Basic Informational Economics

Assignment #4 for Managerial Economics, ECO 351M, Fall 2016
Due, Monday October 31 (Halloween).

The Basic Model

One must pick an action, a in a set of possible actions A, before a random variable, X, takes one of its possible values, $X = x$, at which point, the realized utility is $u(a, x)$. Before on picks the action, one observes a signal, S, taking one of its possible values $S = s$. The signal S is correlated with the random variable X, and this is what makes it useful. The joint distribution of X and S is given by $q(x, s) = Prob(X = x, S = s)$. Here we are assuming that the values of X and S are discrete, changing to continuous random variables makes the math harder without adding a great deal of intuition. In any case, the expected utility maximizer's problem is to pick a function from observations to actions, $s \mapsto a(s)$, so as to maximize

$$(\ddagger) \qquad E\,u(a(S), X) = \sum_{x,s} u(a(s), x)q(x, s).$$

We will use two pieces of notation: $\pi(s) = Prob(S = s) = \sum_x q(x, s)$ is the distribution of the random signal S; $\beta(x) = Prob(X = x) = \sum_s q(x, s)$ is the original (or prior) distribution of the random variable X.

Solving the Basic Model

There are, theoretically, two ways to solve the problem in equation ($\ddagger$). The first, wildly impractical way to solve the problem is to look at all possible policies, that is, to look at each and every function $a : S \to A$, for each function, calculate $E\,u(a(S), X)$, and then pick the one giving the maximal value. The second way is known as "crossing the bridge when you come to it," one waits until observing a signal value, $S = s$, and then figures out the best thing to do. Since you only observe one signal, s, this means that you don't need to think about what you would have done if some other signal, $s' \neq s$, had happened.

> An example demonstrates the comparison between the first and the second approach: the first approach involves dong your homework for this class by figuring out what answer you would give to every problem I could possibly assign, and then once you see what problems I actually assign, just pick your answers from what you've already done; the second approach waits until you see what problems are actually assigned and then solves those.

All that is left is to "figure out the best thing to do" after seeing $S = s$. We do this by re-writing equation ($\ddagger$) above. We now use $\pi(s) = \sum_x q(x, s) = Prob(S = s)$. Using this, rewrite $E\,u = \sum_{x,s} u(a(s), x)q(x, s)$ as

$$\sum_s \pi(s) \sum_x u(a(s), x) \tfrac{q(x,s)}{\pi(s)}$$

where the sum is over the s such that $\pi(s) > 0$ so that we're never dividing by 0. Let us now define your 'beliefs,' denoted by β as a mnemonic for beliefs, about the probability that $X = x$ given that we have seen $S = s$. From Bayes Rule, this is

$$Prob(X = x|S = s) = \beta(x|s) := \tfrac{q(x,s)}{\pi(s)},$$

also known as “the **posterior probability** that $X = x$ after $S = s$ has been observed.

After seeing $S = s$, let $a^*(s)$ be the solution to the problem

$$\max_{a \in A} \sum_x u(a, x)\beta(x|s).$$

Here is the important result.

> If $a^*(s)$ solves $\max_a \sum_x u(a, x)\,\beta(x|s)$ at each s with $\pi(s) > 0$, then $a^*(\cdot)$ solves the original maximization problem in (‡).

The reason is that $E\,u(a^*(S), X)$ is the weighted average $\sum_s \pi(s) \sum_x u(a^*(s), x)\beta(x|s)$, and the only way to maximize this weighted average is to maximize each part of it that receives a positive weight.

Valuing Information

Finally, let $V^\circ = \max_{a\in A} E\,u(a, X) = \max_{a\in A} \sum_x u(a, x)\beta(x)$ be the maximal expected value when the decision maker has no signal and $V^* = E\,u(a^*(S), X)$ be their maximal expected value when they can observe the value of the signal S before choose their action. We know that $V^* \geq V^\circ$ (look at the weighted average argument).

We define the **value of the information structure** as $V^* - V^\circ$.

Problems

For much of this set of problems, your job will be to figure out the best response to different beliefs and then to put them together in the right way.

1. [Infrastructure investment] $X = G$ or $X = B$ corresponds to the future weather pattern, the actions are to Leave the infrastructure alone or to put in New infrastructure, and the signal, s, is the result of investigations and research into the distribution of future values of X. The utilities $u(a, x)$ are given in the following table where c is the cost of the new infrastructure.

