Homework #2, Econometrics III, Spring 2007

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Throughout, $\mathbb{Z} = \{\ldots, 1, 0, 1, \ldots\}$, $\mathbb{N} = \{1, 2, \ldots\} \subset \mathbb{Z}$.

A. Let X_t and Y_t be two white noise processes. Is their sum, $Z_t = X_t + Y_t$, also white noise?

B. Define a random walk as $Y_0 = 0$ and $Y_t = Y_{t-1} + \epsilon_t$, $t \in \mathbb{N}$, where is ϵ_t is white noise with variance s. Show that Y_t, $t \in \mathbb{N}$, is not covariance stationary.

C. Let w_t, $t \in \mathbb{Z}$, be a bounded sequence and consider the following difference equation in y_t:

$$y_t = 0.9y_{t-1} - 0.2y_{t-2} + w_t, t \in \mathbb{Z}.$$

1. Find a solution of this equation in terms of the infinite history by inverting the appropriate lag polynomial.
2. Prove that this solution is bounded.
3. Find an explicit expression for $\partial y / \partial w_{t-j}$.
4. Show that for any $A_1, A_2 \in \mathbb{R}$, the solution you found in part 1 plus $A_1(0.4)^t + A_2(0.5)^t$ is also a solution, and that this new solution is unbounded over $t \in \mathbb{Z}$.

D. Consider a stationary AR(1) process $y_t = \varphi y_{t-1} + \epsilon_t$, $t \in \mathbb{Z}$.

1. Show that if ϵ_t is iid, then $E(Y_t | Y_{t-1}) = \varphi Y_{t-1}$.
2. Show that if ϵ_t is a martingale difference sequence, then $E(Y_t | Y_{t-1}) = \varphi Y_{t-1}$.
3. Show that $E(Y_t | Y_{t-1}) = \varphi Y_{t-1}$ does not hold if $\epsilon_t = z_{t/2}$ if t is even and $\epsilon_t = (z_{(t-1)/2})^2 - 1$ if t is odd where z_t is a Gaussian white noise process.

E. Let X, Y be random variables with finite variance.

1. Solve $\min_{\alpha,\beta} E\left(Y - (\alpha + \beta X)\right)^2$ for (α^*, β^*).
2. Give an example where $E\left(Y \middle| X\right) \neq \alpha^* + \beta^* X$.