Assignment #3 for Managerial Economics, ECO 351M, Fall 2016
Due, Monday October 10.

1. From Ch. 15 of Kreps's *Micro for Managers*, Problem 15.2.
Ans. The gamble is $X = 40,000$ with probability 1/4 and $X = 0$ with probability 3/4. From the graph, $E\,u(X) = \frac{1}{4} \cdot 1 + \frac{3}{4} \cdot 0.5 = \frac{5}{8} = 0.625$. From the graph, the random variable Y with $P(Y = 7,500)$ has $E\,u(Y)$ about halfway between 0.6 and 0.7, that is, slightly above 0.65. Hence Y is the choice.
For Jack, $E\,u(X) = \frac{1}{4}\sqrt{40,000 + 5,000} + \frac{3}{4}\sqrt{0 + 5,000} \simeq 106.066$ while $E\,u(Y) = \sqrt{7,500 + 5,000} \simeq 111.80$, hence Y is the choice.
For Jim, $E\,u(X) = \frac{1}{4}\sqrt{40,000 + 50,000} + \frac{3}{4}\sqrt{0 + 50,000} \simeq 242.705$ while $E\,u(Y) = \sqrt{7,500 + 50,000} \simeq 239.79$, hence X is the choice.
If we thought of the 5,000 in Jack's utility function as his starting wealth and the 50,000 in Jim's as his starting wealth, you see the following effect — a higher willingness to take a given risky bet at higher income levels.
2. From Ch. 15 of Kreps's *Micro for Managers*, Problem 15.3.
Ans. (a) The insurance company takes in \$40,000 and its expected payout is $\$0.05 \cdot 750,000 = 37,500$, so its expected net profit is \$2,500. (b) If the individual is risk neutral, then without insurance his expected payoff is \$962,500, with insurance it is \$960,000, so he would prefer **not** to buy the insurance. Sometimes this is called "self-insurance." (c) With the utility function $u(x) = \sqrt{x}$, the certainty equivalent of the risk is \$950,625, and since this is more than \$9,000 better than the full insurance contract's wealth, the individual would buy the insurance. (d) One can numerically solve using a spread sheet, or one can stop being scared by a little algebra and proceed as follows: buying α coverage yields the random variable X_α with $P(X_\alpha = 1,000,000 - \alpha \cdot 40,000) = 0.05$ and $P(X_\alpha = 1,000,000 - \alpha \cdot 40,000 - 750,000 + \alpha 750,000) = 0.95$; let $M_\alpha = 1,000,000 - \alpha \cdot 40,000$ and $m_\alpha = 1,000,000 - \alpha \cdot 40,000 - 750,000 + \alpha 750,000)$; let $f(\alpha) = E\,u(X_\alpha) = 0.05\sqrt{M_\alpha} + 0.95\sqrt{m_\alpha}$; solve the problem $\max_{\alpha \in [0,1]} f(\alpha)$ by setting $f'(\alpha) = 0$ and solving for α^*; since $\frac{d}{d\alpha} M_\alpha = -40,000$ and $\frac{d}{d\alpha} m_\alpha = 710,000$, the FOCs are
$$0.05 \frac{40,000}{\sqrt{M_\alpha}} = 0.95 \frac{710,000}{\sqrt{m_\alpha}}.$$
Gathering terms yields $\left(\frac{1}{19}\frac{4}{71}\right)^2 = \frac{M_\alpha}{m_\alpha}$, letting $K = \left(\frac{1}{19}\frac{4}{71}\right)^2$, we get $M_\alpha = K \cdot m_\alpha$. Solving for α^* yields $\alpha^* \simeq 0.83$.
3. From Ch. 15 of Kreps's *Micro for Managers*, Problem 15.5.
Ans. This is an example of the idea covered in class — if $E\,X > 0$, then for a small enough α, the expected utility of taking the gamble αX is larger than the expected utility of not taking it. Also, the "12.5859" and the "−7.4267" do not matter for the problem — as in lecture $E\,u(X) > E\,u(Y)$ if and only if for all $a > 0$ and all b, $E\,(au(X) + b) > E\,(au(Y) + b)$, and I can pick a and b so that $v(x) = -e^{-0.000021x}$. Now, for (a), the expected utility of the gamble is $\frac{1}{2}v(50,000) + \frac{1}{2}v(-25,000) \simeq -1.02$ while the expected utilty of not taking the gamble is $v(0) = -1$, so do **not** take the gamble. (b) compare $\frac{1}{2}v(500) + \frac{1}{2}v(-250) \simeq -0.997$ while the expected utilty of not taking the gamble is $v(0) = -1$, so **do** take the gamble.
4. From Ch. 16 of Kreps's *Micro for Managers*,
a. Problem 16.1.

