Homework #3, Micro, Spring 2007

Max Stinchcombe

1. Probability 0 Events: Single Person Decision Theory

We introduce two ways of handling the 0 probability signals. They turn out to be the same for one person games, and in many, but not all, games with more than one person. The terms we'll use are **perfect Bayesian equilibria** and **sequential equilibria**.

S is the set of signals, Ω the set of states, $\pi_S : S \times \Omega \to S$ is defined by $\pi_S(s,\omega) = s$, and $\text{marg}_S(P)(s) := P(\pi_S^{-1}(s))$ is the marginal distribution of P on S. In a similar fashion, $\pi_\Omega : S \times \Omega \to S$ is defined by $\pi_\Omega(s,\omega) = \omega$.

We've got $P \in \Delta(S \times \Omega)$, and the *ex ante* decision problem

$$\max_{s \mapsto a(s)} \sum_{s,\omega} u(a(s),\omega)\, P(s,\omega).$$

For every s with $\text{marg}_S(P)(s) > 0$, let $\beta_s^P := \text{marg}_\Omega P(\cdot|\pi_S^{-1}(s))$ be the posterior distribution after the signal s is observed. Let $a^*(s)$ denote the solution(s) to

$$\max_{a \in A} \sum_{\omega} u(a,\omega)\beta_s^P(\omega).$$

If $\text{marg}_S(P)(s^\circ) = 0$, then *ex ante* expected utility is unaffected by choices after s°. Let S_P° denote the set of s° having probability 0 under P.

The **perfect Bayesian** approach asks that $a^*_{PB}(s)$ be the solution set for

$$\max_{a \in A} \sum_{\omega} u(a,\omega)\beta_s^P(\omega)$$

when $s \notin S_P^\circ$, and $a^*_{PB}(s^\circ)$ be the solution set for

$$\max_{a \in A} \sum_{\omega} u(a,\omega)\beta_s^{Per}(\omega)$$

for some $\beta_{s^\circ}^{Per} \in \Delta(\Omega)$ when $s^\circ \in S_P^\circ$.

The **consistent** approach also asks for best responses to β_s when $s \notin S_P^\circ$, but it asks that $a^*_{con}(s^\circ)$ be the solution set for

$$\max_{a \in A} \sum_{\omega} u(a,\omega)\beta_{s^\circ}^{C}(\omega)$$

for some $\beta_{s^\circ}^C$ that is **consistent** with P.

The distinction is between

(1) "for some $\beta_{s^\circ}^{Per} \in \Delta(\Omega)$," and
(2) "for some $\beta_{s^\circ}^C$ that is **consistent** with P."

Consistency asks that the beliefs at every s come from some strictly positive approximation to, or perturbation of, P. For single person decision theory, we shall see that consistency means nothing at all. This is emphatically **not** true in games.

Problem 1. *Let* $S = \{1, 2, 3\}$*,* $\Omega = \{\omega_1, \omega_2, \omega_3\}$*, and* $A = \{a, b, c\}$*. Let* P *be given by the matrix*

$S = 3$	0	0	0
$S = 2$	0.1	0.1	0.2
$S = 1$	0.2	0	0.4
	ω_1	ω_2	ω_3

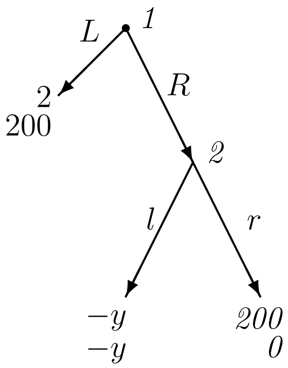
and let $u(a, \omega)$ *be given by the matrix*

d	5	4	3
c	9	0	0
b	6	6	6
a	2	5	8
	ω_1	ω_2	ω_3

(1) *Give* S_P°*, and for every* $s \notin S_P$*, given* β_s*.*
(2) *Give* $a^*_{PB}(s)$ *for each* $s \in S$*.*
(3) *Define* $s \mapsto \beta_s$ *to be* ***consistent with*** P *if there exists a sequence* $P_n \gg 0$ *with* $P_n \to P$*,* $\beta_{n,s} = \beta_s^{P_n}$*, and* $\beta_{n,s} \to \beta_s$*. Give the set of* $s \mapsto \beta_s$ *that are consistent with* P*. [Your calculations should lead you to the observation that* $a^*_{con}(s) \equiv a_{PB}(s)$*.]*

We now apply the perfect Bayesian approach to a simple game.

