Sketches of Solutions to Assignment #2 for Managerial Economics
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1. From the *Notes*, D.2.
Ans. In each of these cases, you can set the problem up as one of comparing the solutions to two different problems that differ by the addition of an extra term. The key to making your life easy is to check sub- or supermodularity by checking that the extra term is monotonic in the decision variable. A subtlety in part c. is that the wages that the bright young folks earn already let the consulting company capture part of the value of the training that they are providing. One could talk about this, partly, as a difference in discount factors between the firm and the youngsters.
2. From the *Notes*, either D.4 or D.5 (prove the statements made in the problem).
Ans. D.4 contains an obvious typo: suppose that $f(x,t)$ is super-modular and that ψ is a strictly increasing function, we want to check that $h(x,t) := \psi(f(x,t))$ is q-supermodular. Suppose that $x' > x$ and $t' > t$. We need to show that
$$[h(x',t) - h(x,t) > 0] \Rightarrow [h(x',t') - h(x,t') > 0] \text{ and}$$
$$[h(x',t) - h(x,t) \geq 0] \Rightarrow [h(x',t') - h(x,t') \geq 0].$$
By supermodularity, we know that $f(x',t') - f(x,t') \geq f(x',t) - f(x,t)$. Now $h(x',t) - h(x,t) > 0$ can only happen if $f(x',t) - f(x,t) > 0$, supermodularity implies then that $f(x',t') - f(x,t') > 0$, and since ψ is strictly increasing, $h(x',t') - h(x,t') > 0$. The argument for "$\geq$" is almost the same.
The argument for $g(x,t) := \varphi(f(x,t),t)$ is almost verbatim.
D.5 has $x^*(t) = 100$ for all $t \in T$. Monotonic transformations of the utility function cannot change this. Taking the indicated derivatives, $\log(f(x,t))$ is neither super- nor sub-modular, and $\log(\log(f(x,t)))$ is strictly sub-modular. This is a reminder that supermodularity is a sufficiently strong condition to guarantee that the best response set does not move down, but even when the best response set does not move down, supermodularity may fail.
3. What is the maximum amount you would pay for an asset that generates an income of \$250,000 at the end of five years if the opportunity cost of using funds is %8?
Ans. $250,000/(1.08)^5 \simeq 170,146$.
4. What is the value of a preferred stock that promises to pay a perpetual dividend of \$125 at the end of each year when the interest rate is %5? In which direction would you move the interest rate you use to evaluate this net present value if the company's prospects begin to look riskier? Why?
Ans. For $0 < \rho < 1$, $\sum_{t=1}^{\infty} \rho^t = \frac{\rho}{1-\rho}$. Setting $\rho = 1/(1-r)$ where $r = 0.05$, $\rho/(1-\rho) = 1/r = 20$. So the answer is $20 \cdot 125 = 2,500$. If the company begins to look riskier, the dividends may be interrupted at some random time T so the expected net present value is $E \sum_{t=1}^{T} 125\rho^t$. Replacing "$\infty$" with a smaller T lowers the value, hence one should use a higher interest rate/lower discount factor to evaluate the value. Note that the assumption here is that we are one year from the first dividend. If you assumed that the dividend was coming tomorrow, then the sum would be $\sum_{t=0}^{\infty} \rho^t = \frac{\rho}{1-\rho}$, which increases the value a little bit.

5. An owner can lease her building for \$120,000 per year for three years. The explicit cost of maintaining the building is \$40,000, and the implicit cost is \$55,000. All revenues are received, and costs borne, at the end of each year. If the interest rate is %5, determine the present value of the stream of accounting profits, and the present value of the stream of economic profits.
   Ans. The net present value of the accounting profits are $\sum_{t=1}^{\infty} \frac{120{,}000-40{,}000}{1.05^t}$, for economic profits, $\sum_{t=1}^{\infty} \frac{120{,}000-95{,}000}{1.05^t}$. One could (reasonably) sum from $t = 0$ as well depending on how you interpret the problem. Faced with a real problem, you would look at when the payments/expenses come due and use those times, and if the payments happened at small intervals, continuous discounting would give a more accurate answer.
6. You are in the market for a new frig and you have narrowed the search to two models. The energy-efficient model sells for \$700 and will save you \$45 per year. For the purposes of use, the standard model is indistinguishable from the energy-efficient model except that it costs \$500. Assuming that your opportunity cost of funds is %6, which frig should you purchase?
   Ans. The net present value of the extra expense is 200. The net present value of the savings is, using discrete discounting and assuming that bills are paid yearly at the end of the year, $\sum_{t=1}^{\infty} \frac{45}{1.06^t}$. The expensive, energy efficient frig wins.
7. From the *Notes*, E.1 (this is a series of calculations very similar to those in the previous four problems, be sure to understand the decisions that would flow from the entries you find for the table).
   Ans. Throughout, use $\int_0^T (-C)e^{-rt}\,dt + \int_T^{\infty} Be^{-rt} = \frac{1}{r}\left[Be^{-rT} + (-C)(1-e^{-rT})\right]$.
8. From the *Notes*, E.2.
   Ans. Use $P = \int_0^T xe^{-rt}\,dt = x\frac{1}{r}(1-e^{-rT})$ and solve. If x is too low or P too high, explicitly if $P > \frac{x}{r}$, then no T will deliver the equality. We will cover result much like part c. in lecture.
9. From the *Notes*, E.3.
   Ans. Solve for $r > 0$ such that $\frac{1}{r}\left[Be^{-rT} + (-C)(1-e^{-rT})\right] = 0$. Since $r > 0$, this is the same as solving $\left[Be^{-rT} + (-C)(1-e^{-rT})\right] = 0$. For the second part, put in $r = 0.10$, $r = 0.20$, and $r = 0.30$ and find the T that makes $\left[Be^{-rT} + (-C)(1-e^{-rT})\right] = 0$.
10. From the *Notes*, E.4.
   Ans. Note that parts c. and d. replicate the first two parts. This makes the problem easier so celebrate. You are looking at the ratio of $\int_0^T x\,dt$ and $\int_0^T xe^{-rt}\,dt$, that is
   $$\frac{xT}{x(1-e^{-rT})/r} = \frac{rT}{1-e^{-rT}}.$$
   This is a function of the product rT, larger rT makes the ratio larger, smaller rT makes the ratio smaller. For fixed T, higher r increases the ratio, for fixed r, higher T increases the ratio. Higher r devalues future payments, higher T means that payments are accruing further in the future, hence worth less in net present value terms.
11. From the *Notes*, F.1.
   Ans. We will do this in class.