.k.a. a Markov scramble) of the auditors’ internal signals (Blackwell 1953). As we are assuming that $H$ firms can generate completely convincing evidence of their true state and would not be willing to settle for anything less than $h_A$ or $h_B$, such transformations are equivalent to $A$ (respectively $B$) increasing their error rate $\varepsilon_A$ to some $\varepsilon'_A$ in the interval $[\varepsilon_A, 1]$ (respectively increasing $\varepsilon_B$ to some $\varepsilon'_B$ in the interval $[\varepsilon_B, 1]$). Thus, an equilibrium is a 6-tuple, $(\sigma^-_{HA}, \sigma^-_{LA}, p^*_A, p^*_B, \varepsilon^*_A, \varepsilon^*_B)$.

If one invests without extra information, one receives $r + r_h$ with probability $\mu$ and $r - r_l$ with probability $(1 - \mu)$. As before, let $\kappa = \frac{\mu - r_h}{1 - \mu}$ be the ratio of expected gains to expected losses (relative to the market rate of return $r$) from investing without extra information about the firm. The following result will be crucial below.

**Lemma 1.** Every $(\varepsilon_A, \varepsilon_B)$ in the set

$$S := \{ (\varepsilon_A, \varepsilon_B) : \varepsilon_B > \kappa, \frac{1-b}{a} < \varepsilon_A < \min \left[ \kappa, \frac{\Delta_a}{a} + \varepsilon_B \left(1 - \frac{\Delta_b}{1-b}\right) \right] \}$$

is consistent with a market equilibrium with certifier competition. $S$ is non-empty iff $\kappa > \frac{1-b}{a}$, and when $S$ is non-empty, industry profits are maximal at the points in $S$ where $\varepsilon_A = \kappa$.

---

Evidence of a drift of auditors’ judgement calls toward the distribution of calls more favored by the buyers of certification can be seen in Carey and Simnett (2006) who study the effect of a regime shift in the auditing rules in Australia. Pre-shift, there was no limit to the tenure of an auditor-client relation, post-shift there was. In the post-shift world, there were several indications that the distribution of judgement calls had shifted against the firms.
Note that, because there is neither liability nor reputation concerns in this part of the analysis, the equilibrium implicit contract is exactly the same as the contracts analyzed in Theorem 1. One then examines the conditions that guarantee the existence of a market equilibrium with certifier competition, and show that the set $S$ is the answer. Further, as noted above, stochastic transformations can increase error rates, not decrease them, so for any point $(\varepsilon_A, \varepsilon_B) \in S$, we look to maximize industry profits over points that are up and to the left in Figure 3. Now, by Theorem 1(c), industry profits are increasing in $\varepsilon_A$ and decreasing in $\varepsilon_B$. This means that there are two possibilities to analyze. Let $\varepsilon_0$ denote the value of $\varepsilon_B$ that solves $\kappa = A - \varepsilon_B (1 - \frac{A}{1-b})$, i.e. $\varepsilon_0 = \frac{\kappa - \Delta_c}{\Delta_c - 1 - \frac{1}{\Delta_c}}$. If $\varepsilon_B > \varepsilon_0$, then industry profits are maximized by auditor $A$ increasing their error rate to $\kappa$ and auditor $B$ engaging in no judgement calls. By contrast, if $\varepsilon_B < \varepsilon_0$, then industry profits are maximized by auditor $A$ increasing their error rate to $\kappa$ and auditor $B$ increasing their error rate to $\varepsilon_0$. This is because moving along the line segment $tu$ increases profits by increasing $\varepsilon_A$ enough to outweigh the losses that arise from increasing $\varepsilon_B$.

We believe that the focus should be on the firm optimal equilibria, that is, the equilibria most preferred by the firms buying the certification. By the last part of Lemma 1 these are the ones with the highest error rate, $\kappa$, for auditor $A$. The problem is that at the exact top end of $S$, the line $uv$, only one certifier is in business. We therefore state the results in terms of all equilibria sufficiently close to $uv$.

We think of these as representing a situation in which the stronger auditor is nearly a monopolist, though the operating firms still have available a credible alternative source of auditing services. It is the existence of this credible, lower cost alternative auditing firm that provides a lower bound for the accuracy of the larger, stronger auditor. The results of Theorem 1(b) and (c) are changed when certifiers are strategic.

