Assignment #2 for **Managerial Economics**
Fall 2017

Due date: Mon. March 4, 2019.

Readings: Klein et al. Ch. II, Kreps Ch. 15, 18.

Problems Involving General Equilibrium and Efficiency

A. This problem is based on the data in the Table below (which reproduces the class handout).

Division	Project	B_k	C_k	B_k/C_k
I	1	600	100	6
	2	1,400	200	7
	3	1,000	200	5
II	4	500	50	10
	5	750	250	3
	6	1,000	200	5
III	7	900	100	9
	8	3,500	500	7
	9	1,600	400	4
IV	10	800	100	8
	11	1,000	250	4
	12	1,200	400	3

(a) Find the profit maximizing allocation of the resource to the different divisions. At that allocation, what is the value of an extra unit of the resource?

(b) Give each division's demand for the resource as a function of the price p charged for the resource, and give the horizontally summed demand curve.

(c) The supply curve is fixed at $1,200$. Find the intersection of the demand and the supply curves. Why and how is this answer related to the your answer for the first part of the problem?

(d) Now suppose that the resource is produced by Division V in the company, and the total cost for x of it is given by $TC(x) = \frac{x}{400} + \frac{x^2}{10{,}000}$. How much should Division V produce? And at what price should it be sold to Divisions I-IV?

B. You have 12 workers and must decide which 6 of them will work on the six machines of type A and which 6 will work on the six machines of type B. If worker i works on the machine of type A they make profits of $\pi_{i,A} > 0$ for the firm, if on B, they make $\pi_{i,B} > 0$ for the firm. This problem uses the data from the following table. (Note that the last row contains the column averages, e.g. $452.8 \simeq (\sum_{i=1}^{12} \pi_{i,A})/12$.)

i	$\pi_{i,A}$	$\pi_{i,B}$	$\pi_{i,A} - \pi_{i,B}$
1	361	273	88
2	522	731	-209
3	823	821	2
4	170	762	-592
5	364	957	-593
6	389	832	-443
7	493	296	197
8	293	779	-486
9	845	757	88
10	442	623	-181
11	239	607	-368
12	493	991	-498
Avg.	452.8	701.6	-248.8

1. Find the assignment of workers to machines that maximizes profit and compare the maximal total profits to the expected profits that would result from a random (uniformly distributed) assignment of workers to machines.
2. Suppose the initial assignment is workers 1 through 6 on machines on type A and 7 through 12 on machines of type B. Further suppose that the workers have ownership rights to these assignments and to the profits they produce for the firm by working at these machines. In this part of the problem, I ask you to imagine a market for these ownership rights, to imagine that there is a price P_A for ownership of the right to work a machine of type A and P_B for machine B.
   a. Graph, as a function of P_A/P_B, the amount of ownership rights for the right to work on machine A that would be supplied.
   b. Graph, as a function of P_A/P_B, the amount of ownership rights for the right to work on machine A that would be demanded.
   c. Find the price or range of prices at which demand is equal to supply and show that the market allocation at these prices is the same as the profit maximizing allocation you found above.
3. Suppose again that the initial assignment is workers 1 through 6 on machines on type A and 7 through 12 on machines of type B. Suppose that the workers own stock options whose value increases as company profit increases. Also, any pair of workers are free to swap assignments if it is mutually agreeable. Provided the workers don't have preferences over which types of jobs they are assigned to, which pair(s) workers would like to swap? Show that after all possible mutually agreeable swaps have happened, the allocation is the same as the profit maximizing allocation you found above.

C. From Kreps, problems 15.1 and 15.5, p. 350-1 and 352, on the basics of producer and consumer surplus and the effects of taxes.

D. From Kreps, problem 15.2, p. 351, on the role of fixed costs and short-run vs. long-run equilibria.

E. From Kreps, problem 15.6, p. 352, on market and non-market allocations.

Problems Involving Corporate Liabilities and Partnerships

F. Some definitional questions.
   1. Explain the difference between a secured creditor and a general creditor.
   2. Explain how a non-recourse loan for capital to invest in a business limits a sole proprietor's personal liability.
   3. A bank loans \$15,000 to the sole proprietor of a business at an interest rate of %8, and unexpectedly, a competitor goes out of business. Explain the subsequent divergence between the book value and market of the bank's note.

G. Partnerships are formed for many reasons, and once formed, they have their own set of incentive problems.
   1. In Ch. 2 of your Klein et al. textbook, what reasons are discussed for the formation of partnerships?
   2. What set of incentive problems special to partnerships are discussed?