Ans. I cannot answer this one for you, your attitudes toward risk are almost certainly different than mine. This exercise was to make you think a bit more concretely about how you think about risks.

b. Problem 16.2.
Ans. Use a spread sheet to calculate $E\,u_\lambda(X_i)$ for $i = 1, \ldots, 6$ and $u_\lambda(x) = -e^{-\lambda x}$. Higher values of λ correspond to higher risk aversion, you should see the pattern of choices that emerges from this.

5. From Ch. 17 of Kreps's *Micro for Managers*,
   a. Problem 17.1.
   Ans. Essentially, this is what we did in lecture when the share retained is θ — let $f(\theta) = E\,u(\theta X)$, if $E\,X > 0$, then $f'(\theta) > 0$ so that $\theta^* > 0$. (a) $f(\theta) = \frac{1}{2}(12.5859 - 7.4267e^{-0.0000211\cdot\theta\cdot 50{,}000}) + \frac{1}{2}(12.5859 - 7.4267e^{-0.0000211\cdot\theta\cdot(-25{,}000)})$, graph this as a function of θ with $0 \leq \theta \leq 1$. (b) Set $f'(\theta) = 0$ and solve for θ^*. (c) $CE(\theta)$ is the y_θ that solves
   $$(12.5859 - 7.4267e^{-0.0000211\cdot y_\theta}) = f(\theta).$$
   This is a monotonic transformation of the function $f(\theta)$ so its maximum will happen at exactly the same θ.
   b. Problem 17.2, (a)-(c).
   Ans. Elaborations of the previous.

6. From Ch. 17 of Kreps's *Micro for Managers*,
   a. Problem 17.3.
   Ans. Let θ be the percentage of the gamble she retains, she believes that this yields her the random variable X_θ with $P(X_\theta = \theta 50{,}000 + (1-\theta)\cdot 125) = 0.7$ and $P(X_\theta = \theta(-25{,}000 + (1-\theta)\cdot 125) = 0.3$. Thus, $f(\theta) = E\,u(X_\theta)$ is the function
   $$f(\theta) = 0.7\cdot(12.5859 - 7.4267e^{-0.0000211\cdot(\theta 50{,}000 + (1-\theta)125)}) + \\ 0.3\cdot(12.5859 - 7.4267e^{-0.0000211\cdot(\theta(-25{,}000)+(1-\theta)125)}).$$
   Set $f'(\theta) = 0$ and solve for θ^*, or use a spreadsheet and its solver.
   b. Problem 17.4.
   Ans. As the the proportion sold $\uparrow$, the quality of the gamble for Jan $\downarrow$. To solve this problem, first check if the previous solution is to keep less than %10, if it is, then this is what she will do. If the previous solution was more than %10 but less than %30, find out what she wants to do with the probabilities above changed to 0.6 and 0.4. If this yields her a higher expected utility, than the previous ..., etc. etc. To solve this kind of problem in full generality is, for once, actually easier with a spread sheet. It turns out that her optimal choice is to sell off %30 of the gamble, keeping the rest for herself.

7. From Ch. 17 of Kreps's *Micro for Managers*, Problem 17.6.
   Ans. The point here is that Biff will pay more for the gamble negatively correlated with his own wealth.
8. When one looks at statistics measuring the competence with which firms are run, after adjusting for the industry, one finds a weak effect in favor of firms with female CEO's, and a much stronger effect in favor of larger firms. In this problem, you are going to investigate a different advantage of being large, the decreasing average cost aspect of simple inventory systems. Decreasing average

costs sometimes go by the name of economies of scale, and economies of scale are a crucial determinant of the horizontal boundary of a firm. In this problem, you will find a power law relating size to costs.

Your firm needs Y units of, say, high grade cutting oil per year. Each time you order, you order an amount Q at an ordering cost of $F + pQ$, where F is the fixed cost of making an order (e.g. you wouldn't want just anybody to be able to write checks on the corporate account and such sytems are costly to implement), and p is the per unit cost of the cutting oil. This means that your yearly cost of ordering is $\frac{Y}{Q} \cdot (F + pQ)$ because $\frac{Y}{Q}$ is the number of orders per year of size Q that you make to fill a need of size Y.

Storing anything is expensive, and the costs include insurance, the opportunity costs of the space it takes up, the costs of keeping track of what you actually have, and so on. We suppose that these stockage costs are s per unit stored. Computerized records and practices like bar-coding have substantially reduced s over the last decades. Thus, when you order Q and draw it down at a rate of Y per year, over the course of the cycle that lasts Q/Y of a year, until you must re-order, you store, on average $Q/2$ units. This incurs a per year cost of $s \cdot \frac{Q}{2}$. Putting this together, the yearly cost of running an inventory system to keep you in cutting oil is

$$C(Y) = \min_{Q} \left[\frac{Y}{Q} \cdot (F + pQ) + s \cdot \frac{Q}{2}\right], \tag{1}$$

and the solution is $Q^*(Y, F, p, s)$.