Problem 2. *In Puccini's Gianni Schicchi, Buoso Donati has died and left his large estate to a monastery. Before the will is read by anyone other than the relatives, they call in a noted mimic, Gianni Schicchi, to play Buoso on his deathbed, re-write the will in front of lawyers, and then convincingly die. The relatives explain, very carefully, to Gianni Schicchi, just how severe are the penalties for anyone caught tampering with a will (at the time, the penalities included having one's hand cut off). The plan is put into effect, but, on the deathbed, Gianni Schicchi, as Buoso Donati, rewrites the will leaving the entire estate to the noted mimic and great artist, Gianni Schicchi. The relatives can expose him, and thereby expose themselves too, or they can remain silent. With player* 1 *being Gianni Schicchi and player* 2 *being the relatives, and some utility numbers with* $y \gg 200$ *to make the point, with Gianni Shicchi being player* 1*, we have*



Gianni Shicchi

	l	r
L	$(2, 200)$	$(2, 200)$
R	$(-y, -y)$	$(200, 0)$

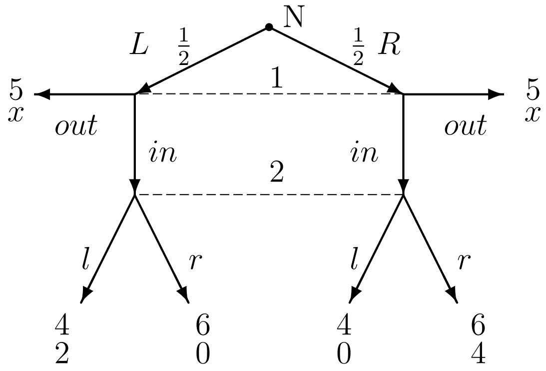
An ***equilibrium outcome*** *for a game is a distribution over the end points of the game that arises from play of equilibrium strategies.*

(1) *Give the normal form for this extensive form game.*
(2) *Give the two sets of equilibria and the two equilibrium outcomes for this game.*
(3) *In one of the sets of equilibria, 2's decision node is reached with probability* 0*, that is, it is a null set. What is* a^*_{PB} *at this null set?*
(4) *A Nash equilibrium is a perfect Bayesian equilibrium if it calls for* a^*_{PB} *at every null set in the game. Which of the equilibria for this game are perfect Bayesian equilibria?*

An equilibrium in which people play optimally with respect to some beliefs at all information sets is called a **sequentially rational** equilibrium, or it is called a **perfect Bayesian equilibrium (pbe)**. The "sequential" does not have to do with sequences such as used in real analysis. Rather, it has to do with rationality in a sequence of decisions. The "perfect" is an attempt to cast aspersions on all equilibria which do not satisfy this criterion.

2. Not all pbes are consistent with the structure of a game

Problem 3. *Consider the following extensive form game.*

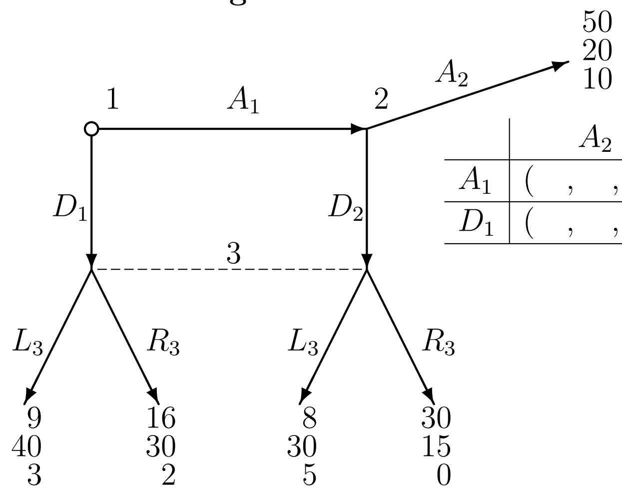


*Beliefs, b, are **consistent** with a strategy σ if there exists a sequence $\sigma_n \gg 0$ with $\sigma_n \to \sigma$, and $b_n \to b$ where b_n are the beliefs one arrives at by applying Bayes' Law with the strictly positive strategies σ_n. An equilibrium is **sequential** if it is sequentially rational with respect to consistent beliefs. [I hate this terminology.]*

