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Introduction

i. Preface

This project considers Avinash K. Dixit's duopoly model, first introduced in his paper The Role of Investment in Entry-Deterrence (The Economic Journal, March 1980). It is a sequential move game with pre-entry capacity investment by an incumbent. The computational model displays the theory of this, and is made to compute equilibrium and critical values. The object of this project is to show the various possible sub-game equilibriums. In addition, solvers are presented which can compute values for any parameter values.

When solving this model, the goal is to find the optimal strategy of the incumbent. The theory of strategic behavior is important to this. This is briefly outlined in the introduction.

ii. Strategic Behavior

To properly consider strategic moves and behavior, the features of these interactions must be mentioned. These were originally published by Thomas Schelling in The Strategy of Conflict (1960). A necessary distinction is made between threats, promises, and commitments. In relation to industrial organization,

a threat means to inflict a penalty on a rival for certain behavior,

a promise means to give a reward to a rival for certain behavior, and

a commitment exists if it is rational for an agent to carry out a threat or promise.

Obviously credibility is necessary if threats and promises are to be made commitments, and an agent can successfully manipulate the behavior of a rival. Schelling (1960) argues that a strategic move can convert talk into commitments. Strategic moves are meant to change the behavior of a rival by changing their expectations of how the agent will act. To effectively do so, four elements are required to make a move strategic.

Rational Expectations: A move must be optimal for an agent to carry out if the rival is to believe it. It must increase an agent's payoff.

Affect Incentives: A move must change the agent's incentives and choices. It must change what is truly optimal for the agent either presently or in the future.

Communication: Rivals must be aware of an agent's move or action before it occurs.

Sequential Moves: The agent is able to move before rivals make their final move.

Engaging in a strategic move requires analysis of the market's structure and payoffs in the future. Anticipation of rival's actions must be considered by the agent in order to maximize payoff. When analyzing Dixit's model, the incumbent is strategy is analyzed to determine its optimal path.
Part 1: The Model

1.1 The Basics

The fundamentals of the Dixit model are:

- 2 firms, the incumbent (firm 1) and the entrant (firm 2)
- Two stages:
  1. Firm 1 chooses investment in capacity of $k_1$
  2. Firm 2 observes $k_1$, and considers entry. If enters, chooses capacity $k_2=q_2$
- Post-entry competition, if entry occurs, is simultaneous (Cournot)
- Homogenous output, requiring 1 unit of labor ($w$) and 1 unit of capacity ($r$) to produce 1 unit of output ($q_i$)
- Startup costs can exist or not, represented by $fc$

Market demand is in the typical form

$$P = A - b(Q)$$

with

$$Q = q_1 + q_2$$

If entry occurs, firm 1 faces two different cost functions which depend on the level of pre-entry capacity investment. If firm 1 installed adequate capacity, it must only pay for labor ($w$) to produce $q_1$. If capacity investment was inadequate to produce the optimal $q_1$, firm 1 must expand its capacity and face additional expansion costs ($r$). Formally,

$$MC_1 = \begin{cases} 
  w & \text{if } q_1 \leq k_1 \\
  w + r & \text{if } q_1 > k_1 
\end{cases}$$

The entrant, if entry is feasible, must install capacity equal to its level of $q_2$. Therefore, it faces the marginal cost function

$$MC_2 = w + r \quad \forall q_2$$

It is evident that the incumbent has an advantage if it installs adequate capacity. The plot of firm 1's graph is shown below.
Figure 1. The MC curve for firm 1 is kinked at $k_1$. Marginal revenue is shown for increasing levels of $q_2$.

The typical procedure for solving quantity games is followed in this model. The best response curves are derived in Mathematica by maximizing profit and solving for quantity, given the rival’s quantity.

Firm 1 has two curves corresponding to its two cost functions. These are

$$q_{1w}[q_{2-}]: = \frac{A - bq_2 - w_1}{2b}$$

if $q_1 \leq k_1$, and

$$q_{1wr}[q_{2-}]: = \frac{A - bq_2 - r_1 - w_1}{2b}$$

if $q_1 > k_1$.

For firm 2, the response function is

$$q_{2wr}[q_{1-}]: = \frac{A - bq_1 - r_1 - w_1}{2b}$$

All of these curves are plotted in the "$q_1$, $q_2$" space. The true response function for firm 1 is a single curve kinked at $k_1$. 
The point $T$ can be thought of as the Cournot point. It is the intersection of the best response curves when firms face identical marginal costs. In this case, market share and quantity is symmetric. Point $V$ is also a Cournot point, but the incumbent captures a larger market share. Where this kink occurs is critical to the resulting equilibrium.
1.2 Strategy

Although only five quantity choices are shown for each firm, there are theoretically infinite choices and resulting sub-games.

