Assignment #6 for Managerial Economics, Fall 2015
Due: Economics Dept. front desk, 2 p.m., Monday December 7'th, 2015

Readings. Baye textbook coverage of: mixed/randomized equilibria; repeated games; competition between firms with market power.

A. Practice with 2×2 games.

1. Find all of the equilibria and their associated equilibrium utility levels for the following game.

	Left	Right
Up	$(25, 45)$	$(38, 78)$
Down	$(3, 63)$	$(71, 55)$

2. Find all of the equilibria and their associated equilibrium utility levels for the following game.

	Left	Right
Up	$(52, 42)$	$(93, 16)$
Down	$(46, 18)$	$(89, 89)$

3. Find all of the equilibria and their associated equilibrium utility levels for the following game.

	Left	Right
Up	$(46, 4)$	$(94, 53)$
Down	$(91, 29)$	$(92, 22)$

4. Find all of the equilibria and their associated equilibrium utility levels for the following game.

	Left	Right
Up	$(37, 86)$	$(58, 3)$
Down	$(16, 47)$	$(68, 99)$

B. Monitoring games.

1. Consider the following game

	Audit	Other
Divert	$(\pi - f, B - C)$	$(\pi + b, 0)$
Clean	$(\pi, -C)$	$(\pi, 0)$

We assume that $B > C > 0$, $\pi, f, b > 0$. The interpretation is the one given in class: player 1 may divert corporate funds to their own uses or run a clean operation; the auditors can audit player 1's operation or do the other parts of their job. The opportunity cost of auditing player 1 is $C > 0$, the benefit to catching diversion is $B > C$, etc.

a. Give the unique equilibrium for this game.
b. For what values of f, b, B, and C is there little diversion despite infrequent auditing? Explain the economics of this.
c. For what values of f, b, B, and C is there a great deal of diversion despite frequent auditing? Explain the economics of this.

2. An office manager is concerned with declining productivity. Despite the fact that she regularly monitors her clerical staff four times each day — at 9:00 AM, 11:00 AM, 1:00 PM, and again at 3:00 PM, office productivity has declined 30 percent since she assumed the helm one year ago. Would you recommend that the office manager invest more time monitoring the productivity of her clerical staff? Explain.

C. Repeated games

1. At a time when demand for ready-to-eat cereal was stagnant, a spokesperson for the cereal maker Kellogg's was quoted as saying, " ... for the past several years, our individual company growth has come out of the other fellow's hide." Kellogg's has been producing cereal since 1906 and continues to implement strategies that make it a leader in the cereal industry. Suppose that when Kelloggs and its largest rival advertise, each company earns \$0 billion in profits. When neither company advertises, each company earns profits of \$8 billion. If one company advertises and the other does not, the company that advertises earns \$48 billion and the company that does not advertise loses \$1 billion. Under what conditions could these firms use Nash reversion trigger strategies to support the collusive level of advertising?
2. A producer can produce a high quality good or they can cut costs and produce a low quality good. The potential buyer cannot ascertain the quality of the good before they buy it. The associated payoffs are

	Low quality	High quality
Don't Buy	$(0, 0)$	$(0, -2)$
Buy	$(-2, 2)$	$(1, 1)$

   a. Give the unique equilibrium for the one-shot version of this game.
   b. Under what conditions could the buyer and producer use Nash reversion trigger strategies to support purchase of high quality products?

D. A product differentiation game. Two firms, unimaginatively i and j, produce partially substitutable goods. The inverse demand functions are

$$q_i(p_i, p_j) = \alpha_i - \beta_i p_i + \gamma_i p_j, \text{ and}$$
$$q_j(p_i, p_j) = \alpha_j - \beta_j p_j + \gamma_j p_i$$

where the parameters, α_i, β_i, γ_i, α_j, β_j and γ_j are all strictly positive. The firms' cost functions are $C_i(q_i) = c_j q_i$ and $C_j(q_j) = c_j q_j$.

a. Find the firms' best response functions.
b. Find the equilibrium prices.
c. Suppose that both firms are acquired by a third firm and that the third firm picks both prices. Give the third firm's profits and explain why they charge higher prices for the two goods.
d. Under what conditions could the two firms use Nash reversion trigger strategies to support the same higher prices and profits?

E. Bonuses and incentives. The next several problems refer to the following situation. Profits for the next fiscal year are a random variable Π. The distribution of Π depends on employee efforts, e. In particular, for any level of profits x and any $e' > e$, $Prob(\Pi > x|e') \geq Prob(\Pi > x|e)$. This means that higher efforts increase the probability that profits are higher than x for any and all values of x.

Employees are paid a salary S and a bonus B if profits reach or exceed a target T. Employee utility if they receive y dollars and put in effort level e is $u(y, e) = v(y) - c(e)$ where $v(\cdot)$ is increasing in y and $c(\cdot)$ is increasing in e. (You could think of e as hours spent on the job, but this is a very crude measure of effort, thought and imagination matter). Employee expected utility is

$$\max_{e \in [0, \overline{e}]} \; [v(S)(1 - P(\Pi > T|e)) + v(S + B)P(\Pi > T|e)] - c(e).$$

There are several reasons to make the bonus depend on Π rather than on e: the employees may know better than the managers what specific forms the effort should take; the managers may not be able to observe e; and we care about outputs, not inputs, a really talented employee may be able to achieve high levels of Π with minimal effort, or may make hard work look effortless.

1. Show that optimal effort, e^*, is increasing in the bonus level, B.
2. Suppose that $v(\cdot)$ demonstrates everywhere positive and everywhere decreasing marginal utility of income.
   a. Show that optimal effort decreases in S.
   b. In professional baseball, we have the observation that, on average, in the year after signing a large new contract, MLB players tend to do worse than they had in the past. In what way is the previous analysis related?
   c. [Harder] Suppose that bonuses, B, are kept constant as a proportion of salary, S, that is $B = \kappa S$ for a fixed number κ. Give conditions on $v(\cdot)$ that would make optimal effort <u>in</u>crease in S, and conditions that would make optimal effort decrease.
3. Setting the target too high or too low is counterproductive.
   a. Explain why we would not expect optimal effort to be monotonic in T.
   b. Explain why the expected utility cannot be (strictly) supermodular in effort, e, and the target level, T.
4. The role of outside options. From the above, you might conclude that lowering S and compensating with a higher B is the optimal strategy for management. This may be short-sighted. Suppose that employees have an "outside option," that is, they can go work for someone else, or go start their own firm, or retire. Suppose that the outside option gives them expected utility of $\overline{v}$. This means that the employee's expected utility maximization problem is now

   $$\max\{\underline{v}, \; \max_{e \in [0, \overline{e}]} \; [v(S)(1 - P(\Pi > T|e)) + v(S + B)P(\Pi > T|e)] - c(e)\}.$$

   The interpretation is that if S and B and the optimal effort that they imply becomes too unrewarding, the employees will take their skills and abilities elsewhere.
   a. Why might optimal effort increase in S?
   b. How does optimal effort depend on B?