(1) *Show that the given 2×2 game is the normal form.*
(2) *Find the sets of Nash equilibria for this game, and the two equilibrium outcomes for this game.*
(3) *Show that every Nash equilibrium for this game is a pbe.*
(4) *Give the only set of beliefs that are consistent with any of the equilibrium strategies.*
(5) *Identify the sequential equilibria.*

Problem 4. *Fill in the payoffs for the normal form representation of the following horse game. Find the closed connected sets of equilibria and the corresponding equilibrium outcomes. Figure out which equilibria are sequential.* [*If you get stuc, there is a worked example of this form below.*]

Another horse game



	A_2	D_2
A_1	(, ,)	(, ,)
D_1	(, ,)	(, ,)

L_3

	A_2	D_2
A_1	(, ,)	(, ,)
D_1	(, ,)	(, ,)

R_3

3. Yet stronger restrictions on beliefs

There are even stronger ways to restrict beliefs than asking that they be consistent with the structure of the game. The most powerful and widely used is a kind of self-referential iterated deletion of dominated strategies test. This is refered to as stability.[1]

A strategy $\sigma_i \in \Delta_i$ **dominates (or strongly dominates)** $t_i \in A_i$ **relative to** $T \subset \Delta$ if

$$(\forall \sigma^\circ \in T)[u_i(\sigma^\circ \backslash \sigma_i) > u_i(\sigma^\circ \backslash t_i)].$$

If $T = \Delta$, this is the usual definition of dominance. Let $D_i(T)$ denote the set of $t_i \in A_i$ that are dominated relative to T. Smaller T's make the condition easier to satisfy, that is,

$$[T' \subset T] \Rightarrow [D_i(T') \supset D_i(T)].$$

A strategy $\sigma_i \in \Delta_i$ **weakly dominates** $t_i \in A_i$ **relative to** $T \subset \Delta$ if

$$(\forall \sigma^\circ \in T)[u_i(\sigma^\circ \backslash \sigma_i) \geq u_i(\sigma^\circ \backslash t_i)], \text{ and}$$

$$(\exists \sigma' \in T)[u_i(\sigma' \backslash \sigma_i) > u_i(\sigma' \backslash t_i)].$$

Let $WD_i(T)$ denote the set of $t_i \in A_i$ that are weakly dominated relative to T.

Lemma 1. *If Γ is finite, then for all $T \subset \Delta$, $A_i \setminus D_i(T) \neq \emptyset$ and $A_i \setminus WD_i(T) \neq \emptyset$.*

This is not true when Γ is infinite.

Problem 5. [Two variants of 'pick the largest integer']

1. $\Gamma = (A_i, u_i)_{i \in I}$ *where* $I = \{1, 2\}$, $A_i = \mathbb{N}$, $u_i(n_i, n_j) = 1$ *if* $n_i > n_j$, *and* $u_i(n_i, n_j) = 0$ *otherwise. Show that every strategy is weakly dominated, and the game has no equilibrium.*
2. $\Gamma = (A_i, v_i)_{i \in I}$ *where* $I = \{1, 2\}$, $A_i = \mathbb{N}$, *and* $v_i(n_i, n_j) = \Phi(n_i - n_j)$, $\Phi(\cdot)$ *being the cdf of a non-degenerate Gaussian distribution. Show that every strategy is strongly dominated (hence the game has no equilibrium).*

We are now going describe an interative sequence of tests. The basic ingredient applied over is **passing at T-test**.

