Assignment #3 for Managerial Economics, Fall 2015
Due, Wednesday October 14, 2015

1. From the *Notes*, Chapter 2, C.1 (diversification).

   Suppose that you have $W = 100$ to invest, and that X and Y have the distribution given in the following table,

$Y \downarrow, X \rightarrow$	1.0	1.3
0.9	$\frac{1}{8}$	$\frac{2}{8}$
1.5	$\frac{4}{8}$	$\frac{1}{8}$

   Thus, investing everything in X, you either get back your investment or you get back your investment plus %30, with Y, you may lose %10, or you may gain %50.

   a. Give EX, EY, σ^2_X, σ^2_Y, and $\sigma_{X,Y}$.
   b. Solve the problem $\max_{0 \leq w \leq W}[f(w) - 0 \cdot g(w)]$ and give the mean and variance of the optimal portfolio.
   c. Solve the problem $\max_{0 \leq w \leq W}[f(w) - 0.1 \cdot g(w)]$ and give the mean and variance of the optimal portfolio.
   d. Solve the problem $\max_{0 \leq w \leq W}[f(w) - 0.2 \cdot g(w)]$ and give the mean and variance of the optimal portfolio.
   e. Solve the problem $\max_{0 \leq w \leq W}[f(w) - 0.3 \cdot g(w)]$ and give the mean and variance of the optimal portfolio.

   Ans.

   - $\mu_X = 1 \cdot \frac{5}{8} + 1.3 \cdot \frac{3}{8} = 1.11250$,
   - $\mu_Y = 0.9 \cdot \frac{3}{8} + 1.5 \cdot \frac{5}{8} = 1.2750$,
   - $\sigma^2_X = (1.0)^2 \cdot \frac{5}{8} + (1.3)^2 \cdot \frac{3}{8} - (\mu_X)^2 = 0.021094$,
   - $\sigma^2_Y = (0.9)^2 \cdot \frac{3}{8} + (1.5)^2 \cdot \frac{5}{8} - (\mu_Y)^2 = 0.084375$, and
   - $\sigma_{X,Y} = EXY - (\mu_X \cdot \mu_Y) = 0.9 \cdot 1.0 \cdot \frac{1}{8} + 0.9 \cdot 1.3 \cdot \frac{2}{8} + 1.5 \cdot 1.0 \cdot \frac{4}{8} + 1.5 \cdot 1.3 \cdot \frac{1}{8} - (1.1125 \cdot 1.275) = -0.019687$.

   With these results and from what is given in the Notes just above this text, the aim is to solve

$$\max_{0 \leq w \leq 100} u_\gamma(w) := (w\mu_X + (W-w)\mu_Y) - \gamma(w^2\sigma^2_X + (W-w)^2\sigma^2_Y - 2w(W-w)\sigma_{X,Y}).$$

   For $\gamma > 0$, $u_\gamma(\cdot)$ is a quadratic that opens downwards. Therefore, finding the solution involves setting the derivative equal to 0 and solving this equation for w°. If $w^\circ < 0$, then the optimum, w^*, is equal to 0, if $w^\circ > 100$, then $w^* = 100$, and if $0 \leq w^\circ \leq 100$, then $w^* = w^\circ$. The calculus is pretty staightforward, here are some graphs to make the points.