**Theorem 2.** If $(\sigma_{HA}^*, \sigma_{LA}^*, p_A^*, p_B^*, \varepsilon_A^*, \varepsilon_B^*)$ is a firm optimal market equilibrium with certifier competition between strategic certifiers, then:

(a) investors treat certifier $A$ as a more reliable guide to investment, $p_A^* > p_B^*$.
(b) if \( \varepsilon_B < \varepsilon_o \), then increases in \( \Delta_c \) have no effect on the investment rate, but if \( \varepsilon_B > \varepsilon_o \), then increases in \( \Delta_c \) increase the investment rate for the industry;

(c) changes in \( \varepsilon_A \) have no effect on investment rates, if \( \varepsilon_B < \varepsilon_o \), then changes in \( \varepsilon_B \) also have no effect on investment rates, but if \( \varepsilon_B > \varepsilon_o \), then increases in \( \varepsilon_B \) decrease investment; and

(d) equilibrium sensitivity to false filings, \( [\sigma_{LA}^\ast t_A + (1 - \sigma_{LA}^\ast) t_B] \), is equal to \( (1 - \kappa) \).

2(a) tells us that strategic behavior of auditors does not change the investor ranking of the investment advice provided by the auditors, even when it is only the stronger auditor that engages in judgement calls. 2(b) reprises the result that willingness to submit to a more costly screening procedure raises investment rates. 2(d) tells us that even with strategic mis-reporting, the market equilibrium does just as good a job at detecting false statements as the non-strategic market structure. Essentially, shifting the higher error rates to higher values shifts the market shares in a counter-vailing fashion that wipes out the effect of the higher error rates.

4.4. Negative Macro Shocks. As before, if macro conditions suffer an adverse shock, modeled as a decrease in \( \mu \) or in \( \tau_h \), only an increase in audit standards can restore investment. Here there is slack in the system — by doing less strategic mis-reporting, the auditors themselves can effect this change. Recall that \( \kappa = \frac{\mu - \tau_h}{1 - \mu \tau_l} \).

Corollary 2.1. If \( \kappa > \frac{1 - b}{a} \) drops to \( \kappa' \in (\frac{1 - b}{a}, \kappa) \), then auditor increases in the strictness of their reviews can partially restore investment. If \( \kappa \) drops below \( \frac{1 - b}{a} \), investment goes to 0 and no changes in auditor behavior or accuracy can restore it.

4.5. Structure of Fines. We re-examine the two fine structures: fining auditors for not detecting or going along with management mis-statements; and fining management for mis-statements.

4.5.1. Fines on Auditors. When auditors are subject to penalties, \( F_{aud} \), be they litigation or other expenses, competition implies that the value of the fines will be passed on to their client firms through higher auditing fees. We denote the implicit contracts as \((w_{hA}, w_{lA})\) for auditor \( A \), and \((w_{hB}, w_{lB})\) for auditor \( B \). An equilibrium with implicit contracts is a 10-tuple, \(((\sigma_{HA}^\ast, \sigma_{LA}^\ast, p_A^\ast, p_B^\ast, \varepsilon_A^\ast, \varepsilon_B^\ast) : (w_{hA}^\ast, w_{lA}^\ast), (w_{hB}^\ast, w_{lB}^\ast))\). Aside from the complexity of the proof, the only change in Corollary 1.2 that happens is a slight qualification of part (c), the result on investment levels.

Corollary 2.2. In any market equilibrium with implicit contracts and auditor fines, the set of equilibria in Lemma 1 changes to the set \( S' \) in Figure 4 from the set \( S \) in Figure 3; and in any firm optimal market equilibrium the

(a) auditor fines are more expensive for the weaker auditor,

(b) fines on auditors have no effect on the behavior of the operating firms, and

(c) there is an increasing linear function \( F_{aud} \mapsto \varepsilon_o(F_{aud}) \) such that, if \( \varepsilon_B < \varepsilon_o(F_{aud}) \) then increases of the level of fines from \( F_{aud} \) lower investment levels, but if \( \varepsilon_B > \varepsilon_o(F_{aud}) \), there is no effect.
4.5.2. **Fines on Management.** We make the same assumptions on payoffs as in §3.4.2: payoff to the \( \text{H} \) firms are unchanged, while the payoff to the \( \text{L} \) firm going to auditor \( A \) is \( \varepsilon_{\text{AP}}Aa' - \Delta_c \), and that of going to auditor \( B \) is \( \varepsilon_{\text{BP}}Ba' \) where \( a' := (a - F_{\text{mgt}}) \); and if the investors have invested in a \( \text{L} \) firm, they receive \( r - r'_l \) where \( r'_l := (r_l + \gamma \cdot F_{\text{mgt}}) > r_l \). The contrast of the following with Corollary 1.3 is informative: because we consider the firm optimal market equilibria, there is no role for \( F_{\text{mgt}} \) in (a); the certifier accuracy result, (b); is unchanged; and the potential for a positive dependence on investment on fines in (c) is reversed.