Definition 2. *For $E \subset Eq(\Gamma)$ and $T \subset \Delta$, E* **passes a T-test** *if*

$$(\forall \sigma \in E)(\forall i \in I)[\sigma_i(D_i(T)) = 0].$$

For example, E passes a Δ-test if no element of E puts mass on a dominated strategy.

[1] Well, not quite. There are several definitions of stable sets of equilibria running around in the literature. The better definitions have the property that a stable set of equilibria exists and satisfies the self-referential tests that are described here. Therefore, if a set fails the self-referential tests, then it must not be stable, and if only one set satisfies the tests, then it is the stable set.

The iterative procedure starts with $S_i^1 = A_i$, defines $\Delta^n = \times_{i\in I}\Delta(S_i^n)$, and if S^n has been defined, set $S_i^{n+1} = S_i^n \setminus D_i(\Delta^n)$. If Γ is finite, then Lemma 1 implies

$$(\exists N)(\forall n, n' \geq N)[S_i^n = S_i^{n'} \neq \emptyset].$$

There are many variations on this iterative-deletion-of-dominated-strategies theme. In all of them, $A_i^1 = \Delta_i$.

(1) $S_i^{n+1} = S_i^n \setminus D_i(\Delta^n)$. If this reduces the strategy sets to singletons, then the game is **dominance solvable** (a term due to Herve Moulin).
(2) $S_i^{n+1} = S_i^n \setminus WD_i(\Delta^n)$ where $WD_i(T)$ is the set of strategies weakly dominated with respect to T.
(3) Set $S_i^2 = S_i^1 \setminus WD_i(\Delta^1)$, and for $n \geq 2$, set $S_i^{n+1} = S_i^n \setminus D_i(\Delta^n)$. Dekel and Fudenberg, and Börgers show that the most that can be justified by appealing to common knowledge of the structure of the game and common knowledge of expected utility maximization is this kind of iterated deletion procedure.

These iterated procedures become really powerful when we make them self-referential. Let $Eq(\Gamma)$ be the set of Nash equilibria of a game Γ. Let us ask if a set of equilibria, $E \subset Eq(\Gamma)$, is "sensible" or "internally consistent" by asking if it passes an E-test. This kind of self-referential test is called an **equilibrium dominance test**. Verbally, this makes (some kind of) sense because, if everyone knows that only equilibria in a set E are possible, then everyone knows that no-one will play any strategy that is either weakly dominated or that is strongly dominated *relative to E itself.* That is, E should survive an E-test.

There is a problem with this idea as stated, if we take $E = Eq(\Gamma)$ and there are many equilibrium outcomes, people do not know enough to determine their own best response. We solve this by applying the iterative procedures only to elements of a particular class, $\mathcal{E}$, of subsets of $Eq(\Gamma)$. This is the class $\mathcal{E}$ of closed and connected subsets of $Eq(\Gamma)$ on which the outcome function is constant.

Formally, fix a set $E \subset Eq(\Gamma)$, set $S_i^1 = A_i$, $E^1 = E$. Given S_i^n and E^n for each $i \in I$, set $\Delta^n = \times_{i\in I}\Delta(S_i^n)$, and define S_i^{n+1} by

$$S_i^{n+1} = S_i^n \setminus \{WD_i(\Delta^n) \cup D_i(E^n)\}.$$

In this step, we eliminate from the game all weakly dominated strategies and all strategies that are dominated relative to E^n.

After this deletion, some of the strategies in E^n may not be playable in the new game. To take care of this, we define

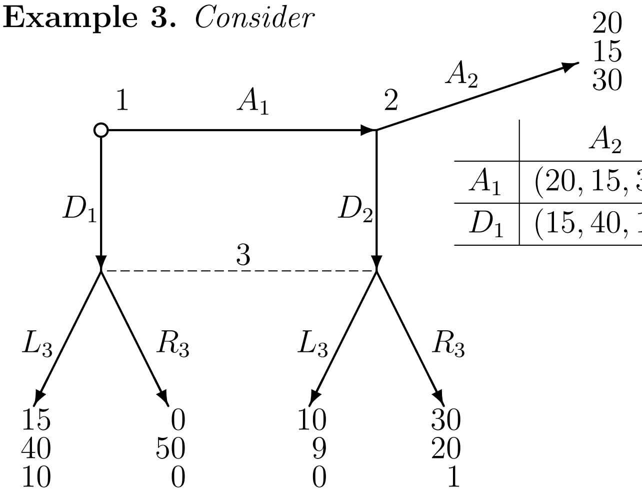
$$E^{n+1} = \{\sigma \in E^n : \forall \sigma \in E^{n+1},\ \forall i \in I,\ \sigma_i(D_i(E^n)) = 0\}.$$

