Sketches of Solutions for Assignment #5 Managerial Economics, Fall 2015 Due: Monday November 30

Readings: Three Baye chapters. Managing in Competitive, Monopolistic, and Monopolistically Competitive Markets. Basic Oligopoly Models. Game Theory: Inside Oligopoly.

A. Questions from intermediate microeconomics.

- 1. A firm sells its product in a perfectly competitive market where other firms charge a price of \$80 per unit. The firms total costs are $C(Q) = 40+8Q+2Q^2$.
 - a. How much output should the firm produce in the short run? **Ans**. $\Pi(Q) = 80Q - (40 + 8Q + 2Q^2)$. Since this is a quadratic opening downwards, this is maximized with $\Pi'(Q) = 0$, that is, when price, P =80, is equal to marginal cost, 8 + 4Q, that is $Q^* = 18$.
 - b. What price should the firm charge in the short run? Ans. It has no choice in the matter, P = 80 — a higher price would result in 0 sales, a lower price would push demand higher than the firm can profitably meet.
 - c. What are the firms short-run profits?
 - **Ans.** $\Pi^* = 80 \cdot 18 (40 + 8 \cdot 18 + 2 \cdot 18^2) = 248.$
 - d. What adjustments should be anticipated in the long run?
 - **Ans.** If all of the firms have access to the same technology as this firm and that is in fact the best technology, then we expect to see entry until $AC(Q) = (40 + 8Q + 2Q^2)/Q$ is minimized and price is equal to that lowest average cost. $AC(\cdot)$ reaches a minimum at $Q = \sqrt{20}$, at which point, average cost is $6 \cdot \sqrt{5} + 8 \simeq 21.4$.
- 2. You are the manager of a firm that produces a product according to the cost function $C(q_i) = 100 + 50q_i 4q_i^2 + q_i^3$. Determine the short-run supply function if:
 - a. You operate a perfectly competitive business.
 - **Ans**. $\pi(q_i) = pq_i C(q_i)$, the FOCs are $\pi'(q_i) = 0$ or $p C'(q_i) = 0$, the usual "price equals marginal cost" equation, but here it is important to check the SOCs as well, $\pi''(q_i) < 0$ (or else you will be at a local minimum for profits). The two expressions are

$$3q_i^2 - 8q_i + (50 - p) = 0$$
, and $-(6q_i - 8) < 0$.

The first yields $q_i = \frac{8 \pm \sqrt{64 - 12(50 - p)}}{6}$, the second yields $q_i > \frac{8}{6}$. Combining the short run supply function is

$$q_i^*(p) = \frac{8 \pm \sqrt{64 - 12(50 - p)}}{6}$$
 for $p > 44\frac{2}{3}$

The fixed cost, 100, is large enough that profits are negative until p > 74.5 (approximately), so the long-run supply function is the short-run for higher p and 0 for lower.

b. You operate a monopoly.

Ans. In this case, there is no supply function, you set either quantity or price.

- c. You operate a monopolistically competitive business.
 Ans. In this case, there is no supply function, you set either quantity or price.
- 3. The CEO of a major automaker overheard one of its division managers make the following statement regarding the firms production plans: "In order to maximize profits, it is essential that we operate at the minimum point of our average total cost curve." If you were the CEO of the automaker, would you praise or chastise the manager? Explain.

Ans. Chastise. Having some market power means that short run profits are maximized when marginal revenue is equal to marginal cost. This in turn may not be a good recommendation for the longer term, e.g. if it prices the division's products out of the market segment, re-design and its consequent change of cost function may be a best response.

4. The second largest public utility in the nation is the sole provider of electricity in 32 counties of southern Florida. To meet the monthly demand for electricity in these counties, which is given by the inverse demand function P = 1,000 - 5Q, the utility company has set up two electric generating facilities: Q_1 kilowatts are produced at facility 1, and Q_2 kilowatts are produced at facility 2 (so $Q = Q_1 + Q_2$). The costs of producing electricity at each facility are given by $C_1(Q_1) = 10,050 + 5Q_1^2$ and $C_2(Q_2) = 5,000 + 2Q_2^2$, respectively. Determine the cost function for the utility, $C(Q) = \min_{Q_1,Q_2} C_1(Q_1) + C_2(Q_2)$ subject to $Q = Q_1 + Q_2$. With this, determine the profit-maximizing amounts of electricity to produce at the two facilities, the optimal price, and the utility companys profits. Setting $Q_2 = Q - Q_1$ turns this into a one variable problem,

$$\min_{Q_1} 10,050 + 5,000 + 5Q_1^2 + 2(Q - Q_1)^2.$$

The solution happens where the two marginal costs are equal, that is, when $10Q_1 = 4(Q - Q_1)$, that is, $Q_1 = \frac{4}{14}Q$ and $Q_2 = \frac{10}{14}Q$. This means that the cost function is

$$C(Q) = (10,050+5,000) + 5(\frac{4}{14}Q)^2 + 2(\frac{10}{14}Q)^2 = 15,050 + \frac{280}{196}Q^2.$$

The profit maximization problem is

$$\max_{Q} Q(1,000-5Q) - (15,050 + \frac{280}{196}Q^2),$$

which has $Q^* = 77\frac{7}{9}$ as its solution, with a price of $1,000 - 388\frac{8}{9} = 611\frac{1}{9}$. B. Oligopoly questions.