**Corollary 2.3.** In any firm optimal market equilibrium with implicit contracts and management fines, when fines are higher

(a) the stronger auditor receives more business,

(b) certifier market accuracy is higher, and

(c) investment levels are lower.

5. **Summary and Future Directions**

This paper examined the role of regulation determining the quality of information produced by certifiers in a competitive market. Both as a benchmark model and as a model reflecting recent policy proposals, we began with non-strategic certifiers, and then turned to the case of strategic certifiers.

The mistakes of non-strategic certifiers are due only to limitations of auditing technology. In this case, market equilibrium accuracy is produced by a combination of firms’ choice of certifiers, which is in turn influenced by the certifiers’ abilities and costs. We found that

\[ w_{h_A}, w_{l_A} = (\Delta_c, \Delta_c) \text{ and } w_{h_B}, w_{l_B} = (0, 0), \]

\[ w_{h_A}, w_{l_A} = (\Delta_c + F_A, \Delta_c + F_A) \text{ and } w_{h_B}, w_{l_B} = (F_B, F_B). \]
penalties levied on certifiers have no effects on firm behavior, essentially because such penalties are passed through, as a fixed cost of doing business, to the firms being audited. However, the presence of such fees effectively reduces investor confidence in the certification process. By contrast, provided it is the decision makers, management and not the shareholders, who pay, penalties levied on firms for their mis-statement have a strong positive effect on the certification market. A useful auxiliary result is that decreasing certifier accuracy is a bad policy response to market downturns. With only relatively minor caveats, these result carry through to the case of strategic certifiers when we focus attention on the equilibria that are optimal for the firms buying the certification.

While this study improves our understanding of the impact of litigation and competition on auditors’ reporting strategy, several other interesting questions remain unanswered and are worth further investigation. Our paper focuses on the immediate effect of audit result on investment decisions and assumes that the firm’s stock price remains constant during this process. It would be interesting to endogenize pricing of stocks and consider how prices adjust as investors receive various indicators about the performance of firms. One basis for such adjustment is the level of audit fees that firms incur. [Huang et al.] (2010) examined a sample of litigation firms as well as non-litigation firms and found that firms with higher audit fees also have higher chance of be sued later on. Recognizing such association, investors will shift their investment towards firms with lower audit fees, which results in price decline for firms with higher audit fees. Auditors’ approval and disapproval of firms’ statement could be another source of price adjustment. From the firms’ perspective, they would be interested in knowing what their stock prices will be if litigation becomes stronger or weaker, or if there are more or less auditors to choose from. This extension can potentially address these questions.

A key element of this model is auditors’ ability to degrade their signals through Markov scrambles. In reality, such degrading are often realized by auditors approving financial statements containing vague information, a process sometimes called “defensive auditing.” After collecting evidence of their clients’ performance, auditors could release only a subset to the investors, or refer to the evidence that they gathered in a tangential fashion designed to protect themselves if litigation should arise. For example, after intentionally forgoing further investigation claims of large increase in sales growth across months, they could de-emphasize this result, but still include it, without noting that it could be due to revenue shifting.

What is fascinating about this vagueness is that investors with different experience and knowledge may derive different messages from the same information. Ordinary investors may take the financial statement and the auditor’s report at the face value, while more sophisticated investors, e.g. the forensic accountants at hedge funds, will look beyond the evidence supplied and search for reasons why other potential evidence is not included. Similar strategic behavior is shown by [Milgrom] (2008) in a setting where a seller tries to persuade a buyer by performing tests of their products. When the seller knows the potential outcomes of different tests, they have incentive to selectively run certain tests and reveal only “good news.” Extending to this direction can generate more thorough understanding of auditors’ strategic reactions to litigation and competition.

A separate extension is to more explicitly consider, in the tradition of financial economics, both how managers make decisions on behalf of the firms and how the interests of extant
shareholders may differ from the interests of new investors. As stakeholders, managers have
their own interest, and it often deviates from that of both the extant and potential future
shareholders in the firm. The literature suggests that management may forgo profitable
investment opportunities for their payoff concerns (Scharfstein and Stein, 1990). Such con-
cerns can also influence firms’ decisions to issuing debt of different seniorities, and hence
their willingness to reveal their true type through the certifiers.

References

Banker, Rajiv D., Hsihui Chang, Ram Natarajan. 2005. Productivity change, technical
progress, and relative efficiency change in the public accounting industry. *Management
Science* **51**(2) 291–304.

Becker, Bo, Todd Milbourn. 2011. How did increased competition affect credit ratings?

35–43.