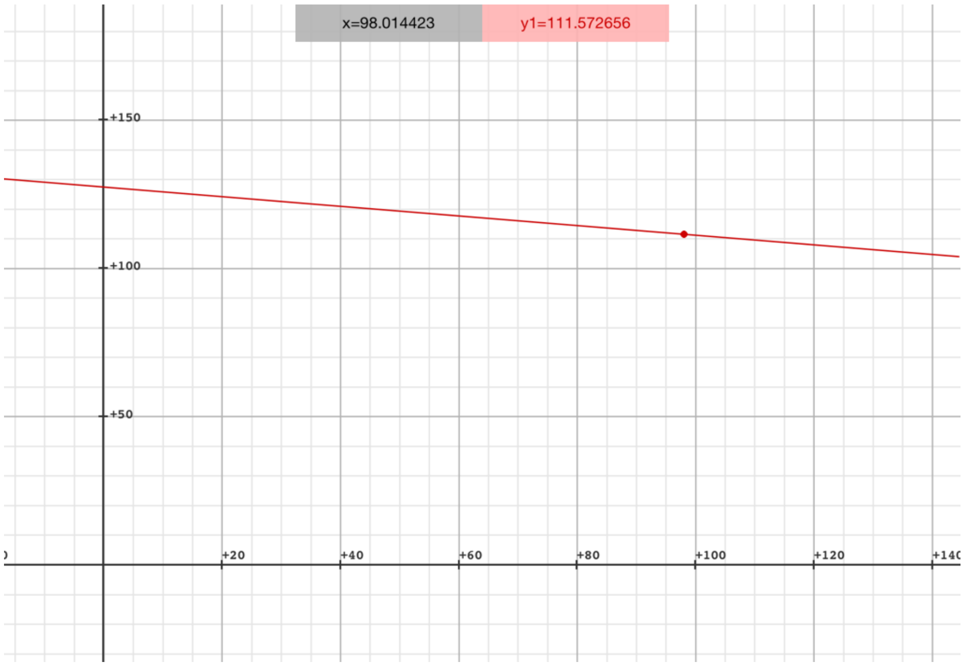

FIGURE 1. The graph of the Mean of the portfolio $w \cdot X + (100 - w) \cdot Y$

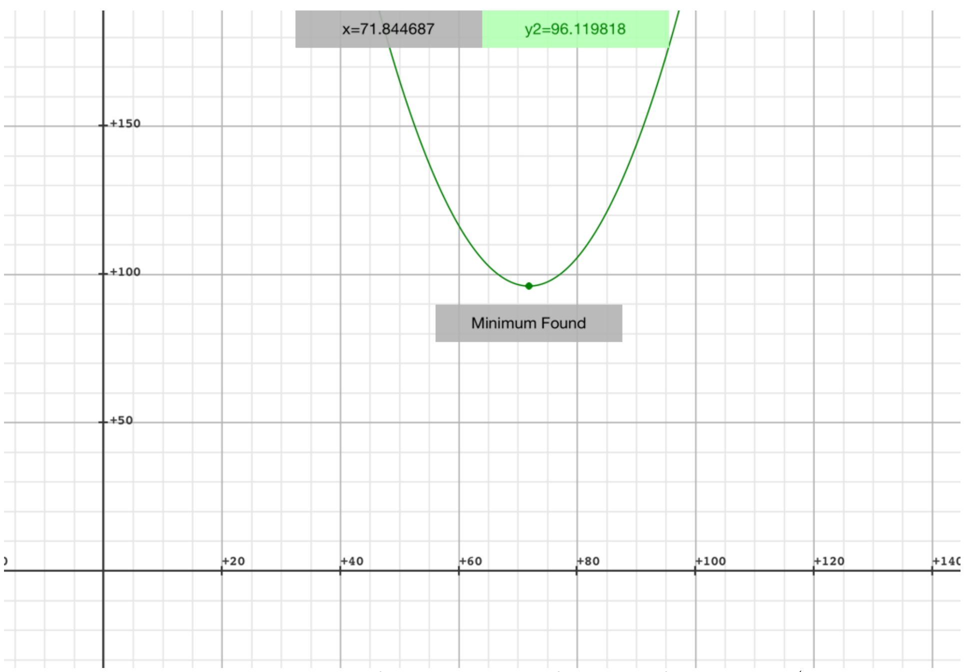

FIGURE 2. The graph of the Variance of the portfolio $w \cdot X + (100 - w) \cdot Y$

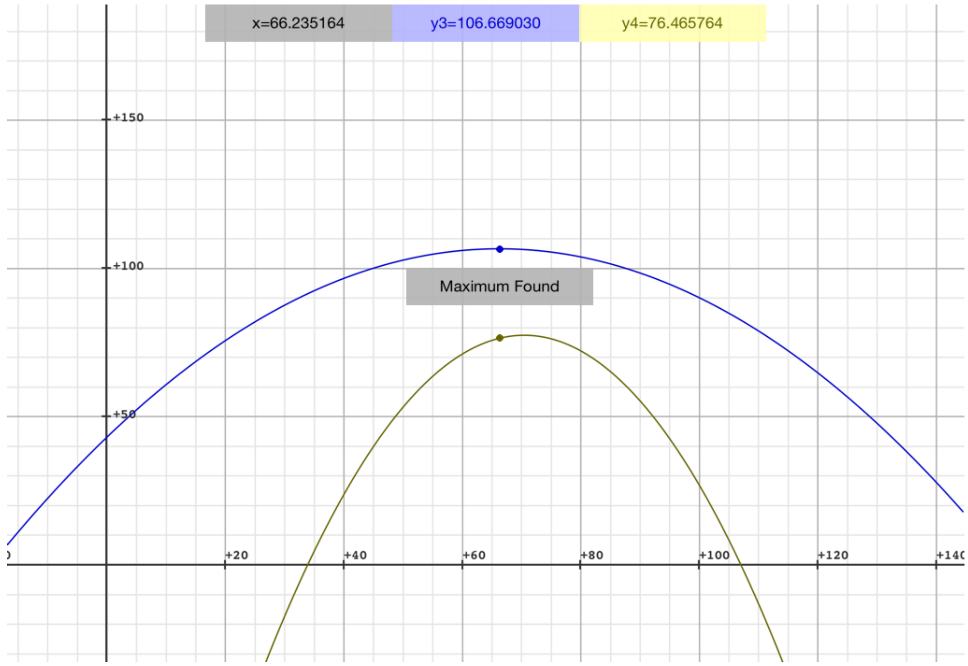

FIGURE 3. $f(w) - \gamma g(w)$ for $\gamma = 0.1$ and 0.4

2. From the *Notes*, Chapter 2, C.2 (certainty equivalents).

   For each of the following utility functions, find the certainty equivalent of X where $P(X = 100) = 0.4$ and $P(X = 200) = 0.6$.

   a. $u(x) = \sqrt{x}$.
   b. $u(x) = x^r$, $0 < r < 1$.
   c. $u(x) = x$.
   d. $u(x) = \log(x)$.
   e. $u(x) = x^2$.