- 1. The inverse market demand in a homogeneous-product Cournot duopoly is $P = 100 2(Q_1 + Q_2)$ and costs are $C_1(Q_1) = 12Q_1$ and $C_2(Q_2) = 20Q_2$.
 - a. Determine the reaction function for each firm.
 - b. Calculate each firms equilibrium output.
 - c. Calculate the equilibrium market price.
 - d. Calculate the profit each firm earns in equilibrium.Ans. The previous were done in lecture.
 - e. Calculate the quantity and profit that maximize the sum of consumer and producer surplus. Use this to calculate the social inefficiency due to the oligopolistic structure of the industry.

Ans. Because the cost and marginal cost for firm 1 is always lower, it is inefficient to have firm 2 produce anything. To find the social optimum, set price equal to 12, the lower marginal cost, and calculate the consumer surplus (area under the demand curve above the price p = 12. The producer surplus will be equal to 0 at that point.

- 2. The inverse demand for a homogeneous-product Stackelberg duopoly is P = 20,000-5Q. The cost structures for the leader and the follower, respectively, are $C_L(Q_L) = 3,000Q_L$ and $C_F(Q_F) = 4,000Q_F$.
 - a. What is the followers reaction function?
 - b. Determine the equilibrium output level for both the leader and the follower.
 - c. Determine the equilibrium market price and the profits of the leader and the follower.
 - d. Calculate the quantity and profit that maximize the sum of consumer and producer surplus. Use this to calculate the social inefficiency due to the oligopolistic structure of the industry.

Ans. Except for the last part, this was done in lecture, the last part mirrors the previous problem.

3. Two firms compete in a market to sell a homogeneous product with inverse demand function P = 400 - 2Q. Each firm produces at a constant marginal cost of \$50 and has no fixed costs. Use this information to compare the output levels and profits in settings characterized by Cournot, Stackelberg, Bertrand, and collusive behavior.

Ans. This is a good review problem.

4. You are the manager of BlackSpot Computers, which competes directly with Condensed Computers to sell high-powered computers to businesses. From the two businesses perspectives, the two products are indistinguishable. The large investment required to build production facilities prohibits other firms from entering this market, and existing firms operate under the assumption that the rival will hold output constant. The inverse market demand for computers is P = 5,100 - 0.5Q and both firms produce at a marginal cost of \$750 per computer. Currently, BlackSpot earns revenues of \$6.38 million and profits (net of investment, R&D, and other fixed costs) of \$1 million. The engineering department at BlackSpot has been steadily working on developing an assembly method that would dramatically reduce the marginal cost of producing these high-powered computers and has found a process that allows it to manufacture each computer at a marginal cost of \$500. How will this technological advance impact your production and pricing plans? How will it impact BlackSpots bottom line?

Ans. This is a Cournot competition problem, find the Cournot equilibrium quantities, the profits being \$1 million means that fixed costs are \$5.38 million. When BlackSpot's marginal cost decreases, their best response curve shifts outward, they produce more, their competitor produces less, and Blackspot's profits increase. You should be able to put numbers on these.

5. The market for a standard-sized cardboard container consists of two firms: CompositeBox and Fiberboard. As the manager of CompositeBox, you enjoy a patented technology that permits your company to produce boxes faster and at a lower cost than Fiberboard. You use this advantage to be the first to choose its profit-maximizing output level in the market. The inverse demand function for boxes is P = 800 - 4Q, CompositeBoxs costs are $C_C(Q_C) = 40Q_C$, and Fiberboard's costs are $C_F(Q_F) = 80Q_F$. Ignoring antitrust considerations, would it be profitable for your firm to merge with Fiberboard? If not, explain why not; if so, put together an offer that would permit you to profitably complete the merger.

Ans. Here is one (of many) easy ways to get to the profitability of merging — if you merge, you can mothball Fiberboard's technology and produce the same quantities using the lower $C_C(\cdot)$, and this yields a higher profit. To put it another way, you at least make $40 \cdot Q_F^S$ where Q_F^S is Fiberboard's Stackelberg quantity. Any offer to buy out Fiberboard that does not exceed the increase in profits from the resulting monopoly is feasible here. Again, you should be able to put numbers on these.

6. During the 1980s, most of the worlds supply of lysine was produced by a Japanese company named Ajinomoto. Lysine is an essential amino acid that is an important livestock feed component. At this time, the United States imported most of the worlds supply of lysine — more than 30,000 tons — to use in livestock feed at a price of \$1.65 per pound. The worldwide market for lysine, however, fundamentally changed in 1991 when U.S.-based Archer Daniels Midland (ADM) began producing lysine — a move that doubled worldwide production capacity. Experts conjectured that Ajinomoto and ADM had similar cost structures and that the marginal cost of producing and distributing lysine was approximately \$0.70 per pound. Despite ADMs entry into the lysine market, suppose demand remained constant at Q = 208 - 80P(in millions of pounds). Shortly after ADM began producing lysine, the worldwide price dropped to \$0.70. By 1993, however, the price of lysine shot back up to \$1.65. Use the theories discussed in the Baye chapter on Basic Oligopoly Models to provide a potential explanation for what happened in the lysine market.¹ Support your answer with appropriate calculations.