Bernheim, B. Douglas, Michael D. Whinston. 1998. Incomplete contracts and strategic

212–223.

265–272.

of Finance* **LXVII**(1) 85–111.

Boot, Arnoud W. A., Todd T. Milbourn, Anjolein Schmeits. 2006. Credit ratings as coordi-

Cahan, Steven F., Wei Zhang. 2006. Auditor litigation and modified reporting

Carey, Peter, Roger Simnett. 2006. Audit partner tenure and audit quality. *The Accounting

Choi, Jong-Hag, Jeong-Bon Kim, Xiaohong Liu, Dan A. Simunic. 2009. Cross-listing audit

Corbae, Dean, Maxwell B. Stinchcombe, Juraj Zeman. 2009. *An Introduction to Mathe-
matical Analysis for Economic Theory and Econometrics*. Princeton University Press,
Princeton, NJ.

Corona, Carlos, Ramandeep S. Randhawa. 2010. The auditor’s slippery slope: An analysis

325–365.


Heron, Randall A., Erik Lie. 2009. What fraction of stock option grants to top executives have been backdated or manipulated? *Management Science* **55**(4) 513–525.


APPENDIX A. PROOFS OF THEOREM 1 AND ITS COROLLARIES

Proof of Theorem 1. The first observation is that $H$-firms choosing different certifiers than $L$-firms cannot be part of a market equilibrium with certifier competition. If certifier choice is a perfect signal of the type of firm, investors can make higher than the market rate of return by directing their capital toward $H$-firms only. By Assumption 1, this means that $L$-firms will benefit from deviating and imitating $H$-firms’ certifier choice. Thus, equilibrium with competing certifiers requires both firm to choose both certifiers with positive probability.

Firm $L$’s and firm $H$’s indifference conditions are

$$\varepsilon_A p_A a - \Delta_c = \varepsilon_B p_B a, \quad \text{and}$$

$$p_A + (1 - p_A) b - \Delta_c = p_B + (1 - p_B) b.$$  \hfill (3)

Solving the $L$ firms’ indifference condition for $p_B$ in terms of $p_A$, then rearranging and substituting yields

$$p_B = p_A \frac{\varepsilon_A}{\varepsilon_B} - \frac{\Delta_c}{a \varepsilon_B}, \quad \text{or} \quad (1 - b) p_A - \Delta_c = (1 - b) p_B$$

$$= (1 - b) p_A \varepsilon_A \varepsilon_B - (1 - b) \frac{\varepsilon_B}{a \varepsilon_B}, \quad \text{so that}$$

$$p_A (1 - b) \left(1 - \frac{\varepsilon_A}{\varepsilon_B} \right) = \Delta_c - (1 - b) \frac{\varepsilon_B}{a \varepsilon_B}, \quad \text{or}$$

$$p_A^\ast = \frac{1}{(\varepsilon_B - \varepsilon_A) \Delta_c} \left[ \frac{\varepsilon_B}{1 - b} - \frac{1}{a} \right]. \quad \text{(8)}$$

Using this expression for $p_A$ in (3) yields

$$p_B = \frac{\varepsilon_A}{\varepsilon_B (\varepsilon_B - \varepsilon_A)} \Delta_c \left[ \frac{\varepsilon_B}{1 - b} - \frac{1}{a} \right] - \frac{\Delta_c}{a \varepsilon_B}$$

$$= \frac{1}{(\varepsilon_B - \varepsilon_A) \Delta_c} \left[ \frac{\varepsilon_A}{1 - b} - \frac{\varepsilon_A}{\varepsilon_B a} \right] - \frac{\Delta_c}{a (\varepsilon_B - \varepsilon_A) \varepsilon_B}, \quad \text{or}$$

$$p_B^\ast = \frac{1}{(\varepsilon_B - \varepsilon_A) \Delta_c} \left[ \frac{\varepsilon_A}{1 - b} - \frac{1}{a} \right]. \quad \text{(10)}$$

Assumption 3 implies $p_A^\ast, p_B^\ast > 0$, and after noting that $(p_A^\ast - p_B^\ast) = \Delta_c \left( \frac{1}{1 - b} \right)$, conclusions (a) and (b) follow from $\varepsilon_B > \varepsilon_A$.

To achieve the market rate of return for investors, certifier $A$, respectively $B$, must have a proportion $\theta_h_A$, respectively $\theta_h_B$, of $H$-firms, and investors are indifferent between investing or not investing after seeing an “$h$” report. We now show that this can be satisfied when and only when $\theta_h_A < \mu < \theta_h_B$.