   Ans. $E\,X^r = 100^r \cdot 0.4 + 200^r \cdot 0.6$ so solve for κ such that $u(\kappa) = \kappa^r = (100^r \cdot 0.4 + 200^r \cdot 0.6)$, which yields $\kappa = (100^r \cdot 0.4 + 200^r \cdot 0.6)^{1/r}$ for $0 < r < 1$. The first problem has $r = \frac{1}{2}$. The third problem has $r = 1$, which yields $\kappa = (100 \cdot 0.4 + 200 \cdot 0.6) = E\,X$. For the fourth problem, solve $\log(\kappa) = \log(100) \cdot 0.4 + \log(200) \cdot 0.6$, for the last problem solve $\kappa^2 = 100^2 \cdot 0.4 + 200^2 \cdot 0.6$ and note that $\kappa > E\,X$ in this case.

3. From the *Notes*, Chapter 2, C.3 (certainty equivalents).

   For each of the following utility functions, find the certainty equivalent of X where X is uniformly distribution on the interval $[100, 200]$.

   a. $u(x) = \sqrt{x}$.
   b. $u(x) = x^r$, $0 < r < 1$.
   c. $u(x) = x$.
   d. $u(x) = \log(x)$.
   e. $u(x) = x^2$.

   Ans. Repeat the previous using $\int_{100}^{200} u(x) \frac{1}{100}\, dx$ to calculate the expected utilities. E.g. solving $\kappa^r = \int_{100}^{200} x^r \frac{1}{100}\, dx$ yield $\kappa = \left[\frac{1}{r+1}(200^{r+1} - 100^{r+1})\right]^{1/r}$.

4. From the *Notes*, Chapter 2, C.4 (warranties and certainty equivalents).

You are selling a good that may fail and inflict a random loss $L \geq 0$ on the consumer. You offer the potential consumers, a warranty that costs C, and, if they purchase it, they will suffer no loss. Show that if the expected cost to you of making good the losses is less than $E\,L$, you can offer this warranty and increase your expected profits. [You need to think through which consumers will and which will not buy the insurance, and note that there may also be an effect of increasing the demand.] How might your answers change if those who buy the warranty become more careless and increase the probability of loss (moral hazard)?

Ans. If a customer has purchase the good and is considering the warranty, they are comparing $E\,u(W-L)$ to $E\,u(W-C)$ (where W is the wealth that they have). For any risk averse expected utility maximizer, $u(E\,(W-L)) > E\,u(W-L)$, and $E\,(W-L) = W - E\,L$. If $C < E\,L$, then $u(W-C) > u(W-L) > E\,u(W-L)$. If your average cost of offering the warranty is some $C < L$, you can offer this the warranty at price C and have it accepted by risk averse consumers. However, the pricing decision should be made with information about the elasticity of demand with respect to the price you charge.

Notice that people who did not buy the product because of the risk might now be motivated to buy it. We now use the following notation for the utility function with and without the product, $u(\$,1)$ and $u(\$,0)$, 1 representing having 1 unit of the product, etc.

People who did not by the product at price p satisfied $u(W,0) > E\,u((W-p)-L,1)$, but if the buy the product and the warranty, their utility is $u(W-p-C,1)$ and we know that $u(W-p-C,1) > E\,u((W-p)-L,1)$. For some of them, it is possible that $u(W-p-C,1) > u(W,0)$, and they are the new consumers attracted because of the possibility of buying a warranty.

The moral hazard increases $E\,L$, if it raises it so far that it is now higher than what you are charging, C, you are losing money on average.

5. From the *Notes*, Chapter 2, C.5 (moral hazard and adverse selection).

By putting in effort $0 \leq e \leq \bar{e}$, a business can reduce the probability of fire in the warehouse to $P(e)$ where $P'(e) < 0$ and $P''(e) > 0$. If there is a fire, there will be a loss $L > 0$ and profits will be $(R-L) > 0$, if there is not a fire, they will be R. The utility costs of effort to firm θ, $0 < \theta < 1$, are $(1-\theta)c(e)$ where $c'(e) > 0$ and $c''(e) > 0$. Higher θ's correspond to lower costs of prevention, so these are "better" firms.