	Good	Bad
L	10	6
N	$(10 - c)$	$(9 - c)$

   a. After you see a signal $S = s$, you form your posterior beliefs $\beta(G|s)$ and $\beta(B|s)$. For what values of $\beta(G|s)$ do you optimally choose to leave the infrastructure as it is? Express this inequality as “L if $c > M$,” give the value of M in terms of $\beta(G|s)$, and interpret. [You should have something like “leave the old infrastructure as it is if costs are larger than expected gains.”]

   b. If $c = 0.60$, that is, if the new infrastructure costs 20% of the damages it prevents, give the set of $\beta(G|s)$ for which it optimal to leave the old infrastructure in place.
   For the rest of this problem, assume that $c = 0.60$.

   c. Suppose that the original or prior distribution has $\beta(G) = 0.75$ so that, without any extra information, one would put in the New infrastructure. We now introduce some signal structures. Suppose that we can run test/experiments that yield $S = s_G$ or $S = s_B$ with $P(S = s_G|G) = \alpha \geq \frac{1}{2}$ and $P(S = s_B|B) = \gamma \geq \frac{1}{2}$. The joint distribution, $q(\cdot, \cdot)$, is

	Good	Bad
s_G	$\alpha \cdot 0.75$	$(1-\gamma) \cdot 0.25$
s_B	$(1-\alpha) \cdot 0.75$	$\gamma \cdot 0.25$

Give $\beta(\cdot|S = s_G)$ and $\beta(\cdot|S = s_B)$. Verify that the average of the posterior beliefs is the prior, that is, verify that $\sum_s \pi(s)\beta(\cdot|x) = \beta(\cdot)$.

d. Show that if $\alpha = \gamma = \frac{1}{2}$, then the signal structure is worthless.

e. Give the set of $(\alpha, \gamma) \geq (\frac{1}{2}, \frac{1}{2})$ for which the information structure strictly increases the expected utility of the decision maker. [You should find that what matters for increasing utility is having a positive probability of changing the decision.]

2. [A more theoretical problem] We say that signal structure A is unambiguously better than signal structure B if every decision maker, no matter what their action set and what their utility functions are, would at least weakly prefer A to B. This problem asks you to examine some parts of this definition.

Let us suppose that the random variable takes on two possible values, x and x', with probabilities $p > 0$ and $p' = (1-p) > 0$, that there are two possible actions, a and b, that the signals take on two values s and s', that the signal structure is given by

	x	x'
s	αp	$(1-\gamma)p'$
s'	$(1-\alpha)p$	$\gamma p'$

and that the utilities are given by

	x	x'
a	$u(a, x)$	$u(a, x')$
b	$u(b, x)$	$u(b, x')$

We assume that $\alpha > \frac{1}{2}$ and $\gamma > \frac{1}{2}$ and we let $V(\alpha, \gamma)$ denote the maximal expected utility when the signals are distributed as above and the utilities are above.

a. Show that $V(\alpha, \gamma) \geq V(\frac{1}{2}, \frac{1}{2})$, that is, the information structure with $\alpha > \frac{1}{2}$ and $\gamma > \frac{1}{2}$ is unambiguously better than the structure with $\alpha = \gamma = \frac{1}{2}$.

b. Give utilities such that $V(\alpha, \gamma) > V(\frac{1}{2}, \frac{1}{2})$, that is, show that some decision maker strictly prefers the information structure with $\alpha > \frac{1}{2}$ and $\gamma > \frac{1}{2}$ to the structure with $\alpha = \gamma = \frac{1}{2}$.

c. Show that for any utilities, if $\alpha' > \alpha \geq \frac{1}{2}$ and $\gamma' > \gamma \geq \frac{1}{2}$, then $V(\alpha', \gamma') \geq V(\alpha, \gamma)$.

d. Give utilities such that $V(\alpha', \gamma') \geq V(\alpha, \gamma)$ for $\alpha' > \alpha \geq \frac{1}{2}$ and $\gamma' > \gamma \geq \frac{1}{2}$.

3. [The value of repeated independent observations] This problem continues where the first problem left off. Now suppose that the test/experiment can be run twice and that the results are independent across the trials. Thus, $P(S = (s_G, s_G)|G) = \alpha^2$, $P(S = (s_G, s_B)|G) = P(S = (s_B, s_G)|G) = \alpha(1-\alpha)$, and $P(S = (s_B, s_B)|G) = (1-\alpha)^2$ with the parallel pattern for B.

a. Fill in the probabilities in the following joint distribution $q(\cdot, \cdot)$. Verify that the average of posterior beliefs is the prior belief.