Investor indifference conditions are

$$\theta_h_A = \frac{\mu \sigma_H A}{\mu \sigma_H A + (1 - \mu) \sigma_L A}, \quad \text{and}$$

$$\theta_h_B = \frac{\mu (1 - \sigma_H A)}{\mu (1 - \sigma_H A) + (1 - \mu)(1 - \sigma_L A)}.$$  \hfill (13)
Solving for the firms’ choice of certifiers, $\sigma_{HA}$ and $\sigma_{LA}$, yields, as an intermediate step,

$$\mu\sigma_{HA}(1 - \theta_{ha}) = (1 - \mu)\sigma_{LA}\theta_{ha}, \text{ and}$$

$$\mu(1 - \sigma_{HA})(1 - \theta_{hb}) = (1 - \mu)(1 - \sigma_{LA})\theta_{ha}. \tag{14}$$

The first of these yields,

$$\sigma_{HA} = \left(\frac{1}{\kappa}\right)\varepsilon_A \cdot \sigma_{LA}. \tag{16}$$

Substituting this into the second and solving yields

$$\sigma_{HA}^* = \varepsilon_A \left(\frac{1}{\kappa} \varepsilon_B - 1\right) \frac{1}{(\varepsilon_B - \varepsilon_A)} \text{ and } \sigma_{LA}^* = \varepsilon_B - \kappa \left(\frac{1}{\varepsilon_B - \varepsilon_A}\right). \tag{17}$$

Now, $\theta_{ha} < \mu < \theta_{hb}$ iff $\varepsilon_A < \kappa < \varepsilon_B$ iff $\sigma_{HA} \in (0, 1)$ and $\sigma_{LA} \in (0, 1)$. To see that (c) holds, note that $\partial\sigma_{HA}^*/\partial\varepsilon_A > 0$, $\partial\sigma_{HA}^*/\partial\varepsilon_B < 0$, $\partial\sigma_{LA}^*/\partial\varepsilon_A < 0$, and $\partial\sigma_{LA}^*/\partial\varepsilon_B > 0$.

Finally, for (d), using (17), equilibrium accuracy can be re-written as $\sigma_{LA}^*t_A + (1 - \sigma_{LA}^*)t_B = (1 - \kappa)$. □

Proof of Corollary 1.1. When $0 < \mu' < \theta_{ha}$, both firms choosing certifier $A$ is not an equilibrium as any investor response to the choice of auditor $B$ is strictly prefered by the $L$-firms to investor best response to both choosing $A$.

We now show that there is no mixed-strategy equilibrium with $H$-firms choosing $A$ and $L$-firms mixing between $A$ and $B$. For this to be an equilibrium, it requires $p_A + (1 - p)b - \Delta_c \geq b$ and $\varepsilon_F - \Delta_c = 0$. In other words, $p_A = \Delta_a \geq \frac{\Delta_a}{1+b}$. Since $\varepsilon_A \geq \frac{1-b}{a}$, there is a contradiction.

The other mixed strategy possibilities being even easier to eliminate, the only equilibrium has both firms choosing $B$, and investor best response is not to invest. The rest follows directly from Theorem 1. □

Proof of Corollary 1.2. Reorganizing the expression for the expected fines, we have

$$F_A = (1 - \theta_{ha})\varepsilon_AF_{aud} = \frac{r_h}{r_l + \varepsilon_A} F_{aud}, \text{ and}$$

$$F_B = (1 - \theta_{hb})\varepsilon_BF_{aud} = \frac{r_h}{r_l + \varepsilon_B} F_{aud}. \tag{18, 19}$$

As $\varepsilon_B > \varepsilon_A$, $F_B > F_A$, so that (a) holds.

In the presence of auditor liability, defining $\Delta_c' = (\Delta_c - (F_B - F_A))$, firms’ indifference conditions are

$$\varepsilon_A p_a - \Delta_c' = \varepsilon_B p_B a, \text{ and}$$

$$p_A + (1 - p_A)b - \Delta_c' = p_B + (1 - p_B)b, \tag{20, 21}$$

which are the equations (3) and (4) with $\Delta_c'$ instead of $\Delta_c$. Solving just as before yields the new investment probabilities,

$$p_{aud}^A = \left(1 - \frac{F_B - F_A}{\Delta_c}\right) p_A^* \text{ and } p_{aud}^B = \left(1 - \frac{F_B - F_A}{\Delta_c}\right) p_B^*. \tag{22}$$

where $p_A^*$ and $p_B^*$ are the equilibrium investment rates with no fines from Theorem 1. Since $\frac{F_B - F_A}{\Delta_c} > 0$ and $(F_B - F_A)$ is linear and increasing in $F_{aud}$, higher fines lead to lower investment so that (e) holds.
Investors’ indifference conditions remain the same as in the case without liability, and this is what determines the proportions of $H$ and $L$ firms picking the stronger auditor. Thus, $\sigma_{HA}^{aud} = \sigma_{LA}^{*}$ and $\sigma_{LA}^{aud} = \sigma_{LA}^{*}$, and both are independent of $F_{aud}$, so that neutrality, (b) holds.