The problem for firm θ without insurance is

$$E\,\Pi = \max_{0 \leq e \leq \bar{e}} [P(e)(R-L) + (1-P(e))R] - (1-\theta)c(e). \tag{1}$$

One simple form of insurance policy costs C and has a deductible D, that is, it reduces losses from L to $(L-D)$. As is only sensible, $C \ll D$ and $D < L$. If the firm θ buys such an insurance policy, their problem becomes

$$E\,\Pi_{ins} = \max_{0 \leq e \leq \bar{e}} [P(e)((R-C)-(L-D)) + (1-P(e))(R-C)] - (1-\theta)c(e). \tag{2}$$

a. If the firms do not have insurance, which ones put in the higher preventive efforts? Explain.
b. If the firms do have insurance, which ones put in the higher preventive efforts? Explain.

c. The company buys insurance if $E\,\Pi_{ins} > E\,\Pi$. Show that it is the lower type firms that buy insurance. Explain why we get this form of adverse selection.
d. Most insurance policies will only pay out if the insured has exercised due diligence. How would your answer to the previous change if the insurance company would only pay out for damages if $e \geq e^{\circ}$?

a. Consider the function

$$f(e,\theta) = [P(e)(R-L) + (1-P(e))R] - (1-\theta)c(e) = m + \theta c(e) \tag{3}$$

where m is all of the terms that **do not** have both θ and e in them. Because $\partial^2 f/\partial\theta\partial e = c'(e) > 0$, this function is supermodular, hence higher θ firms put in higher efforts.

b. Consider the function

$$g(e,\theta) = [P(e)((R-C)-(L-D)) + (1-P(e))(R-C)] - (1-\theta)c(e) = m' + \theta c(e) \tag{4}$$

where m' is all of the terms that **do not** have both θ and e in them. Because $\partial^2 g/\partial\theta\partial e = c'(e) > 0$, this function is supermodular, hence higher θ firms put in higher efforts.

c. This is a rather advanced question. We will return to the technique used here, the **envelope theorem**, in the future.
We want to find the set of θ such that $E\,\Pi_{ins}(\theta) > E\,\Pi(\theta)$ where $E\,\Pi_{ins}(\theta)$ is the maximized quantity in equation (2 and $E\,\Pi(\theta)$ is the maximized quantity in equation (1. A really good intermediate microeconomics course would have taught you the tool to solve such problems — the **envelope theorem**. It tells us how to calculate the derivative of the optimized utility.
Define the "value" as a function of θ by

$$V(\theta) = \max_{e \in [0,\overline{e}]} h(e,\theta),$$

let $e^*(\theta)$ satisfy $\frac{\partial h(e^*(\theta),\theta)}{\partial e} \equiv 0$ and suppose that $V(\theta) = h(e^*(\theta),\theta)$ (this means that we are supposing that solving the FOCs characterizes an optimum). Basic calculus rules tell us that

$$\frac{dV(\theta)}{d\theta} = \frac{\partial h(e^*(\theta),\theta)}{\partial e}\frac{de^*(\theta)}{d\theta} + \frac{\partial h(e^*(\theta),\theta)}{\partial \theta}.$$

The first term on the right-hand side of this equation is identically equal to 0. This means that $\frac{dV(\theta)}{d\theta}$ is equal to $\frac{\partial h(e^*(\theta),\theta)}{\partial \theta}$.
With this in place, return to equations (3) and (4) above and notice that the solution to $\max_e f(e,\theta)$ involves a larger e^*_f than the solution, e^*_g, to $\max_e g(e,\theta)$. Add to this the observation that $\partial f/\partial\theta = c(e^*_f)$ and $\partial g/\partial\theta = c(e^*_g)$. Since $c(\cdot)$ is an increasing function, we know that $\frac{dV_{ins}(\theta)}{d\theta} < \frac{dV(\theta)}{d\theta}$. This says that expected profits have a higher slope in θ when the company does not buy insurance. From which we know that if any group stays away from insurance, it is the group with the higher θ's.

d. With this restriction, the insurance company is avoiding (part of) the moral hazard problem. Now the comparison is between the without insurance problem,

$$E\,\Pi = \max_{0 \leq e \leq \overline{e}} [P(e)(R-L) + (1-P(e))R] - (1-\theta)c(e). \tag{5}$$

and the modified insurance problem,

$$(6)\ \ E\,\Pi_{ins} = \max_{e^\circ \le e \le \overline{e}} [P(e)((R-C)-(L-D)) + (1-P(e))(R-C)] - (1-\theta)c(e).$$

The modification is in the constraint set, now it is $e \in [e^\circ, \overline{e}]$, before it was $e \in [0, \overline{e}]$. This lowers the expected profits for those low type firms that would have, before, chosen insurance and then chosen an $e^* < e^\circ$.