It is possible than $E^{n+1} = \emptyset$. This tells us that the original set E has failed the test. In other words, we say that $E \in \mathcal{E}$ **passes the iterated equilibrium dominance test** if at each stage in the iterative process, there exists a non-empty $E^{n+1} \in \mathcal{E}$, $E^{n+1} \subset E^n$, such that $\forall \sigma \in E^{n+1}$, $\forall i \in I$, $\sigma_i(D_i(E^n)) = 0$.

You will now examine this workings of this logic, first in a "horse" game, then in a couple of signaling games.

Problem 6. *Returning to the previous horse game, which of the closed connected sets of equilibrium strategies that you found satisfy the self-referential test? [If you need it, and I would if I were seeing this for the first time, you can find a worked example of this procedure in a different horse game.]*

Example 3. *Consider*



	A_2	D_2
A_1	$(20, 15, 30)$	$(10, 9, 0)$
D_1	$(15, 40, 10)$	$(15, 40, 10)$

L_3

	A_2	D_2
A_1	$(20, 15, 30)$	$(30, 20, 1)$
D_1	$(0, 50, 0)$	$(0, 50, 0)$

R_3

There are three closed connected sets of equilibria for this game. Listing probabilities of playing down and left first, they are

$$E_A = \{((0, 1), (0, 1), (\gamma, 1 - \gamma)) : \gamma \geq 5/11\},$$

where the condition on γ comes from $15 \geq 9\gamma + 20(1 - \gamma)$,

$$E_B = \{((1, 0), (\beta, 1 - \beta), (1, 0)) : \beta \geq \frac{1}{2}\},$$

where the condition on β comes from $15 \geq 10\beta + 20(1 - \beta)$, and

$$E_C = \{((0, 1), (1, 0), (0, 1))\}.$$

Note that for any $\sigma, \sigma' \in E_k$, $k \in \{A, B, C\}$, the outcome is the same, hence the utilities are the same.

There are no weakly dominated strategies for this game, which means that iterated deletion of weakly dominated strategies changes nothing.

(1) $u_1(s \backslash D_1) = (15, 15, 0, 0)$ *while* $u_1(s \backslash A_1) = (10, 20, 30, 20)$ *so no weakly dominated strategies for* 1,

(2) $u_2(s\backslash D_2) = (40, 9, 50, 20)$ *while* $u_2(s\backslash A_2) = (40, 15, 50, 15)$ *so no weakly dominated strategies for* 2*,*

(3) $u_3(s\backslash L_3) = (10, 0, 10, 30)$ *while* $u_3(s\backslash R_3) = (0, 1, 0, 3)$ *so no weakly dominated strategies for* 3*.*

