**ECO 387L.26, Advanced Microeconomic Analysis**
**Large Games and Continuous Time Games**
FALL 2013

**Instructor:** Maxwell B. Stinchcombe,

**Time and Location:** M, W 12:30-1:50pm, BRB 1.120

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**Office Hours:** T, Th 9:30-11:00am, BRB 2.102A

## OVERVIEW AND OBJECTIVES

We are going to cover the results and techniques behind recent advances in the theory of games with large player sets, and for games played in continuous time. The work for the earlier part of the semester will involve problem sets designed to develop and strengthen your command of the techniques, in the latter part of the semester, it will be more focused on using the results and techniques on problems you are working on.

## REQUIREMENTS

Graduate Microeconomics I and II, a graduate course in Probability and Statistics, and a semester of Mathematics for Economists or the equivalent, are necessities. Having had a course in game theory is desirable, but not strictly necessary.

## ADMINISTRATIVE ISSUES

I will both e-mail and post (on my web-page) assignments, papers, and other material.

## REQUIRED TEXTS

- Corbae, D., Stinchcombe, M. B., and Zeman, J. (2009). *An introduction to mathematical analysis for economic theory and econometrics*. Princeton University Press, Princeton, NJ
- Nelson, E. (1987). *Radically elementary probability theory*, volume 117 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ

## RECOMMENDED TEXTS

- Fudenberg, D. and Tirole, J. (1991). *Game theory*. MIT Press, Cambridge, MA
- Wolff, M. and Loeb, P. A., editors (2000). *Nonstandard analysis for the working mathematician*, volume 510 of *Mathematics and its Applications*. Kluwer Academic Publishers, Dordrecht
- Nelson, E. (1987). *Radically elementary probability theory*, volume 117 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ
- Lindstrøm, T. (1988). An invitation to nonstandard analysis. In *Nonstandard analysis and its applications (Hull, 1986)*, volume 10 of *London Math. Soc. Stud. Texts*, pages 1–105. Cambridge Univ. Press, Cambridge

## 1. Introduction

We will start by introducing infinitesimals and re-doing several (hopefully) familiar pieces of game theory with them, perfect, proper, and stable equilibria. Infinitesimals are the quantities denoted $df$, $dg$, $dx$, $dt$, or similar, from your calculus class. These might have been presented as being nothing more than "devices," although they are devices that, often enough, let you calculate the answer. Along the way, we will re-do big parts of calculus and give a very different proof of the implicit function theorem.

The real line plus the infinitesimals plus the infinite numbers is denoted ${}^*\mathbb{R}$, read "star-R." With $\mathcal{P}_F(X)$ denoting the finite subsets of a set $X$, ${}^*\mathcal{P}_F(X)$ is the class of star-finite sets. The ones that we will be interested in are at infinitesimal distance for the set $X$.

1. In the first part of the course, we will use $^*$-finite sets to analyze compact and continuous normal form games. Along the way, we will go through the basics of probability measures on metric spaces and give a very different proof of one of the basic results — the Riesz representation theorem.
2. In the second part, we will use $^*$-finite sets to model large populations. We will look at some older core theory and some recent advances in implementation theory. In both cases, the asymptotic interpretations of the star-finite sets will be crucial.
3. In the third part, we will use $^*$-finite sets to model time increments, replace e.g. $[0, T]$ with $^*$-finite time line $\{0, dt, 2 \cdot dt, \ldots, T - dt, T\}$. Our starting point will some results in stochastic process theory: existence of Brownian motion; Itô's lemma; point processes; infinitely divisible laws and Levy processes; the martingale convergence theorem. With the $^*$-finite time line, these results are much easier to understand, prove, and use, the "only" tricky part is going back to the model of the time line that most people use.

## 2. Star-Finite Models of Large Sets (8/28 - 9/18)

Our first task in this part of the course is to model infinitely small, non-zero numbers. Our first game theoretic use of these numbers will be to model perturbations in perfect, proper, and (some version of) stable equilibria.

Our second task in this part of the course is to model large finite subsets of compact sets that are an infinitesimal distance away from the compact sets. Particularly important for our later work is the set of asymptotic interpretations that we will have. Our game theoretic use of these sets will be to define and analyze the perfect and proper (and stable if you feel like it) equilibria of games with compact strategy sets and jointly continuous payoffs. If time permits, we will replace "jointly continuous payoffs" with bounded payoffs. While the $^*$-finite theory is the same, going back to the model of strategies that most people use is, as described above, a bit "tricky."

Our third in this part of the course is to understand just how tricky it is to get back to the model that that most people use when we analyze the $^*$-finite versions of extensive form games.