6. From the *Notes*, Chapter 2, E.1 (when information is valuable).

This and the next problem has two states, $X = G$ood and $X = B$ad with prior information telling us that $P(X = G) = 0.75$ and $P(X = B) = 0.25$. It also has two actions Leave alone and New infrastructure. The payoffs are

	Good	Bad
L	10	6
N	$(10-c)$	$(9-c)$

Beliefs/evidence yields $\beta_G = Prob(G)$, $\beta_B = (1-\beta_G) = Prob(B)$. One optimally leaves the infrastructure as it is if

$$10\beta_G + 6(1-\beta_G) > (10-c)\beta_G + (9-c)(1-\beta_G), \text{ that is, if}$$
$$c > 3(1-\beta_G).$$

We suppose that the new infrastructure costs 20% of the damages it prevents, that is, $c = 0.60$ so that one strictly prefers to Leave the infrastructure alone iff $\beta_G > 0.8$, i.e. iff there is less than one chance in five of the bad future weather patterns.

Because prior information puts $\beta_G = 0.75$, without any extra information, one would put in the New infrastructure. Let us now think about signal structures. First let us suppose that we can run test/experiments that yield $S = s_G$ or $S = s_B$ with $P(S = s_G|G) = \alpha \ge \frac{1}{2}$ and $P(S = s_B|B) = \beta \ge \frac{1}{2}$. The joint distribution, $q(\cdot,\cdot)$, is

	Good	Bad
s_G	$\alpha \cdot 0.75$	$(1-\beta)\cdot 0.25$
s_B	$(1-\alpha)\cdot 0.75$	$\beta \cdot 0.25$

Beliefs or **posterior beliefs** are given by

$$P(G|S = s_G) = \beta_{s_G}(G) = \frac{\alpha \cdot 0.75}{\alpha \cdot 0.75 + (1-\beta)\cdot 0.25}$$

and

$$P(G|S = s_B) = \beta_{s_B}(G) = \frac{(1-\alpha)\cdot 0.75}{(1-\alpha)\cdot 0.75 + \beta \cdot 0.25}.$$

Note that the average of the posterior beliefs is the prior, one has beliefs $\beta_{s_G}(\cdot)$ with probability $\alpha \cdot 0.75 + (1-\beta)\cdot 0.25$ and beliefs $\beta_{s_B}(\cdot)$ with probability $(1-\alpha)\cdot 0.75 + \beta \cdot 0.25$.

Here is the statement of the problem:

If $\alpha = \beta = \frac{1}{2}$, the signal structure is worthless. Give the set of $(\alpha, \beta) \ge (\frac{1}{2}, \frac{1}{2})$ for which the information structure strictly increases the expected utility of the decision maker. [You should find that what matters for increasing utility is having a positive probability of changing the decision.] Verify that the average of the posteriors is the prior.

Ans. If $\alpha = \beta = \frac{1}{2}$, then $\beta_{s_G}(G) = \beta_{s_B}(G) = 0.75$, and your decision never changes, no matter what signal you observe, hence this information structure is worthless.

Suppose now that $\alpha, \beta > \frac{1}{2}$. In this case $\beta_{s_B}(G) < 0.75 < 0.8$, so the signal s_B will not change the action from N. By contrast, for $\alpha, \beta > \frac{1}{2}$, we have $\beta_{s_G}(G) > 0.75$. Provided $\beta_{s_G}(G) > 0.80$, your optimal choice of action will be chosen. Thus, the condition on α and β is

$$\frac{\alpha \cdot 0.75}{(\alpha \cdot 0.75 + (1-\beta) \cdot 0.25} > \frac{8}{10},$$

first multiplying the top and bottom of the left-hand side by 4 and then cross-multiplying, this is

$$30\alpha > 24\alpha + 8(1-\beta), \text{ that is, } \frac{\alpha}{(1-\beta)} > \frac{4}{3},$$

or $\beta > 1 - \frac{3}{4}\alpha$.

7. From the *Notes*, Chapter 2, E.2 (the value of independent observations).

Now suppose that the test/experiment can be run twice and that the results are independent across the trials. Thus, $P(S = (s_G, s_G)|G) = \alpha^2$, $P(S = (s_G, s_B)|G) = P(S = (s_B, s_G)|G) = \alpha(1-\alpha)$, and $P(S = (s_B, s_B)|G) = (1-\alpha)^2$ with the parallel pattern for B. Fill in the probabilities in the following joint distribution $q(\cdot, \cdot)$ and verify that the average of posterior beliefs is the prior belief.

	Good	Bad
(s_G, s_G)	$\frac{3}{4}\alpha^2$	$\frac{1}{4}(1-\beta)^2$
(s_G, s_B)	$\frac{3}{4}\alpha(1-\alpha)$	$\frac{1}{4}\beta(1-\beta)$
(s_B, s_G)	$\frac{3}{4}\alpha(1-\alpha)$	$\frac{1}{4}\beta(1-\beta)$
(s_B, s_B)	$\frac{3}{4}(1-\alpha)^2$	$\frac{1}{4}\beta^2$

If $\alpha = \beta = \frac{1}{2}$, the signal structure is worthless. Give the set of $(\alpha, \beta) \geq (\frac{1}{2}, \frac{1}{2})$ for which the information structure strictly increases the expected utility of the decision maker.