It is evident, due to first-mover advantage, that the incumbent is in a position to threaten the entrant with zero or negative profits if it enters. It must also consider the general strategy tree for entry games if this threat is credible and to be made a commitment.
Figure 1.4 The incumbent may try to capture more market share and be 'aggressive'. If entry deterrence is successful, it will receive monopoly rents minus the cost of deterring entry. If the entrant is not deterred, the incumbent may 'accommodate' or 'fight' with different payoffs.

The 'passive' strategy by the incumbent is not often considered in the Dixit model. The main branches considered are shown below.

Figure 1.5 Incumbent chooses to be aggressive, and must consider the relative value of payoffs.

The symbols for payoffs take the form (entrant, incumbent) and mean:
- $\pi^m$: monopoly profits
- $\pi^d$: symmetric duopoly profits
- $\pi^w$: price war profits

Although price war does not occur in a quantity game, it can be thought of as profit from a quantity competition. If an incumbent installs an adequate level of capacity, the market share will favor the incumbent. However, the actual value of these payoffs depend on market conditions and chosen level of $k_1$. 
In order to determine the sub-game perfect Nash equilibrium, backward induction must be used. Various cases may arise due to the relative values of payoffs.

- If $\pi^w > \pi^d - c$, 'fight' is the optimal strategy for the incumbent after choosing 'aggressive'
- If $\pi^m - c > \pi^d - c$, or $\pi^m - c > \pi^w$, 'aggressive' strategy which induces 'out' strategy for the entrant is optimal

In order to properly solve this game, the second stage must be solved for any value of $k_1$. Three different situations may arise depending on the level of $k_1$ in relation to points $T$ and $V$. 
1.3 Three Nash Sub-games for an arbitrary $k_1$

1.) Inadequate capacity

This is defined as the **Capacity Expansion** outcome. In this case, $\text{MR}_1$ exceeds $\text{MC}_1$ at $k_1$. It is optimal for firm 1 to expand production in this case.

**Figure 1.5** $\text{MR}_1 > \text{MC}_1$ at $k_1$ (left). Firm 1 will install additional capacity in order for $\text{MR}=\text{MC}$ (right)
2.) Too much capacity

This is defined as the **Excess Capacity** outcome. MR₁ intersects MC₁ before k₁, and production should occur where MR₁ = MC₁. Firm 1 cannot credibly threaten to produce at this level since MC₁ > MR₁ at this point. Firm 1 will produce at q₁^V.

*Figure 1.6* k₁ exceeds q₁^V

*Figure 1.7* Overinvestment in capacity, production occurs at q₁^∗
3) Adequate Capacity

This is defined as the **Full Utilization** outcome. Firm 1 will produce exactly at capacity.

![Diagram](image1)

**Figure 1.8** $q_1^V > k_1 > q_1^T$

![Diagram](image2)

**Figure 1.9** Various MR curves are shown for different levels of $q_2$. Regardless of $q_2$, firm 1 will produce at capacity.

The **Full Utilization** outcome represents many different possible equilibria, as the level of capacity investment can be altered depending on fixed costs.
Part 2: Experiments

Now that the potential outcomes for relative values of capacity investment have been established, the question of optimal capacity investment remains. For an arbitrary value of capacity investment, it is clear that firm 1 has three distinct choices. However, when analyzing the potential outcomes after the second stage, it is evident that firm 1 has much more to consider. If firm 1 invests at the full utilization outcome, there is an even greater set of sub-game Nash equilibriums that can occur. The value and position of these equilibriums depend on profits, which in turn depend on fixed costs.

To describe and test these, additional points are added to the graphs in the "q₁, q₂" space. These are

- Point B: where firm 2's profits are non-positive, with corresponding $q_L^T$ (limit quantity)
- Point S: the Stackelberg outcome, with corresponding $q^K_S$.

The position of these points relative to $T$ and $V$ will reveal firm 1's optimal strategy.

The results section will determine the optimal values of capacity investment for many unique sub-games that can occur between $T$ and $V$. The explanation of these sub-games is detailed graphically. In the discussion section, Exercise 14.2 from Church & Ware (1999) is solved using actual values. Calculators are used to obtain solutions.

The experiments are not fully comprehensive. There are many sub-games that can occur, and when parameters are changed there are even more. The experiments will find the most common of these sub-games, and the most important factors which effect them.