**Proof of Corollary 1.3.** Set $r'_{l} = (r_{l} + \gamma F_{mgt})$ and $a' = (a - F_{mgt})$ so that investor return to putting their money in an $L$ firm $r - r'_{l}$ and management payoff is $(a - F_{mgt})$ if their auditor passes on their report and they receive investment. Setting $\kappa'_{l} = \frac{\mu}{1 - \mu} r'_{l} < \kappa$, we have, exactly as in the proof of Theorem 1,

\[
\sigma_{HA}^{mgt} = \frac{\varepsilon_{A} (\frac{1}{\varepsilon_{B}} - 1)}{(\varepsilon_{B} - \varepsilon_{A})}, \quad \sigma_{LA}^{mgt} = \frac{\varepsilon_{B} - \kappa'}{(\varepsilon_{B} - \varepsilon_{A})}
\]

\[
p_{A}^{mgt} = \left(\frac{1}{\varepsilon_{B} - \varepsilon_{A}}\right) \Delta_{c} \left[ \frac{\varepsilon_{B}}{1 - b} - \frac{1}{a} \right], \quad \text{and}
\]

\[
p_{B}^{mgt} = \left(\frac{1}{\varepsilon_{B} - \varepsilon_{A}}\right) \Delta_{c} \left[ \frac{\varepsilon_{A}}{1 - b} - \frac{1}{a} \right].
\]

The only difference between the solutions here and the solutions in Theorem 1 are the substitutions of $\kappa'$ for $\kappa$ and $a'$ for $a$. As $\kappa > \kappa'$, we have the market share results, (a), which directly yields the result about certifier market accuracy, (c).

The investment rate for the clientelle for auditor $A$ is $\mu \sigma_{HA}^{mgt} p_{A}^{mgt} + (1 - \mu) \sigma_{LA}^{mgt} \varepsilon_{A} p_{A}^{mgt}$, which is, modulo suppressing inessential detail about the $\mu$’s, equal to

\[
\frac{\Delta_{c}}{(\varepsilon_{B} - \varepsilon_{A})^{2}} \left[ \frac{\varepsilon_{A} \varepsilon_{B}}{1 - \kappa} - \varepsilon_{A} \kappa \right] \left[ \frac{\varepsilon_{B}}{1 - b} - \frac{1}{a} \right],
\]

and for larger values of $\gamma$, the increasing term dominates the decreasing one. \qed

**Appendix B. Proofs of Theorem 2 and its Corollaries**

*Graphical arguments for Lemma 1.* Because there is neither liability nor reputation concerns in this part of the analysis, the equilibrium implicit contract is exactly the same as the contracts analyzed in Theorem 1. Being above the line segment $\overline{st}$ corresponds to $\varepsilon_{A}$ satisfying Assumption 3, which guarantees that $p_{*}^{A}, p_{*}^{B} > 0$. Being to the right of the line segment $\overline{st}$ corresponds to $\varepsilon_{B} > \kappa$, which, by (17), guarantees that $\sigma_{HA}^{*}, \sigma_{LA}^{*} > 0$. $(\varepsilon_{A}, \varepsilon_{B})$ being below the line segment $\overline{tu}$ corresponds to $p_{A}^{*} < 1$. Finally, being below the line segment $\overline{uv}$ corresponds to $\varepsilon_{A} < \kappa$, which, by (17) again, guarantees that $\sigma_{HA}^{*}, \sigma_{LA}^{*} < 1$. \qed

*Proof of Lemma 1.* In this version of the model, there is no payoff consequence to mistakes by the auditor. As a result, competition drives the implicit contracts to the costs of the firms in all contingencies. This is the same contract considered in Theorem 1 hence the same indifference conditions must hold. From the proof of Theorem 1 Assumption 3 and $\varepsilon_{B} > \varepsilon_{A} > 0$ guarantee that that $p_{*}^{A} > p_{*}^{B} > 0$. Thus, to keep both $p_{*}^{A}$ and $p_{*}^{B}$ in the interval
[0, 1], it is sufficient that \( p_A^* \leq 1 \), i.e.