To check the first statement (about worthlessness), note that if $\alpha = \beta = \frac{1}{2}$, then $P(G|s_1, s_2) = \frac{3}{4}$ for any signals s_1, s_2. To figure out which $\alpha, \beta > \frac{1}{2}$ strictly increase the expected utility of the decision maker, note first that $P(G|s_B, s_B) < 0.75 < 0.80$ so that the decision, N, will not change if the signal is s_B, s_B. The easiest of the three other cases is s_G, s_G, in which case α and β must satisfy

$$P(G|s_G, s_G) = \frac{0.75\alpha^2}{0.75\alpha^2 + 0.25(1-\beta)^2} > 0.8.$$

Rearranging yields

$$\left(\frac{\alpha}{(1-\beta)}\right)^2 > \frac{4}{3}, \text{ or } \left(\frac{\alpha}{(1-\beta)}\right) > \sqrt{\frac{4}{3}}.$$

By way of comparison, in the previous problem, the condition was $\left(\frac{\alpha}{(1-\beta)}\right) > \frac{4}{3}$, so this condition is easier to satisfy, independent repetition of the testing magnifies the strength of the good signal.

The slightly more complicated cases involve receiving the mixed signals s_G, s_B or s_B, s_G. If $\alpha = \beta > \frac{1}{2}$, these mixed signals are completely uninformative because $P(G|s_G, s_B) = P(G|s_B, s_G) = 0.75$. However, if $\alpha \neq \beta$, the signal can be informative. For it to be informative enough to change the action, we must have $P(G|s_G, s_B) > 0.8$, equivalently,

$$\frac{3\alpha(1-\alpha)}{3\alpha(1-\alpha)+\beta(1-\beta)} > \frac{4}{5},$$

which reduces (much as above) to

$$\frac{\alpha(1-\alpha)}{\beta(1-\beta)} > \frac{4}{3}.$$

Since $x(1-x) \equiv \frac{1}{4} - (x - \frac{1}{2})^2$, this in turn reduces to

$$3\left(\frac{1}{4} - (\alpha - \frac{1}{2})^2\right) > 4\left(\frac{1}{4} - (\beta - \frac{1}{2})^2\right).$$

A little extra work shows that the set of α and β satisfying this condition is a subset of the set of α and β satisfying the previous inequality, $\left(\frac{\alpha}{(1-\beta)}\right) > \frac{4}{3}$.

8. From the *Notes*, Chapter 2, E.3 (when more information adds nothing).

   Now suppose that the test/experiment can be run twice but that the results are dependent across the trials. Thus, $P(S = (s_G, s_G)|G) = \alpha$, $P(S = (s_G, s_B)|G) = P(S = (s_B, s_G)|G) = 0$, and $P(S = (s_B, s_B)|G) = (1-\alpha)$ with the parallel pattern for B. Fill in the probabilities in the following joint distribution $q(\cdot, \cdot)$.

	Good	Bad
(s_G, s_G)	$\frac{3}{4}\alpha$	$\frac{1}{4}(1-\beta)$
(s_G, s_B)	0	0
(s_B, s_G)	0	0
(s_B, s_B)	$\frac{3}{4}(1-\alpha)$	$\frac{1}{4}\beta$

   If $\alpha = \beta = \frac{1}{2}$, the signal structure is worthless. Give the set of $(\alpha, \beta) \geq (\frac{1}{2}, \frac{1}{2})$ for which the information structure strictly increases the expected utility of the decision maker.

   Ans. This is the same as the first of the information problems if: we read "(s_G, s_G)" here as being the same as "s_G" in that problem; and we read "(s_B, s_B)" here as being the same as "s_B" in that problem.

9. From the *Notes*, Chapter 2, E.4 (when more information adds something). **In the Notes, there was a typo in the following.**

   Now suppose that the test/experiment can be run twice but that the results are γ-independent across the trials. That is, $P(S = (s_G, s_G)|G) = \gamma\alpha^2 + (1-\gamma)\alpha$, $P(S = (s_G, s_B)|G) = P(S = (s_B, s_G)|G) = \gamma\alpha(1-\alpha) + (1-\gamma)0$, and $P(S = (s_B, s_B)|G) = \gamma(1-\alpha)^2 + (1-\gamma)(1-\alpha)$ with the parallel pattern for B. 1-independence is the independent signal structure given two problems above, 0-independence is the signal structure given in the previous problem. Fill in the probabilities in the following joint distribution $q(\cdot, \cdot)$ and verify that the average of the posterior beliefs is the prior belief.