	Good	Bad
(s_G, s_G)		
(s_G, s_B)		
(s_B, s_G)		
(s_B, s_B)		

b. Give $\beta(G|(s_G, s_G))$, $\beta(G|(s_G, s_B))$, $\beta(G|(s_B, s_G))$, and $\beta(G|(s_B, s_B))$.
c. Show that if $\alpha = \beta = \frac{1}{2}$, then the signal structure is worthless.
d. Give the set of $(\alpha, \beta) \geq (\frac{1}{2}, \frac{1}{2})$ for which the information structure strictly increases the expected utility of the decision maker.
e. Explain why the set is larger here than it was in the previous problem.

4. You are the only builder of houses for a planned sub-division. There is a %40 chance that the demand function for houses will be low, $P = 300,000 - 300 \cdot Q$, and a %60 chance that it will be high, $P = 400,000 - 250 \cdot Q$. Your cost function is $C(Q) = 1,400,000 + 120,000Q$. You have to make the decision about how many to build before you know whether the demand will be low or high,
a. How many houses should you build to maximize your expected profits? What are your maximal expected profits?
b. How much would you be willing to pay for a "crystal ball" forecast of demand conditions, that is, a perfectly accurate forecast?
c. An econometric forecaster offers to do a study of the market. The forecasts are not guaranteed accurate, rather they have the probability distribution given in the following table.

	Low Demand	High Demand
Low forecast	0.3	0.2
High forecast	0.1	0.4

If you had access to this forecasting service, what would your maximal expected profits be? What would your willingness to pay for this forecast be?

5. From Ch. 18 of Kreps's *Micro for Managers*, Problem 18.2(a) and (b).

6. From Ch. 18 of Kreps's *Micro for Managers*,
a. Problem 18.3,
b. Problem 18.5, and
c. Problem 18.6.

7. By putting in effort $0 \leq e \leq \overline{e}$, a business can reduce the probability of fire in the warehouse to $P(e)$ where $P'(e) < 0$ and $P''(e) > 0$. If there is a fire, there will be a loss $L > 0$ and profits will be $(R - L) > 0$, if there is not a fire, they will be R. The utility costs of effort to firm θ, $0 < \theta < 1$, are $(1 - \theta)c(e)$ where $c'(e) > 0$ and $c''(e) > 0$. Higher θ's correspond to lower costs of prevention, so these are "better" firms. One simple form of insurance policy costs C and has a deductible D, that is, it reduces losses from L to D by paying $(L - D)$ to an insured firm in case of a fire causing loss L. As is only sensible, $C < D < L$.

The problem for firm θ without insurance is

$$E\,\Pi(\theta) = \max_{0 \leq e \leq \overline{e}} [P(e)(R - L) + (1 - P(e))R] - (1 - \theta)c(e),$$

while the problem for firm θ with insurance is

$$E\,\Pi_{ins}(\theta) = \max_{0 \leq e \leq \overline{e}} [P(e)((R - C) - D) + (1 - P(e))(R - C)] - (1 - \theta)c(e).$$

a. For a firm with a given θ, compare their optimal effort if they have insurance to their optimal effort if they do not have insurance. Explain. [The difference you find is due to what is called 'moral hazard.']
b. For a given insurance policy with cost C and deductible D, which firms, indexed by their θ's, buy insurance? What happens to their riskiness after they buy insurance?
c. An actuarially fair insurance policy is one in which the cost exactly covers the expected payments. In more detail, if p is the average probability of loss among the firms that buy insurance, and $(L - D)$ is what the insurance company pays out every time there is a fire, then $C = p(L - D)$. Verbally explain why the previous results imply that an actuarially fair policy may not insure very many firms.

8. From Ch. 18 of Kreps's *Micro for Managers*, Problem 18.8.
9. Mazzucato argues, in Ch. 2 of *The Entrepreneurial State*, that when one looks across firms, the correlation between R&D expenditures and growth is very small unless one considers a special subset of the firms. Why should the selection bias exist and how does she solve it?
10. Mazzucato argues, in Ch. 3 of *The Entrepreneurial State*, that venture capitalists tend to come into the process of turning new scientific knowledge into new products very late in the process and that they tend to have a time horizon much shorter than is usually needed to bring the products to market. What mechanisms does she describe for venture capitalists that allow them to get out, with a profit, before the product comes to market?
11. Mazzucato argues, in Ch. 7 of *The Entrepreneurial State*, that high aggregate levels of R&D is not enough for a country to benefit from innovation. What other factors does she name that have the property that $\partial^2 Ben(R, x)/\partial R \partial x > 0$? Here R measures R&D and x measures the other factors.