a. Without actually solving the problem in equation (1), find out whether Q^* depends positively or negatively on the following variables, and explain, in each case, why your answers makes sense: Y; F; p; and s.
Ans. We did the super/submodularity analyses of this in class.

b. Now explicitly find the optimal tradeoff between fixed costs and storage costs to solve for $Q^*(Y, F, p, s)$ and $C(Y)$.
Ans. We did this algebra in class.

c. Find the marginal cost of an increase in Y. Verify that the average cost, $AC(Y)$, is decreasing and explain how your result about the marginal cost implies that this must be true.
Ans. As we saw in class, the marginal cost is everywhere decreasing, hence the average cost must be everywhere decreasing.

d. With the advent and then lowering expenses of computerized inventory and accounting systems, the costs F and s have both been decreasing. Does this increase or decrease the advantage of being large?
Ans. Let $C(Y; F, s, p)$ be the optimal cost. Suppose that $F' > F$, $s' > s$ and $Y' > Y$. At the older, higher values F' and s', the cost advantage was $C(Y'; F', s', p) - C(Y; F', s', p)$, and the newer, lower values, the cost advantage is $C(Y'; F, s, p) - C(Y; F, s, p)$. One can explicitly check which difference is larger using the formulate for $C(\cdot; F, s, p)$ given in class, or, more easily, one can calculate the marginal cost, $C'(\cdot; F, s, p)$, ask if lowering from F' to F and from s' to s increases or decreases marginal cost.

9. When one looks at historical statistics about R&D rates, one finds that it is concentrated in the larger firms. Such figures do not include a recent phenomenom, the growth in the number of firms that specialize in doing contract R&D, often

for the government, but increasingly in the recent past, for the large pharmaceutical firms who have been "outsourcing their brains." In this problem, you are going to investigate a simple case of how being large can give a decreasing risk-adjusted average cost of doing R&D. Behind the results you will find here is the notion of portfolio diversification.

We are going to suppose that research projects cost C, that C is "large," and that research projects succeed with probability p, that p is "small," and that if the project does not succeed, then it fails and returns 0. Thus, the distribution of returns for a project are $(R-C)$ with probability p and $0-C$ with probability $1-p$. Since R, if it happens, will be off in the future and the costs, C, must be borne up front, we are supposing that R measures the net present value of the eventual success if it happens.

The expected or average return on a research project is $p(R-C)+(1-p)(-C)$ which is equal to $pR-C$, expected returns minus expected costs. We assume that $pR > C$, that is, that expected returns are larger than expected costs. We are also going to assume that success on different projects are independent of each other. Specifically, if you take on two projects, then the probability that both succeed is p^2, the probability that both fail is $(1-p)^2$, and the probability that exactly one of them succeeds is $[1-p^2-(1-p)^2]$, that is, $2p(1-p)$.

A heavily used measure of the risk of a random return is its standard deviation, which is the square root of the average squared distance of the random return from its average. We let $\mu = pR-C$ be the average or expected return of a single project, the standard deviation is then $p\sqrt{(R-C)-\mu} + (1-p)\sqrt{(-C)-\mu}$, which is denoted σ. Of particular interest is the ratio $\frac{\sigma}{\mu}$, a unitless measure giving the risk/reward ratio for the project. Of interest is the comparison of the risk/reward ratio when you have one project and when you have two. Its inverse, $\frac{\mu}{\sigma}$ is a risk adjusted measure of the average return.

a. If $R = 10^7$ and $C = 100,000$, find the set of p for which the expected value, μ, is positive. For these p, give the associated σ and $\frac{\mu}{\sigma}$. Graph your answers in an informative fashion.

Ans. We want the set of μ such that $\mu_p := 10^7p - 10^5 > 0$, which is $p > 1/100$. For such p,

$$\sigma_p = p\sqrt{(R-C)-\mu_p} + (1-p)\sqrt{(-C)-\mu_p},$$

and the ratio is

$$\frac{\mu_p}{p\sqrt{(R-C)-\mu_p} + (1-p)\sqrt{(-C)-\mu_p}}.$$

Graph this ratio for some representative values of p.

b. Now suppose that your research budget is expanded, and you can afford to undertake two projects. Verify that the expected value is now $2\cdot\mu$. Verify that the new σ for the R&D division is $\sqrt{2}$ times the answer you previously found. What has happened to the risk adjusted measure of the average return?

Ans. It has increased from $\frac{\sigma}{\mu}$ to $\sqrt{2}\frac{\sigma}{\mu}$.

c. Repeat the previous two problems with $R = 10^8$ and $C = 200,000$.

Ans. Yup.

d. In the inventory problem above, there was a power law giving the advantage of being larger. Give the general form of the power law relating the research budget to the risk adjusted rate of return.

Ans. If the research budget allows x times as large a project, then the new risk adjusted rate of return is $\sqrt{x} \cdot \frac{\mu}{\sigma}$.