### 2.1. Finite Game Equilibrium Refinement via Infinitesimals.

- Corbae, D., Stinchcombe, M. B., and Zeman, J. (2009). *An introduction to mathematical analysis for economic theory and econometrics*. Princeton University Press, Princeton, NJ (Chapter 11), alternately Lindstrøm, T. (1988). An invitation to nonstandard analysis. In *Nonstandard analysis and its applications (Hull, 1986)*, volume 10 of *London Math. Soc. Stud. Texts*, pages 1–105. Cambridge Univ. Press, Cambridge
- Fudenberg, D. and Tirole, J. (1991). *Game theory*. MIT Press, Cambridge, MA (Chapter 8.4, 11)
- Blume, L., Brandenburger, A., and Dekel, E. (1991a). Lexicographic probabilities and choice under uncertainty. *Econometrica*, 59(1):61–79

- Blume, L., Brandenburger, A., and Dekel, E. (1991b). Lexicographic probabilities and equilibrium refinements. *Econometrica*, 59(1):81–98

2.2. **Star-Finite Analysis of Compact and Continuous Games.**

A. Basic tools
   1. Corbae, D., Stinchcombe, M. B., and Zeman, J. (2009). *An introduction to mathematical analysis for economic theory and econometrics*. Princeton University Press, Princeton, NJ (Chapter 9, 11)
   2. Cutland, N. J. and Feng, H. Q. (1993). An infinitesimal proof of the implicit function theorem. *Glasgow Math. J.*, 35(2):163–166
   3. Ross, D. (1989). Yet another short proof of the Riesz representation theorem. *Math. Proc. Cambridge Philos. Soc.*, 105(1):139–140

B. Existence and refinement, Simon, L. K. and Stinchcombe, M. B. (1995). Equilibrium refinement for infinite normal-form games. *Econometrica*, 63(6):1421–1443

C. Extensions via compactification, Harris, C. J., Stinchcombe, M. B., and Zame, W. R. (2005). Nearly compact and continuous normal form games: characterizations and equilibrium existence. *Games Econom. Behav.*, 50(2):208–224

2.3. **Star-Finite Analysis of Infinite Extensive Form Games.**

A. Overview handout (from AMES 2013)

B. Optional, Stinchcombe, M. B. (2005). Nash equilibrium and generalized integration for infinite normal form games. *Games Econom. Behav.*, 50(2):332–365

C. Optional, Stinchcombe, M. B. (2000). The finitistic theory of infinite extensive form games. *Working paper, Department of Economics, U.T. Austin*

## 3. Large Population Models

3.1. **Non-Cooperative Games.**

A. Khan, M. A. and Sun, Y. (2002). Non-cooperative games with many players. *Handbook of Game Theory with Economic Applications*, 3:1761–1808

B. Stinchcombe's game theory notes

C. Khan, M. A. and Sun, Y. (1999). Non-cooperative games on hyperfinite Loeb spaces. *J. Math. Econom.*, 31(4):455–492

3.2. **General Equilibrium Theory and Implementation.**

A. Core theory and implementation for economies with $^*$-finite sets of agents

B. Asymptotic implementation theory Hashimoto, T. (2013). The generalized random priority mechanism with budgets. *Working paper, Stanford Business School*

## 4. Continuous Time Games

We begin with some "basic" stochastic process theory and dynamic optimization, both of which are useful far beyond game theory. We then turn to some recent work that has focused on using stochastic signals of actions in continuous time games. This has been especially productive in studying models of reputations. Hopefully we will go beyond this.

4.1. **Calculus of Variations, Optimal Control Theory.**

A. Still a good introduction, Kamien, M. I. and Schwartz, N. L. (1991). *Dynamic optimization*, volume 31 of *Advanced Textbooks in Economics*. North-Holland Publishing Co., Amsterdam, second edition. The calculus of variations and optimal control in economics and management

B. Basic $^*$-finite analysis, Cutland, N. J. (1983). Internal controls and relaxed controls. *Journal of the London Mathematical Society*, 2(1):130–140

### 4.2. *-finite Stochastic Process Theory.

A. Basics, Nelson, E. (1987). *Radically elementary probability theory*, volume 117 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ
B. Stochastic integrals from Anderson course notes
C. Anderson, R. M. (1976). A non-standard representation for Brownian motion and Itô integration. *Israel J. Math.*, 25(1-2):15–46

### 4.3. Games: Monitoring and Reputation.

A. Why continuous time game theory can be so difficult, Stinchcombe, M. B. (1992). Maximal strategy sets for continuous-time game theory. *J. Econom. Theory*, 56(2):235–265
B. Sannikov, Y. (2008). A continuous-time version of the principal-agent problem. *Rev. Econom. Stud.*, 75(3):957–984
C. Sannikov, Y. and Skrzypacz, A. (2010). The role of information in repeated games with frequent actions. *Econometrica*, 78(3):847–882
D. Faingold, E. and Sannikov, Y. (2011). Reputation in continuous-time games. *Econometrica*, 79(3):773–876
E. Bohren, J. A. (2012). Stochastic games in continuous time: Persistent actions in long-run relationships. *Working paper, Department of Economics, UCSD*