\[
p_A^* = \Delta \frac{\varepsilon_B - 1}{a} \leq 1,
\]

that is

\[
\varepsilon_A \leq \frac{\Delta_c}{a} + \varepsilon_B \left(1 - \frac{\Delta_c}{1 - b}\right).
\]

(27)

(28)

Also from the proof of Theorem 1, we know that keeping \( \sigma_{HA}^*, \sigma_{LA}^* \) in the interval (0, 1) is equivalent to having \( \varepsilon_A < \kappa < \varepsilon_B \) (which is in turn equivalent to \( \theta_{hA} < \mu < \theta_{hB} \)). Thus, the set of \( (\varepsilon_A, \varepsilon_B) \) consistent with a market equilibrium with certifier competition is the set

\[
S := \{ (\varepsilon_A, \varepsilon_B) : \varepsilon_B > \kappa, \frac{1-b}{a} < \varepsilon_A < \min \{ \kappa, \frac{\Delta_c}{a} + \varepsilon_B \left(1 - \frac{\Delta_c}{1 - b}\right) \} \}.
\]

(29)

As profits of both the \( H \) and \( L \) firms are increasing in \( \varepsilon_A \) and decreasing in \( \varepsilon_B \), for \( \varepsilon_B > \varepsilon_o \), the maximal \( \varepsilon_A \) is \( \kappa \), for \( \varepsilon_B < \varepsilon_o \), the calculating the derivatives shows that increases in \( \varepsilon_B \) are worth the corresponding increases in \( \varepsilon_A \).

\( \square \)

The calculations and arguments for the points ‘close to’ the boundary can be formally written with statements of the form “for points at a distance sufficiently small.” Instead, we use “\( \approx 0 \)” or “\( \approx 1 \)” to indicate infinitesimal closeness, show that our results hold for any strictly positive \( \delta \approx 0 \), and then appeal to the spillover principle (Corbae et al., 2009, see e.g. Chap. 11.2.c).

**Proof of Theorem 2** Part (a) is directly from the corresponding part of Theorem 1

For the first part of (b), if \( \varepsilon_B < \varepsilon_o \), the most profitable auditor behavior is \( (\varepsilon_A^*, \varepsilon_B^*) \approx (\kappa, \varepsilon_o) \). From (17), we know that \( \sigma_{HA}^* \approx \sigma_{LA}^* \approx 1 \). Therefore, the industry investment rate is determined by \( p_A^* \), and \( p_A^* \approx 1 \) at all points of the form \( (\kappa, \varepsilon_o) \). By contrast, if \( \varepsilon_B > \varepsilon_o \), the most profitable auditor behavior is \( (\varepsilon_A^*, \varepsilon_B^*) \approx (\kappa, \varepsilon_B) \). While this does not change with \( \Delta_c \), \( p_A^* \approx \Delta_c \frac{\varepsilon_B - \frac{1}{a}}{(\varepsilon_B - \kappa)} < 1 \) is a strictly increasing function of \( \Delta_c \) with positive, non-infinitesimal slope.

For part (c), the initial observation is that \( \sigma_{HA}^* \approx \sigma_{LA}^* \approx 1 \) so that only \( p_A^* \) matters for determining the industry investment rate. Now \( p_A^* \approx 1 \) if \( \varepsilon_B < \varepsilon_o \), or \( p_A^* \approx \Delta_c \frac{\varepsilon_B - \frac{1}{a}}{(\varepsilon_B - \kappa)} \) if \( \varepsilon_B > \varepsilon_o \). In either case, \( p_A^* \) does not depend on \( \varepsilon_A \), in the first case, its slope with respect to \( \varepsilon_B \) is infinitesimal, in the second it is strictly negative.

Finally, for (d), \( \sigma_{LA}^* \approx 1 \), so that \( [\sigma_{LA}^* l_A + (1 - \sigma_{LA}^*) l_B] \approx l_A = (1 - \varepsilon_A^*) \approx (1 - \kappa). \)

\( \square \)

**Proof of Corollary 2.1** Suppose that \( \kappa' \in (\frac{1-b}{a}, \kappa) \). If the stronger auditor does not change behavior, investor best responses are to not invest. Define \( \varepsilon_o' \frac{\kappa' - \frac{\Delta_c}{a}}{1 - \frac{\Delta_c}{1 - b}} < \varepsilon_o \). The new maximal profits happen at \( \varepsilon_A' = \kappa' \) and \( \varepsilon_B' = \max(\varepsilon_B, \varepsilon_o') \), and these involve lower investment rates and lower profits. The second part is immediate from Lemma 1.