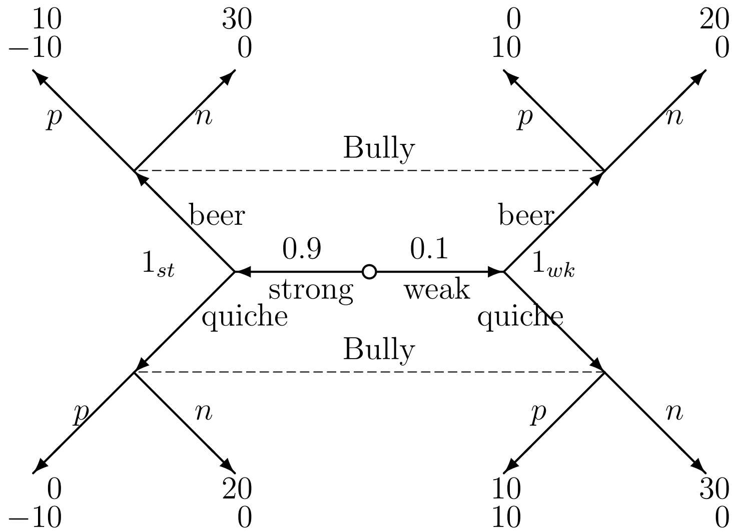
There are consistent beliefs that 3 *could hold to make* E_A *sequential because the beliefs associated with* $((\epsilon, 1-\epsilon), (\delta, 1-\delta), (\gamma, 1-\gamma))$ *are* $b = (\frac{\epsilon}{\epsilon+(1-\epsilon)\delta}, \frac{(1-\epsilon)\delta}{\epsilon+(1-\epsilon)\delta})$. *For* $\epsilon, \delta \downarrow 0$, b *can converge to anyplace in* $[0, 1]$. *There are beliefs,* b*, for which any of the given, strictly positive* γ*'s are best responses.*

However, such b*'s are not* ***stable*** *— if everyone knows that the outcome associated with* E_A *obtains, they must believe in some equilibrium strategy in* E_A. *However, against all of the strategies in* E_A*,* D_1 *is dominated for* 1*. Stable beliefs would survive the removal of such a strategy from the game, and none of the beliefs making* E_A *sequential are stable.*

It is not just the first round of arguments about beliefs that is needed for stability, suppose that we have eliminated a strategy and we look at the new game. Pbe, sequentiality, and stability should also apply here, in the new smaller game. However, the strategy R_3 *is dominated for* 3 *relative to* E_B*, removing* R_3 *make* D_2 *weakly dominated for* 2*, meaning that every* $\sigma \in E_B$ *puts mass on the deleted strategy.*

Therefore, since at least one closed connected set of equilibria survive the self-referential tests, E_C *does. Something that can be directly verified.*

The following is a famous example due Cho and Kreps (1987).



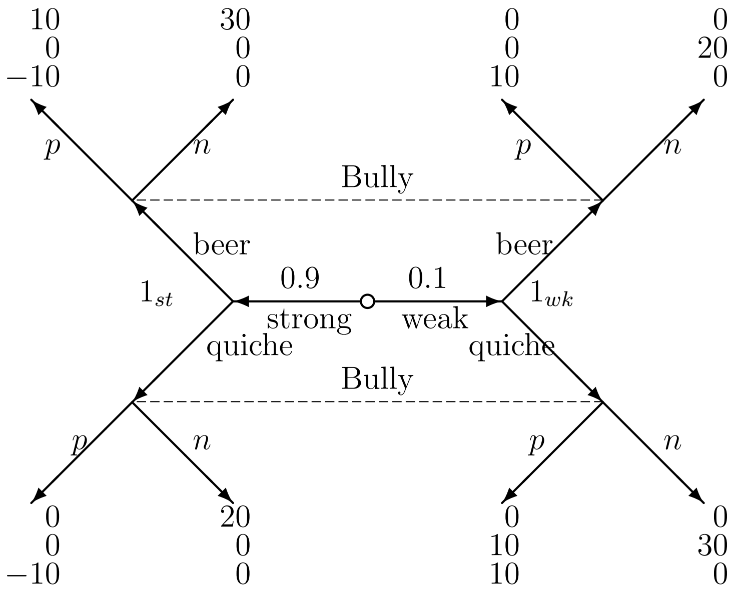
There are (at least) two versions of the story behind this game:

(1) There is a fellow who, on 9 out of every 10 days on average, rolls out of bed like Popeye on spinach. When he does this we call him "strong." When strong, this fellow likes nothing better than Beer for breakfast. On the other days he rolls out of bed like a graduate student recovering from a comprehensive exam. When he does this we call him "weak." When weak, this fellow likes nothing better than Quiche for breakfast. In the town where this schizoid personality lives, there is also a swaggering bully. Swaggering bullies are cowards who, as a result of deep childhood traumas, need to impress their peers by picking on others. Being a coward, he would rather pick on someone weak. He makes his decision about whether to pick, p, or not, n, after having observed what the schizoid had for breakfast.