Three general outcomes may occur:

- Blockaded Monopoly: High fixed costs naturally deter entry
- Stackelberg-type: Where the incumbent accommodates, but quantity is asymmetric
- Strategic Accommodation or Deterrence: Depending on optimal choice, either may occur
Part 3: Results

3.1 High Fixed Costs: Blockaded Monopoly

If fixed costs are very large, the limit quantity $q^L_1$ occurs to the left of $q^T_1$. Even if no capacity is installed in the first stage, firm 1 does not fear entry by firm 2. The best outcome firm 2 could hope for is at point $T$, which results in negative profits. Firm 1 will invest and produce at the monopoly quantity.

Figure 3.1 Blockaded Monopoly: $q^L_1 < q^T_1$
3.2 Stackelberg outcome 1: \( \pi_2 > 0 \) at \( q_1^V \)

In this sub-game, the profits of firm 2 are positive at point \( V \), and the limit output lies beyond \( q_1^V \). As mentioned, firm 1 cannot commit to any output beyond \( q_1^V \). As a result, firm 1 must accommodate firm 2. Since the Stackelberg outcome lies before \( q_1^V \), and it is optimal for firm 1 to produce at \( q_1^S = k_1 \).

\[ \text{Figure 3.2 Accommodation 1: } q_1^S < q_1^V, \text{ production occurs at } q_1^S = k_1 \]
3.3 Stackelberg outcome 2: $\pi_2 > 0$ at $q_1^V$, $q_1^S > q_1^V$

Again, the profits of firm 2 are positive at point $V$. Now the Stackelberg point lies beyond point $V$. Since firm 1 cannot commit to any quantity greater than $q_1^V$, and production will occur at $q_1^V = k_1$.

**Figure 3.3** Accommodation 2: $q_1^S > q_1^V$, production occurs at $q_1^V = k_1$
3.4 Strategic Entry Deterrence: Blockaded Monopoly

If the limit quantity lies between point $T$ and $V$, meaning $\pi_2<0$ at $V$ and $\pi_2>0$ at $T$, two situations arise. The first is where the limit quantity is less than the monopoly quantity. Firm 1 can easily install capacity equal to the monopoly output and enjoy monopoly rents. This is another case of Blockaded Monopoly.

**Figure 3.4 Blockaded Monopoly 2: $q_1^L < q_1^M$**
3.5 Strategic Accommodation or Deterrence

If firm 1 cannot install monopoly capacity, it must analyze the relative profitability of alternatives. These two alternatives are installing the Stackelberg quantity or the limit quantity. Profitability of both must be determined. By plotting the iso-profit curves, this can be seen.

Figure 3.5 Accomodation is more profitable
In *Figure 3.5*, a higher profit (lower iso-profit curve) is obtained by producing the Stackelberg output than producing the limit quantity. Economies of scale must be low for the limit output to be so far to the right. Accommodation will occur.

If economies of scale exist, and are sufficient, producing the limit quantity can be a credible threat if it is at least as profitable as producing the Stackelberg quantity. In this case, entry will be deterred.

*Figure 3.6* Producing $q_1^L$ is profitable. Entry is deterred.
Part 4: Discussion

4.1 Solving the model

The optimal strategy for firm 1 relies heavily upon the given market parameters. Where the points $T$ and $V$ occur is of primary interest to the incumbent. When the response curves are solved simultaneously for $q_1$ and $q_2$,

$$q_T^1 = \frac{A - r - w}{3b}$$
$$q_T^2 = \frac{A - r - w}{3b}$$

and

$$q_V^1 = \frac{A + r - w}{3b}$$
$$q_V^2 = \frac{A - 2r - w}{3b}$$

The difference between these two points relies primarily on the cost of capacity ($r$). A solver is presented below which can calculate these values.

<table>
<thead>
<tr>
<th>Equilibrium Quantities at T &amp; V</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 68</td>
</tr>
<tr>
<td>b 1</td>
</tr>
<tr>
<td>Cost of Capacity</td>
</tr>
<tr>
<td>Cost of Input</td>
</tr>
</tbody>
</table>

\[
\begin{bmatrix}
q_T^1 & 28 \times 3, \\
q_T^2 & 28 \times 3, \\
q_V^1 & 104 \times 3, \\
q_V^2 & 10 \times 3
\end{bmatrix}
\]

Figure 4.1 Solving for points $T$ and $V$

It is important to note that sunk fixed costs do not affect the position of these points.

