ECONOMETRICS III: TIME SERIES

MAXWELL B. STINCHCOMBE

This course covers some of the basic tools for data that accrues over time. During the previous two quarters you should have seen the basic linear econometric model and some non-linear models with diseases. In this course, a particular set of diseases is comes from various forms of the failure of the independence of errors that arises because stuff happening now influences stuff happening later.

The first set of tools involve some new results about asymptotic (or limit) properties of statistics, both for estimating and for testing. We will develop these at the same time that we develop the properties of a number of the heavily used models and techniques. These usually take the form of parametric descriptions of the data generating process. These allow us to pretend, after some manipulation and often quite successfully, that the innovations in the data are really something like independent after all.

Typical features in the time series we are interested in include "trends," "seasonality" and "cyclicality." The quotes are there because the words are so vague. Making them sharper, sometimes by providing several alternative, non-equivalent definitions of them, is part of our job.

Sources:

1. The textbook for the class is James Hamilton's *Time Series Analysis*, Princeton University Press, New York, 1994.
2. Much of the advanced mathematics that we will sometimes use is well presented in Kolomogorov and Fomin [3], which is a very good place to start.
3. Much more comprehensive is Dudley [2], a revised and improved, paperback version of a 1989 classic.

Evaluation: 50% on homeworks that will be given out approximately once every two weeks, 50% on final exam.

Date: Spring 2007.

Topics

I. Conditional expectations and probabilities.
   A. Finite probability spaces.
      1. Definition 1, from intro prob/stats.
      2. Definition 2, using projection.
      3. Definition 3, using the Radon-Nikodym Theorem.
   B. General probability spaces.
      1. $L^2(\Omega, \mathcal{F}, P)$ spaces, inner products, norm, completeness, denseness of simple functions.
      2. Doob's Theorem and $L^2(\Omega, \mathcal{G}, P) = L^2(X)$, $\sigma(X) = \mathcal{G} \subset \mathcal{F}$.
      3. Definition 1, for simple functions, from intro prob/stats.
      4. Definition 2, using projection.
      5. Relation to linear regression.
      6. Definition 3, using the Radon-Nikodym Theorem.

II. Review of linear (and non-linear) regression.
   A. Projection onto the affine functions in $L^2(X)$. Independence and large n approximations.
   B. Projection onto compactly generated sets of functions in $L^2(X)$. Independence and large n approximations.
   C. Regressing on lagged variables, errors can be correlated with explanatory variables.

III. Linear dynamic systems
   A. Difference equations (Hamilton, Chapter 1).
   B. Lag operators and lag polynomials (Hamilton, Chapter 2).

IV. Classes of time series (Hamilton, Chapter 3).
   A. White noise.
   B. Stationarity, strict and weak.
   C. Martingale difference sequences.
   D. AR models.
   E. MA models.
   F. ARMA models.
   G. GARCH models (brief intro).

V. Forecasting stationary processes (Hamilton, Chapter 4).
   A. AR processes, infinite and finite histories.
   B. MA processes, infinite and finite histories.
   C. ARMA processes, infinite and finite histories.
   D. Sums of processes.
   E. Wold decomposition.
      1. The previous examples as special cases.
      2. From projections in Hilbert spaces (notes).

VI. Failures of stationarity

A. Models of nonstationary series (Hamilton, Chapter 15.1-3).
B. Structural breaks, and finding them (Chu, Stinchcombe, and White [1]).
C. Modeling regime Changes (Hamilton, Chapter 22).

VII. Flavors of causality.
A. Reduced form simultaneous equations (Hamilton, Chapter 9).
B. Directed acyclic graphs, and settable systems (White and Chalak [4]).

References

1. Stinchcombe Maxwell Chu, Chia-Shang James and Halbert White, *Monitoring structural change*, Econometrica **64** (1996), no. 5, 1045–1065.
2. R. M. Dudley, *Real analysis and probability*, Cambridge Studies in Advanced Mathematics, vol. 74, Cambridge University Press, Cambridge, 2002, Revised reprint of the 1989 original. MR MR1932358 (2003h:60001)
3. A. N. Kolmogorov and S. V. Fomīn, *Introductory real analysis*, Dover Publications Inc., New York, 1975, Translated from the second Russian edition and edited by Richard A. Silverman, Corrected reprinting. MR MR0377445 (51 #13617)
4. Halbert White and Karim Chalak, *A unified framework for definining and identifying causal effects*, 2006.

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