(2) 9 out of every 10 days on average, a stranger who feels like Glint Westwood[2] comes into town. We call such strangers "strong." Strong strangers like nothing better than Beer (and a vile cigar) for breakfast. On the other days, a different kind of stranger comes to town, one who feels like a graduate student recovering from a comprehensive exam. We call such strangers "weak." Weak strangers like nothing better than Quiche for breakfast. Strong and weak strangers are not distinguishable to anyone but themselves. In the town frequented by breakfast-eating strangers,

[2]A mythical Hollywood quasi-hero, who, by strength, trickiness and vile cigars, single-handedly overcomes huge obstacles, up to and including bands of 20 heavily armed professional killers.

there is also a swaggering bully. Swaggering bullies are cowards who, as a result of deep childhood traumas, need to impress their peers by picking on others. Being a coward, he would rather pick on someone weak. He makes his decision about whether to pick, p, or not, n, after having observed what the stranger had for breakfast. With payoffs listed in the order 1_{st}, 1_{wk}, 2 and normalizing strangers' payoffs to 0 when they are not breakfasting in this town, the game tree is



This game has three players (four if you include Nature), 1_{st} (aka Glint), 1_{wk} (aka the weak stranger), and 2 (aka the Bully). In principle, we could also split the Bully into two different people depending on whether or not they observed Beer or Quiche being eaten. The logic is that we are the sum of our experiences, and if our experiences are different, then we are different people. If we did this second agent splitting, we would have the game in what is called **agent normal form**. In this game, instead of putting 0's as the utilities for the strangers' when they are not breakfasting in this town, we could have made 1_{st}'s utility equal to 1_{wk}'s even when they are out of town. Since we are changing utilities by adding a function that depends only on what someone else is doing, this cannot change anything about the equilibrium set.

Problem 7. *At the end of this problem, you will have found that only one equilibrium outcome of the Beer-Quiche game survives the self-referential tests.*

1. *Show that* 2 *has a dominated strategy,* (p, p).
2. *Show that by varying* 2*'s strategy amongst the remaining* 3*, we can make either Beer or Quiche be a strict best response for both strangers.* [*This means that no strategies are dominated for the strangers, and iterated deletion of dominated strategies stops after one round.*]
3. *Show that the equilibrium set for this game can be partitioned into two sets,* E_1 *and* E_2*, but note that we must now specify* 3 *strategies,*

$$E_1 = \{((q), (q), (0, \beta, 0, 1-\beta)) : 21 \geq 12\beta + 30(1-\beta),\ \textit{i.e.}\ \ \beta \geq \frac{1}{2}\},$$

and

$$E_2 = \{((b), (b), (0, 0, \beta, 1-\beta)) : 29 \geq 28\beta + 30(1-\beta)\ \ \textit{i.e.}\ \ \beta \geq \frac{1}{2}\}.$$

Note that the outcome function, $\mathbb{O}(\cdot)$*, is constant on the two closed and connected sets* E_1 *and* E_2.

4. *Show that the self-referential tests do eliminate* E_1. [*For* 1_{wk}*, Beer is dominated relative to* E_1*, after removing Beer for* 1_{wk}*,* (p, n) *is weakly dominated for* 2*, implying that no* $\sigma \in E_1$ *survives the iterative steps.*]
5. *Show that* E_2 *does survive the iterative steps of the self-referential tests.*

4. Some optional stuff

Here are a variety of signaling games to practice with if you are so inclined. I am not requiring that you do these. I offer them in case you get hooked on signaling games.

The presentation of the games is a bit different than the extensive form games we gave above, part of your job is to draw extensive forms. Recall that a pooling equilibrium in a signaling game is an equilibrium in which all the different types send the same message, a separating equilibrium is one in which each types sends a different message (and can thereby be separated from each other), a hybrid equilibrium has aspects of both behaviors.