When these quantities are found, the rational range of capacity investment is evident. In addition to these quantities, total quantity and price can be found. Monopoly quantity is also an important value to consider.
### Figure 4.2 Total Quantity, Price and \( Q_m \)

<table>
<thead>
<tr>
<th></th>
<th>T &amp; V</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Equilibrium Quantity</strong></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>68</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
</tr>
<tr>
<td>Cost of Capacity</td>
<td>38</td>
</tr>
<tr>
<td>Cost of Input</td>
<td>2</td>
</tr>
<tr>
<td>(Q_T)</td>
<td>(\frac{56}{3})</td>
</tr>
<tr>
<td>(Q_V)</td>
<td>(\frac{94}{3})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>T &amp; V</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equilibrium Price</strong></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>68</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
</tr>
<tr>
<td>Cost of Capacity</td>
<td>38</td>
</tr>
<tr>
<td>Cost of Input</td>
<td>2</td>
</tr>
<tr>
<td>(Price_T)</td>
<td>(\frac{148}{3})</td>
</tr>
<tr>
<td>(Price_V)</td>
<td>(\frac{110}{3})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>T &amp; V</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Monopoly</strong></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>68</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
</tr>
<tr>
<td>Cost of Capacity</td>
<td>38</td>
</tr>
<tr>
<td>Cost of Input</td>
<td>2</td>
</tr>
<tr>
<td>(Q_m)</td>
<td>14</td>
</tr>
</tbody>
</table>
As revealed in the results section, if investment occurs between these points, production will occur at capacity. The variable $q_1$ can be replaced with $k_1$ in the quantity and profit equations. Fixed entry costs must be considered for optimal investment to be found. The value of this cost will only affect firm 2, since firm 1 is already established.

Point B, and the corresponding limit quantity, is found by solving

$$\pi_2 = 0$$

for $q_1$, which is

$$q_1^L = \frac{A - 2\sqrt{b}\sqrt{r_c} - r - w}{b}$$

Evidently, the larger the fixed cost is, the smaller the limit quantity is. A solver for this quantity is shown which allows for different fixed costs. If this lies between $T$ and $V$, this value can be input as $k_1$.

![Figure 4.3 Limit quantity and proof](image)
When startup costs are 49, the limit quantity lies between points $T$ and $V$. Firm 1 would produce 14, and if firm 2 still wishes to produce, it would do so at 7.

\[ \text{Figure 4.4 The lowest iso-profit curve is obtained when } k_1 = q_1^L \]

4.2 Extensions

So far, the solution to this model has mainly considered the incumbent’s strategy. A worthy extension to consider is the welfare effects of this market structure. All of the information to do this is contained within the notebooks. It would require the calculation of

\[ TS = \frac{Q^2}{2} + \pi \]

where $\pi$ is aggregate profits (producer surplus) and $Q$ is market quantity.

Church & Ware (1999, p.503) detail strategic accommodation as welfare enhancing. However, in the case of strategic entry deterrence, the welfare effect depends on parameter values. For example, if fixed costs are high, strategic deterrence and the creation of a natural monopoly is welfare enhancing.

Another extension would be to consider the strategic effects of the strategy employed by the incumbent. In the Cournot-style Dixit model, products are strategic substitutes. Whether investment is a
'tough' or 'soft' strategy depends on the relationship between firm 2's profit and firm 1's investment. This relationship is best approximated by

\[
\frac{\partial \pi_2}{\partial k_1} = \frac{\partial \pi_1}{\partial q_1} \left( \frac{\partial \pi_1}{\partial k_1} \right)
\]

If \( \frac{\partial \pi_2}{\partial k_1} < 0 \), investing is a 'tough' strategy and 'top dog' strategy. It would seem intuitive that capacity investment would be a 'top dog' strategy, but this would need proof. It might not be true for all parameter values.

A final extension would be to analyze this game structure under Bertrand competition. Products would be strategic complements, and the outcome would change dramatically. This is briefly outlined in *The Role of Investment in Entry-Deterrence (1980)*.
Part 5: Computational Model

5.1 Using Mathematica

The strategy trees outlined in section 1.2 are plotted in Mathematica. It might not be the best medium for displaying graphics, but it is capable of doing so. To better display the graphics in a word document, they are saved as .bmp and copied via Paint.

Mathematica does have a built-in "TreePlot[" function for making tree graphics, but adjusting the appearance and accurately labeling the vertices is not simple. Instead, the "Graphics[" command with "Circle[", "Arrows[", and "Text[" is used to create a diagram of the game. Coordinates are defined to position the graphics appropriately.

