Syllabus for Math for Economists

ECO 385D, Fall 2016

Maxwell B. Stinchcombe

Basics

We meet Mondays and Wednesdays, 2 -3:30 pm in BRB 1.118. The TA's are Peter Toth and Daria Pus. Stinchcombe's office and hours are Tu. 9:30-11 and 3:30-5 pm, during the morning hours, the undergraduates have priority, in the afternoons, you do. Peter Toth and Daria Pus share the office BRB 4.122, Daria's office hours are W. 9:30-11:30, Toth's are tbd.

Overview and objectives

The first aim of this class is to cover the mathematical background necessary for your first year in graduate school in economics. This is optimization theory and a bit of probability theory: existence and characterization of optima; parametrized changes in optima; optima in stochastic situations; optima in dynamic situations; and optima in stochastic dynamic situations. A secondary aim is to introduce you to tools and perspectives that will be useful in the later years in your study of economics.

Texts

There is one required book and two recommended books for the course.

- [Req.] D. Corbae, M. B. Stinchcombe, and J. Zeman, (2009). *An introduction to mathematical analysis for economic theory and econometrics.* Princeton University Press
- [Rec.] Sundaram, R. (1996). *A First Course in Optimization Theory.* Cambridge University Press
- [Rec.] A. Mas-Colell, M. D. Whinston, and J. R. Green (MWG), (1995) *Microeconomic Theory.* Oxford University Press

Schedule

The following is a rough guide to the topics and sources.

- Weeks 1-2, survival knowledge: derivatives and their notation; concavity, differentiable concavity and first order conditions for optima; turning constrained optimization problems into unconstrained problems using Lagrangean functions. Sources: Ch. 5-10 in Corbae *et al.*, Ch. 1-6 in Sundaram; Appendices M.A, M.C, M.D, M.J, and M.K in MWG.
- Week 3, the space $\mathbb{R}$: completeness; separability; contraction mappings; monotone sequences and the basic growth model. Sources: Ch. 3 in Corbae *et al.* and/or your favorite real analysis textbook, Ch. 1-2 in Sundaram.
- Weeks 4-5, metric spaces, especially $\mathbb{R}^\ell$: completeness; closure; compactness; connectedness; continuity; contractions; Lipschitz and uniform continuity. Sources: Ch. 4.1-9 in Corbae *et al.*, appendix M.H in MWG, Ch. 3 in Sundaram.
- Weeks 6-7, correspondences and contraction mappings: upper- and lower hemi-continuity; Theorem of the Maximum; Banach's contraction mapping theorem and dynamic programming. Sources: Ch. 4.10-11 in Corbae *et al.*, Ch. 9 in Sundaram, and appendix M.N in MWG.
- Weeks 8-9, convex structures: convex sets; concave and convex functions; quasi-concave and quasi-convex functions; maximization and concavity/convexity; separation Theorems and the Kuhn-Tucker theorem; envelope theorem and geometric interpretations of Lagrange multipliers. Sources: Ch. 5.1-10 in Corbae *et al.*, Ch. 5-8 in Sundaram, and appendices M.J-L in MWG.
- Weeks 10-11, fixed point theorems: Tarski's fixed point theorem, matching, and equilibrium sets for supermodular games; Brouwer's and Kakutani's fixed point theorems, existence results for general equilibrium and game theory. Sources: Ch. 2.7-9 and 5.11-12 in Corbae *et al.*, appendix M.I in MWG, and Ch. 10 in Sundaram.
- Weeks 12-14, metric and measure spaces: the space of compact sets; spaces of continuous functions and probability distributions on $\mathbb{R}$; approximation theory; the implicit function Theorem; the metric completion Theorem and Lebesgue spaces; stochastic dynamic programming. Sources: Ch. 6 and 7.1-4 in Corbae *et al*, Ch. 12 in Sundaram.

Evaluation

- 60% on homework assignments, one per each of the last 6 sets of topics listed above.
- 40% on a final exam, exact format to be determined.

I encourage you to work together on the homework assignments. The aim of this course is to ease your work in mastering the knowledge offered in your other courses this year. This means that you should know how and when to use the various tools taught in this course, and how and when not to use them. Mostly, but not always, achieving this working knowledge is easier when work is done in groups.

However, I strongly advise you to avoid the temptation to "free ride" on the work of others. You need to know how to use this material on your own, when facing your own problems, when facing the comprehensive exams. Learn the material, use it, absorb the use of it, but be sure to make it yours.