Ans. Price dropped to marginal cost, a Bertrand result, and then went back up to the price that prevailed when there was just one firm, the monopoly price, collusion. One could understand the initial drop as ADM proving to Ajinomoto that they could and would deliver enough lysine to drive prices down to marginal cost.

- C. Games.
 - 1. While there is a degree of differentiation among general merchandise retailers like Target and Kmart, weekly newspaper circulars announcing sales provide evidence that these firms engage in price competition. This suggests that Target and Kmart simultaneously choose to announce one of two prices for a given product: a regular price or a sale price. Suppose that when one firm announces the sale price and the other announces the regular price for a particular product, the firm announcing the sale price attracts 50 million extra customers to earn a profit of \$5 billion, compared to the \$3 billion earned by the firm announcing the regular price. When both firms announce the sale price, the two firms split the market equally (each getting an extra 25 million customers) to earn profits of \$1 billion each. When both firms

 $^{^{1}}$ For fun, find business press coverage of the scandal. While under-prosecuted, white collar crime is not completely un-prosecuted.

announce the regular price, each company attracts only its 50 million loyal customers and the firms each earn \$3 billion in profits.

a. If you were in charge of pricing at one of these firms, would you have a clear-cut pricing strategy? If so, explain why. If not, explain why not.

Ans. The one-shot game has the following form,

	Sale	Regular
Sale	(1,1)	(5,3)
Regular	(3,5)	(3,3)

Clarity would be achieved if either firm had a dominant strategy and one could work out the implications, but neither firm has a dominant strategy. There are two pure strategy equilibria, but there is disagreement about which of the possible pure strategy equilibria is best, payoffs of (5,3) versus (3,5). The mixed strategy equilibrium gives payoffs (3,3), which is Pareto dominated by either pure strategy equilibrium. It is not clear, from the little given here, what the firms will do.

b. Solve for the three equilibria for the one-shot game the describes a "oneoff" pricing decision.

Ans. The pure strategy equilibria are as just given, (Regular,Sale) and (Sale,Regular), with payoffs (3,5) and (5,3), the mixed equilibrium has both randomizing $(\frac{1}{2}, \frac{1}{2})$, with payoffs (3,3).

c. Propose a mechanism that might solve your dilemma. (Hint: Unlike Walmart, neither of these two firms guarantees "Everyday low prices.")

Ans. If the two firms alternated week-ends for their sales, then on average the payoffs would be (4, 4). There are many many other possible correlating devices.

2. Two firms, a supplier and a manufacturer can invest in expensive, complementary technologies, and if they both do this, they will achieve the high quality output that will guarantee both high profits. The problem is that if one of them has invested, the other firm would be better "free riding" on their investment, it's an expensive investment for both of them, and the improvements on just one side will improve profits somewhat, at no cost to the non-investor. Putting numbers on the payoffs, let us suppose they are

	Don't invest	Invest
Don't invest	(5,7)	(32, 0)
Invest	(0, 22)	(28, 19)

a. Suppose that a vertical merger or acquisition is arranged and that the joint firm receives the sum of the payoffs to the two firms. What is the optimal investment pattern for the joint firm?

Ans. Both invest, profits are 28 + 19 = 47, which is greater than 0 + 22, 32 + 0, or 5 + 7.

b. Suppose that one of the firms hasn't had the advantage of your experience with the idea of solving dynamic interactions by "looking forward and solving backwards." Not knowing this cardinal principle, they decide that they will move first, invest, and give the other firm every incentive to invest. What will be the result?

Ans. If they try moral suasion, we rather expect the other firm to freeride. If firm 1 has invested and they can commit to paying firm 2 at least x > 3 for investing, firm 2 will invest and profits will be (28 - x, 19 + x). If firm 2 has invested and they can commit to paying firm 1 at least y > 4 for investing, firm 1 will invest and profits will be (28 + y, 19 - y). The commitment can take many forms, perhaps signing an exclusive supply/demand agreement at a price that transfers the x or the y.

c. Consider contracts of the form: "I will invest, and if I do not invest while you have invested, I owe you damages of x. You will invest, and if you do not invest while I have invested, you owe me damages of x. Further, this contract is not valid unless both of us have signed it." For what values of x will the contract have the property that signing the contract and then investing becomes the dominant strategy?

Ans. Consider the game

	Don't invest	Invest
Don't invest	(5,7)	(32-z,z)
Invest	(z, 22 - z)	(28, 19)

We wish make Invest a dominant strategy for both, this requires z > 5, 28 > 32 - z, z > 7, and 19 > 22 - z. So any z > 7 satisfies all of these requirements. Note the contrast between the payoffs (28, 19) and (28 - x, 19 + x) for x > 3 or (28 + y, 19 - y) for y > 4.