A sample:

```mathematica
extgame = Graphics[{Arrow[{{0, 0}, {20, 20}}],
    Thick, Circle[{-3, 0}, 3], Directive[Black, Thick], Text[1, {-3, 0}] ...}
labels = Graphics[{Text[StyleForm[Choice of k1, FontFamily -> Times, FontSize -> 12, FontWeight -> Bold], {1, -15}] ...}
Show[extgame, labels]
```

After defining multiple graphics, they can be combined using the "Show["] function.

The MR/MC graph shown in Figure 1.1 was mainly used to display the theory of this model. To do so, equations for the lines were given arbitrary values to accommodate a 10x10 space. A plot was created separately from the labels, then combined with the "Show["] function.

A sample:

```mathematica
plot1 = Plot[If[0 <= q1 <= 5, w] ... 
labelwr = Graphics[Text["MC_{w_1+r_1}", {12, 6}]];
```

It was soon evident that changing this graph for different situations would require a lot of manipulation to the coordinates given to labels and lines. Using the "Manipulate[""] function allows for a much more dynamic graph. This was used for the graphs in section 1.3.

A sample:

```mathematica
Manipulate[Show[Plot[... brf1w, brf1wr, brf2,...
... Graphics[... Line[...]]] ...
... Item["Quantity Subgames", Alignment -> Center], {k1, 2, "Capacity Investment"}, {2 -> "Capacity Expansion", 4 -> "Full Utilization", 6.3 -> "Excess Capacity"]}]`
This yields a nice graph that can be manipulated for value of capacity investment. To make it more dynamic, the coordinates of lines and labels are changed when the value of \( k_1 \) is changed. This graph was still not dynamic enough; it was plotted for certain values of the parameters \( A, b, w, \) and \( r \).

To be able to show a more dynamic graph, the actual mathematics of this model must be plotted. These equations must first be found.

Functions in *Mathematica* may take the form:

Using this form, the best response curves are derived and plotted. Solving took much the same form as Kendrick et. al (2006, ch9-10) describes.

To allow these functions to be manipulated within a plot, they must be expressed as their equation and not their assignment. For example,

\[
\text{Manipulate[Show[Plot[}}\left[\frac{A - 2bq1 - w}{b}, \frac{A - 2bq1 - r - w}{b}\right]\text{, ...}
\]

will not allow for the manipulation of the parameters, but

\[
\text{Manipulate[Show[Plot[}}\left[\frac{A - 2bq1 - w}{b}, \frac{A - 2bq1 - r - w}{b}\right]\text{, ...}
\]

will allow any of the parameters to be manipulated. This can be problematic if the basic functions change form, since the plot definitions must also be changed. Many different options are used correct range and display of the plot, and are shown in the notebook files.

To enable the labels and ticks to move with the manipulation of parameters, they must also use equations.

... Ticks \[\left\{\left\{\frac{A - r - w}{3b}, q_1^L\right\}, \left\{z, q_1^L\right\}, \left\{A + r - w, q_1^V\right\}\right\}, \left\{\frac{A - r - w}{3b}, q_2^L\right\}, \left\{A - 2bq1 - r - w}{2b}\right\}\text{, ...}

/. \text{q1} \rightarrow z, q_2^L, \left\{\frac{A - 2r - w}{3b}, q_2^V\right\}\text{, ...}

... Thin, Dashed, Black, Line \[\left\{\left\{0, \frac{A - r - w}{3b}\right\}, \left\{A - r - w, \frac{A - r - w}{3b}\right\}\right\}\text{, ...}

... Text \[\text{StyleForm[R}\text{1}^w, \text{FontFamily} \rightarrow \text{Times}, \text{FontSize} \rightarrow 12, \text{FontWeight} \rightarrow \text{Bold}]\left\{10, \left(\frac{A - 2bq1 - w}{b} / \text{q1} \rightarrow 1\right) + 1\right\}, \text{...}

Once a highly dynamic plot is created, the many different sub-games can easily be illustrated. Additional labels can be added to suit any situation.
The calculators shown in section 4 are very similar in form to the dynamic graphs. With use of the "Manipulate" function, and in some cases the "Solve" and "Simplify" functions, the calculators are created.

\[
\text{Manipulate[Simplify[Solve][(-A + bk1 + r + w)^2 = 4b] = fc, k1][[1]]}, \text{Item["Limit Quantity"}, \text{Alignment} \rightarrow \text{Automatic}]], \text{Item["A"}, 68], \text{Item["b"}, 1], \text{Item["r"}, 38], \text{Item["w"}, 2]], \text{Item["fc"}, 4], \text{Item["k1"}, 24]]; \\
\]

This creates a calculator with both value-entry and a drop down list for manipulation.

![Calculator Image]

Clearly, generalization allows for a few graphs to display many different situations.
References