	Good	Bad
(s_G, s_G)	$\frac{3}{4}(\gamma\alpha^2 + (1-\gamma)\alpha)$	$\frac{1}{4}(\gamma(1-\beta)^2 + (1-\gamma)(1-\beta))$
(s_G, s_B)	$\frac{3}{4}\gamma\alpha(1-\alpha)$	$\frac{1}{4}\gamma\beta(1-\beta)$
(s_B, s_G)	$\frac{3}{4}\gamma\alpha(1-\alpha)$	$\frac{1}{4}\gamma\beta(1-\beta)$
(s_B, s_B)	$\frac{3}{4}(\gamma(1-\alpha)^2 + (1-\gamma)(1-\alpha))$	$\frac{1}{4}(\gamma(\beta)^2 + (1-\gamma)\beta)$

Suppose that $(\alpha, \beta) \gg (\frac{1}{2}, \frac{1}{2})$ has the property that the 1-independent structure is strictly valuable. Find the set of γ such that γ-independence is strictly valuable.

Ans. What kind of perverse mind would ask you to solve this?!? My apologies, this leads through a thicket of algebra that is less informative than I hoped for.

10. From the *Notes*, Chapter 2, F.1 (expected profits and risk sensitivity).

You must make your production quantity decision before you know the price that will prevail in the market (and you are too small to have any effect on the price). Your cost function is $C(Q) = 200 + 2Q + 3Q^2$. There is %10 chance that the price will be \$300, and %20 chance that it will be \$400 and a %70 chance that it will be %600.

a. Find the quantity that maximizes your expected profit, give your optimal mean profits as well as the variance and standard deviation of your profits.

b. Repeat the previous to maximize $E\,\Pi(Q) - \frac{1}{6}\mathrm{Var}(\Pi(Q))$ where $\Pi(Q)$ is the random profit you make if you choose Q.

Ans. If you choose a quantity Q, your profits are the random variable

$$\Pi(Q) = \begin{cases} 300 \cdot Q - (200 + 2Q + 3Q^2) & \text{with probability } 0.10 \\ 400 \cdot Q - (200 + 2Q + 3Q^2) & \text{with probability } 0.20 \\ 600 \cdot Q - (200 + 2Q + 3Q^2) & \text{with probability } 0.70 \end{cases}$$

From this, $E\,\Pi(Q) = 530 \cdot Q - (200 + 2Q + 3Q^2)$ and $\mathrm{Var}(\Pi(Q)) = Q^2 \cdot \mathrm{Var}(P)$ where P is the random price, and $\mathrm{Var}(P) = 300^2 \cdot 0.10 + 400^2 \cdot 0.20 + 600^2 \cdot 0.70 - (530)^2 = 12,100$.

To maximize expected $\Pi(Q)$, solve

$$\max_{Q \geq 0}\ E\,\Pi(Q) = 530 \cdot Q - (200 + 2Q + 3Q^2),$$

since this is a quadratic in Q opening downwards, set the derivative equal to 0 and solve,

$$\frac{d}{dQ} E\,\Pi(Q) = 530 - 2 - 6Q = 0, \text{ or } Q^* = 88.$$

Plugging this back in, $E\,\Pi(Q^*) = 23,032$ and $\mathrm{Var}(\Pi(Q^*)) = 88^2 \cdot 12,100 = 93,704,400$.

To maximize $E\,\Pi(Q) - \frac{1}{6}\mathrm{Var}(\Pi(Q))$, solve

$$\max_{Q \geq 0}\ E\,\Pi(Q) - \frac{1}{6}12,100 \cdot Q^2.$$

Again, this is a quadratic in Q opening downwards, now the FOCs are

$$530 - 2 - 6Q - \frac{12,100}{6}Q = 0,$$

which has the solution $Q^* = 0.26$. Here the $\frac{1}{6}$ models a decision maker who is scared out of their mind about risk, they reduce their production levels by more than a factor of 300!

11. From the *Notes*, Chapter 2, F.2 (expected profits and the value of information).
    You are the only builder of houses for a planned sub-division. There is a %60 chance that the demand function for houses will be $P = 300,000 - 400Q$ and a %40 chance that it will be $P = 500,000 - 275Q$. Your cost function is $C(Q) = 140,000 + 240,000Q$. You have to make the decision about how many to build before you know whether the demand will be low or high,

    a. How many houses should you build to maximize your expected profits?
    Ans. As a function of Q, your profits are the random variable

$$\Pi(Q) = \begin{cases} (300,000 - 400Q)Q - (140,000 + 240,000Q) & \text{with probability } 0.60 \\ (500,000 - 275Q)Q - (140,000 + 240,000Q) & \text{with probability } 0.40 \end{cases}$$