The presentation method is taken directly from Banks and Sobel's (1987) treatment of signaling games. Signaling games have two players, a Sender S and a Receiver R. The Sender has private information, summarized by his type, t, an element of a finite set T. There is a strictly positive probability distribution ρ on T; $\rho(t)$, which is common knowledge, is the ex ante probability that S's type is t. After S learns his type, he sends a message, m, to R; m is an element of a finite set M. In response to m, R selects an action, a, from a finite set $A(m)$; S and R have von Neumann-Morgenstern utility functions $u(t,m,a)$ and $v(t,m,a)$ respectively. Behavioral strategies are $q(m|t)$, the probability that S sends the message m given that his type is t, and $r(a|m)$, the probability that R uses the pure strategy a when message m is received. R's set of strategies after seeing m is the $\#A(m)-1$ dimensional simplex Δ_m, and utilities are extended to $r \in \Delta_m$ in the usual fashion. For each distribution λ over T, the receiver's best response to seeing m with prior λ is

$$Br(\lambda, m) = \text{arg max}_{r(m)\in\Delta_m} \sum_{t\in T} v(t,m,r(m))\lambda(t). \tag{1}$$

Examples are represented with a bi-matrix $B(m)$ for each $m \in M$. There is one column in $B(m)$ for each strategy in $A(m)$ and one row for each type. The (t,a)'th entry in $B(m)$ is $(u(t,m,a), v(t,m,a))$. With t_1 being the strong type, t_2 the weak, m_1 being beer, m_2 being quiche, a_1 being *p*ick a fight, and a_2 being *n*ot, the Beer-Quiche game is

$B(m_1)$	a_1	a_2
t_1	$10,-10$	$30,0$
t_2	$0,10$	$20,0$

$B(m_2)$	a_1	a_2
t_1	$0,-10$	$20,0$
t_2	$10,10$	$30,0$

You should carefully match up the parts of this game and the extensive form of B-Q given above.

Here is a simple example to start on:

$B(m_1)$	a_1
t_1	$2,2$
t_2	$2,2$

$B(m_2)$	a_1	a_2
t_1	$3,3$	$0,0$
t_2	$0,0$	$3,3$

Problem 8. *Draw the extensive form for the game just specified. Find the* 3 *connected sets of equilibria. Show that all equilibria for this game are sequential. Show that the* 3 *connected sets of equilibria satisfy the self-referential tests.*

The following game is Cho's (1987, Example 2.1): the types are A, B, and C, $\rho(A) = \rho(C) = 3/8$, $\rho(B) = 1/4$, the messages are L and R, and the actions are as given.

$B(L)$	U	D
A	$2,1$	$-1,0$
B	$2,1$	$0,0$
C	$2,0$	$0,1$

$B(R)$	U	M	D
A	$0,2$	$0,0$	$0,2$
B	$0,2$	$4,3$	$1,-1$
C	$0,-3$	$1,-2$	$4,0$

Problem 9. *Draw the extensive form for the game just specified and analyze the equilibrium set.*

The following is a sequential settlement game of a type analyzed by Sobel (1989): There are two types of defendants, S: type t_2 defendants are negligent, type t_1 defendants are not, $\rho(t_1) = 1/2$. S offers a low settlement, m_1, or a high settlement, m_2. R, the plaintiff, either accepts, a_1, or rejects a_2. If R accepts, S pays R an amount that depends on the offer but not S's type. If R rejects the offer, S must pay court costs and a transfer depending only on whether or not S is negligent. With payoffs, the game is

$B(m_1)$	a_1	a_2
t_1	$-3,3$	$-6,0$
t_2	$-3,3$	$-11,5$

$B(m_2)$	a_1	a_2
t_1	$-5,5$	$-6,0$
t_2	$-5,5$	$-11,5$

Problem 10. *Draw the extensive form for the game just specified. Analyze the equilibria of the above game.*

One more game! This one has $\rho(t_1) = 0.4$.

$B(m_1)$	a_1
t_1	$0,0$
t_2	$0,0$

$B(m_2)$	a_1	a_2	a_3	a_4
t_1	$-1,3$	$-1,2$	$1,0$	$-1,-2$
t_2	$-1,-2$	$1,0$	$1,2$	$-2,3$

Problem 11. *Draw the extensive form for the game just specified and analyze the equilibrium set.*