EXTENDED ABSTRACT FOR THE VON NEUMANN/MORGENSTERN APPROACH TO AMBIGUITY

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The paper has three main parts: one about decriptive completeness; one about representing continuous linear preferences defined on partial descriptions of probability distributions; and one giving applications of the new preferences encompassed in this approach.

I. Descriptive completeness.
   A. For modeling risky choice problems, a prior p is a distribution on the state space Ω. The choice between a pair of functions $\omega \mapsto f(\omega)$ and $\omega \mapsto g(\omega)$ from states to utility relevant consequences models a situation in which the decision maker believes that they are choosing between the two image laws, $\mu := f(p)$ and $\nu := g(p)$. The details of p and the functions f and g do not matter provided that the prior is non-atomic, because this guarantees that there always exist many functions f and g giving rise to μ and ν. Priors that fail to be non-atomic rule out a huge part of the models of risky choice, for example, ruling out all Gaussian models.
   B. Many models of choice under ambiguity substitute a set S of priors for the single prior p and use this set to rank the choice between f and g. Now the choice between the pair of functions f and g models a situation in which the decision maker believes that they are choosing between two sets of distributions, $A := f(S)$ and $B := g(S)$. We give a sufficient (and nearly necessary) condition for S to be **descriptively complete**. These conditions guarantee that the details of S and the functions f and g do not matter because there always exist many functions f and g giving rise to A and B. Priors that fail to be non-atomic rule out a huge part of the models of risky choice, analogously, sets of priors that fail to be descriptively complete sets always rule out a dense set of models of ambiguous choice, and may only allow a nulls set of problems to be modeled.

II. Representation theory.
   A. In the von Neumann/Morgenstern approach to risky choice problems, the choice between the distributions μ and ν is given by which of the expected utilities, $\int u(x)\, d\mu(x)$ or $\int u(x)\, d\nu(x)$, is larger. The function $u(\cdot)$ is continuous, so these are preferences representabale by a weak*-continuous linear functionals on the image laws. This structure arises from the probability distributions (spanning the) dual space of the continuous functions, equivalently, from the continuous functions being the predual of (the span of the) space of probability distributions.
   B. In the von Neumann/Morgenstern approach to ambiguous choice problems, the choice between the sets of distributions A and B is given weak*-continuous linear functionals on *sets* of image laws. These functionals can be given a representation by identifying the predual of (the span of the) space of sets of probability distributions. The functional forms that represent the extant multiple prior preferences are either weak*-continuous linear functionals or else preferences that are locally approximated by these functionals.

III. New preferences and old puzzles.
   A. As Machina's (2012) work on ambiguity aversion with three or more outcomes has shown, preferences over functions from a state space to consequences that have a Choquet integral representation require that ambiguity aversion be constant across wealth levels. This papers gives a class of weak*-continuous linear preferences with ambiguity aversion that is decreasing (or increasing) with wealth.

B. First and second order stochastic dominance are broadly useful tools for analysing risky choice problems. The representation theory allows these to be generalized to ambiguous problems. It also gives rise to a new class of dominance relations that can distinguish between sources of uncertainty.
C. Identifying ambiguity aversion as dislike of an expansion of a set of distributions around its center, this paper gives complete separations between risk and ambiguity attitudes. This solves an old puzzle about the situations in which the α-minmax EU preferences are and are not ambiguity averse for $\alpha > \frac{1}{2}$.
D. Modeling ambiguous information structures as ones in which the decision maker will face a distribution over sets of distributions, the representation theory yields a Bayesian theory of the value of ambiguous information structures through a complete theory of updating for convex sets of priors.