    Therefore $E\,\Pi(Q) = (380,000 - 350Q)Q - (140,000 + 240,000Q)$, a quadratic in Q opening downwards. Maximizing yields $700Q = 240,000$, or $Q^* = 342.86$, since only integer numbers of houses make sense, $Q^* = 343$, the size of a cozy little sub-division. Here, expected profits are $6,702,850$, or roughly, 6.7 million dollars.

    b. How much expected profits are you losing because you must make your decision before you know the demand? [This is the cost of inflexibility.]
    Ans. If you knew the demand, e.g. if you had a crystal ball, then with probability 0.6 you would solve $\max_{Q \geq 0}(300,000 - 400Q)Q - (140,000 + 240,000Q)$ for $Q^* = 300$ and a **negative** profit of $-18,140,000$, and with probability 0.4 you would solve $\max_{Q \geq 0}(500,000 - 275Q)Q - (140,000 + 240,000Q)$ for $Q^* = 436$ and a profit of $60,943,600$, roughly 61 million.
    To answer the question of what your expected profits are, you need to figure out of the builder will go ahead and lose the 18 million if they knew the demand was going to be lousy. There are several possible ways to think about this: they are committed to the project and swallow the loss of 18 million; they haven't yet lined up financing and can back out with no loss, hence receive 0 instead of losing the 18 million; they decide to put up cheaper houses (lowering the marginal cost of $240,000$) in a fashion that does not hurt the demand curve too badly. If they decide to swallow the loss, their expected profits improve to $13,493,440$, or roughly, 13.5 million. If they can walk away with no loss, their expected profits improve to 24.3 million.

    c. An econometric forecaster offers to do a study of the market. The forecasts are not guaranteed accurate, rather they have the probability distribution given in the following table.

	Low Demand	High Demand
Low forecast	0.4	0.1
High forecast	0.2	0.3

    If you had access to this forecasting service, what would your maximal expected profits be? [The difference between this and the first answer gives the value of this kind of forecast.]
    Ans. If you receive the Low forecast, which happens with probability $\frac{1}{2}$, your beliefs become 0.8 on low demand and 0.2 on high demand, if you receive the High forecast, your beliefs become 0.4 on low demand and 0.6 on high demand.

If Low, $Q^* = 320$, $\Pi^* =$etc.

12. From the *Notes*, Chapter 2, F.3 (experience goods and quality uncertainty).

You become hungry while driving through a small town in west Texas. The next town is so far away that you will drive off the road in hypoglycemic shock if you do not stop and eat. There are two restaurants to choose from, a national chain burger joint with known quality, $q = 0.3$. There is also a small diner. Suppose that you know that the quality of diners in small west Texas towns has a uniform distribution on an interval $[\underline{q}, \underline{q} + 0.4]$.

a. If your utility function from quality is $u(q) = q$, find how large $\underline{q}$ would have to be in order for you to choose to go to the diner.
Ans. Here is one way to answer the question, it involves direct calculations. In the subsequent problems, we will use certainty equivalents. If the quality of diners is uniformly distributed on $[q, q+0.4]$ and $u(q) = q$, then your expected utility is $\int_q^{q+0.4} x \frac{10}{4}\,dx = q + 0.2$. Thus, for any q such that $q + 0.2 > 0.3$, you will surely go to the diner.

b. If your utility function from quality is $u(q) = \sqrt{q}$, find how large $\underline{q}$ would have to be in order for you to choose to go to the diner.
Ans. The certainty equivalent of a random variable uniformly distributed on $[q, q + 0.4]$ is the κ that solves $\sqrt{\kappa} = \int_q^{q+0.4} \sqrt{x} \frac{10}{4}\,dx$. As long as $\kappa > 0.3$, you will choose the diner. Since the integral is equal to $\frac{5}{3}\left[(q + 0.4)^{3/2} - q^{3/2}\right]$, setting this equal to $\sqrt{0.3}$ and solving yields $q \simeq 0.1116$ so for any q greater than 0.1116 you will go to the diner. In the previous problem, the you were risk neutral, hence $q = 0.1$ was the answer. Here you are risk averse, they need to be facing a better distribution in order to go to the diner.

c. If your utility function from quality is $u(q) = q^2$, find how large $\underline{q}$ would have to be in order for you to go to the diner.
Ans. Here you are a risk lover, variability in the quality of the food you get makes you happy. Using risk equivalents, find κ to solve $\kappa^2 = \int_q^{q+0.4} x^2 \frac{10}{4}\,dx$, that is, for any $q > 0.076887$ you will go to the diner. Here, being a risk lover, you are willing to tolerate worse distributions.

d. Suppose now your utility for quality is u^r for r in the interval $(0, 1)$. Find the set of r for which you would choose to go to the diner if quality is uniformly distributed on the interval $[0.2, 0.6]$.
Ans. Unfortunately, there is no r (except $r = 0$ which is a silly utility function) for which this is true for this problem. The issue is that the uniform distribution on 0.2 to 0.6 is just too good for the range of risk aversions given.