\( \square \)

**Proof of Corollary 2.2** Set \( r = (w_{hA} - w_{hB}), s = (\varepsilon_A w_{hA} + (1 - \varepsilon_A) w_{lA}), \) and \( t = (\varepsilon_B w_{hB} + (1 - \varepsilon_B) w_{lB}) \). The \( L \) and the \( H \) firm indifference conditions are

\[
\varepsilon_A p_A a - (s - t) = \varepsilon_B p_B a, \quad \text{and}
\]

\[
p_A + (1 - p_A) b - r = p_B + (1 - p_B) b.
\]

(30)

(31)
Solving yields

\[
p_A^* = \frac{1}{(\varepsilon_B - \varepsilon_A)} \left[ \varepsilon_B \frac{1}{1 - b} - \frac{(s - t)/r}{a\varepsilon_B} \right] \quad \text{and} \quad \frac{\varepsilon_A}{(\varepsilon_B - \varepsilon_A)} \left[ \varepsilon_A \frac{1}{1 - b} - \frac{(s - t)/r}{a\varepsilon_B} \right].
\]

(32) (33)

The investor indifference conditions lead to

\[
\sigma_{HA}^* = \frac{\varepsilon_A (\frac{1}{b}\varepsilon_B - 1)}{(\varepsilon_B - \varepsilon_A)} \quad \text{and} \quad \sigma_{LA}^* = \frac{\varepsilon_B - \kappa}{(\varepsilon_B - \varepsilon_A)}.
\]

(34)

which delivers \boxed{[b]}. Auditor indifference requires \(w_{hA} = w_{lA} + F_{aud}\) and \(w_{hB} = w_{lB} + F_{aud}\), and since competition drives profits to zero, we also have

\[
\mu(1 - \sigma_{HA})w_{hB} + (1 - \mu)(1 - \sigma_{LA})[(1 - t_B)(w_{hB} - F_{aud}) + t_B w_{lB}] = 0, \quad \text{and} \quad \mu \sigma_{HA}(w_{hA} - \Delta_c) + (1 - \mu)\sigma_{LA}[(1 - t_A)(w_{hA} - F_{aud} - \Delta_c) + t_A(w_{lA} - \Delta_c)] = 0.
\]

(35) (36)

Together, these yield

\[
w_{hA}^* = (1 - \theta_{hA})F_{aud} + \Delta_c, \quad \text{and} \quad w_{hB}^* = (1 - \theta_{hB})F_{aud},
\]

(37) (38)

from which we have

\[
r = (w_{hA}^* - w_{hB}^*) = \Delta_c + F_{aud}(\theta_{hB} - \theta_{hA}), \quad s = (\varepsilon_A w_{hA}^* + (1 - \varepsilon_A)w_{lA}^*) = w_{lA}^* + \varepsilon_A F_{aud}, \quad t = (\varepsilon_B w_{hB}^* + (1 - \varepsilon_B)w_{lB}^*) = w_{lB}^* + \varepsilon_B F_{aud}, \quad \text{and} \quad (s - t) = (w_{lA} - w_{lB}) - (\varepsilon_B - \varepsilon_A)F_{aud}.
\]

(39) (40) (41) (42)

As in the proof of Lemma \boxed{[1]} the constraints for the existence of a market equilibrium with certifier competition are \(\varepsilon_B > \kappa > \varepsilon_A\), and \(p_A^* \leq 1\), which in this case leads to \(\varepsilon_A \leq (1 - \frac{r}{1-t}) \varepsilon_B + (\frac{s-t}{a})\). Because \(r = \Delta_c + F_{aud}(\theta_{hB} - \theta_{hA})\) and \((s-t) = (w_{lA} - w_{lB}) - (\varepsilon_B - \varepsilon_A)F_{aud}\), this change has shifted downwards and rotated clockwise the comparable line from Lemma \boxed{[1]} and Figure 3, changing \(\overline{u}\) to \(\overline{UT}\) as in Figure 4.

The linear function \(F_{aud} \mapsto \varepsilon_0(F_{aud})\) is defined by solving \(\kappa = (1 - \frac{r}{1-t}) \varepsilon_B + (\frac{s-t}{a})\), and \boxed{[c]} follows as in Lemma \boxed{[1]}

Proof of Corollary \boxed{[2,3]} Claim (a) is the observation that at firm optimal market equilibria, the stronger auditor’s share of the business is \(\sim 1\). The increase in certifier market accuracy is the observation that increasing \(r_t\) to \(r_t'\) decrease \(\kappa\) to \(\kappa' < \kappa\), and this is the industry
error rate because the stronger auditor’s share of the business is $\simeq 1$. For (c), note that the investment rate is $(\mu + (1 - \mu)\kappa) p_A^{mgt}$, a product of positive terms that decrease in $F_{mgt}$. □