H. [Sequential, simultaneous, and efficient decisions] A software designer, s, and marketer, m, form a partnership to which they contribute their efforts, respectively $x \geq 0$ and $y \geq 0$. Both have quasi-linear utility functions,

$$u_s = \$_s - x^2 \text{ and } u_m = \$_m - y^2,$$

where $\$_s$ and $\$_m$ are monies received by s and m respectively. The twice continuously differentiable, strictly concave profit function π satisfies

$$\begin{aligned} &\pi(x,0) = \pi(0,y) = 0, \text{ and} \\ &(\forall x, y > 0)[\partial\pi(x,y)/\partial x > 0,\ \partial\pi(x,y)/\partial y > 0,\ \partial^2\pi(x,y)/\partial x \partial y > 0]. \end{aligned}$$

   1. Give the FOCs for the equilibrium for the scenario in which the partners choose their efforts simultaneously, and share the profits equally.
   2. Give the FOCs for the equilibrium for the scenario in which the software designer chooses her effort before the marketer chooses hers, and the marketer observes the designer's effort before choosing her own.
   3. Give the FOCs for the efficient situation, where the efforts are chosen to maximize the profits available to be equally shared between the two.
   4. Supposing now that a third party can buy the partnership, employ the two to continue their jobs, and, very importantly, pick levels of effort and reward for them. Show that the buyer can make a profit while simultaneously making the two at least as well off as they were in either of the first two scenarios.
   5. Compare the equilibrium levels of effort and payoffs in the first three scenarios. If you need to make extra assumptions to get definite answers, be explicit about them.
   6. Identify if/how the material from the previous problem on the formation of partnerships relates to the first five parts of this problem.

Problems Involving Risk and Reactive Risk

I. A sole proprietor invests \$120,000 in a small business, \$80,000 of their own money and \$40,000 borrowed from a bank at a %10 rate of interest (so that \$4,000 must be paid to the bank at the end of the fiscal year).
   1. If, net after all expenses and allowances, including a reasonable amount for her own services but before interest payments, the sole proprietor makes a fiscal year profit of \$18,000 (or a %15 rate of return on investment). What is the proprietor's rate of return on her own investment?
   2. If, net after all expenses and allowances, including a reasonable amount for her own services but before interest payments, the sole proprietor makes a fiscal year profit of \$6,000 (or a %5 rate of return on investment). What is the proprietor's rate of return on her investment?
   3. Give the word used to describe the financial consequences of the use of debt and equity that you have just found.

J. For this problem, suppose that, R, the rate of return on money invested in a firm has a uniform distribution over the interval $[0.8, 1.6]$. The firm is financed using D in debt, in the form of a non-recourse loan, and E in equity. At the end of the year, payment due on D is $0.08 \cdot D$. This must be paid out of the total returns, $R \cdot [D + E]$.
   1. Give the expected value (mean), variance and standard deviation of total returns, $E\,R \cdot [D + E]$, and of the entrepreneur's profits, $E\,(R \cdot [D + E] - 0.08D)$.
   2. Give the expected value (mean), variance and standard deviation of the entrepreneur's rate of return on equity, $E\,(R \cdot [D + E] - 0.08D)/E$.

K. From Kreps, problem 18.6, p. 427, on the securitization of risky projects.

L. The "mean-variance" preferences are a very simple way to represent the utility of various risky options (and you should read Kreps's coverage of them). Facing the random variable X, utility is given by

$$U(X) = E\,X - \lambda \mathrm{Var}\,(X)$$

where $\lambda > 0$ is a measure of the sensitivity to variance/risk.

Give the cdfs, the means, the variances, and, as a function of $\lambda > 0$, the utility of the following random variables.
   1. X is uniformly distributed on the interval $[0.5, 1.7]$.
   2. X' is equal to -0.2 with probability 0.05, and is uniformly distributed on the interval $[0.5, 1.7]$ with probability 0.95.
   3. Y is uniformly distributed on the interval $[0.7, 1.9]$.
   4. Y' is equal to -0.2 with probability 0.05, and is uniformly distributed on the interval $[0.7, 1.9]$ with probability 0.95.
   5. R is uniformly distributed on the interval $[-2, 6]$.
   6. $S = \max(0, R)$, that is, S has the cdf

$$F_S(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{4} + \frac{x}{8} & \text{if } 0 \le x \le 6. \end{cases}$$

M. Before the invention of LLC's (limited liability corporations), it was possible to lose more than one's investment in the firm. This problem provides a comparison of the incentives to invest before and after.

Suppose that R is uniformly distributed on the interval $[-1, 4]$ and that $S = \max(0, R)$, that is,

$$S = \begin{cases} 0 & \text{if } R \leq 0 \\ R & \text{if } 0 < R \end{cases}.$$

Having wealth w, an investor with mean-variance preferences and risk sensitivity λ is looking at investing x into the firm, $0 \leq x \leq w$. Let $x_R^*(\lambda)$ be the solution to

$$\max_{0 \leq x \leq w} E\left[(w - x) + Rx\right] - \lambda \text{Var}\left((w - x) + Rx\right),$$

and let $x_S^*(\lambda)$ be the solution to

$$\max_{0 \leq x \leq w} E\left[(w - x) + Sx\right] - \lambda \text{Var}\left((w - x) + Sx\right).$$

1. For $\lambda' > \lambda$, which is larger, $x_R^*(\lambda')$ or $x_R^*(\lambda)$? Give both a mathematical and an economic argument for your answer.
2. For $\lambda' > \lambda$, which is larger, $x_S^*(\lambda')$ or $x_S^*(\lambda)$? Give both a mathematical and an economic argument for your answer.
3. For fixed $\lambda > 0$, which is larger, $x_R^*(\lambda)$ or $x_S^*(\lambda)$? Give both a mathematical and an economic argument for your answer.

N. Suppose that the probability of loss $L = 100,000$ from self-dealing behavior by a manager can be reduced at a monitoring cost of c, specifically, that $P(c) = \gamma \cdot e^{-c}$ where $\gamma = 0.2$ and c is measured in units of thousands of dollars. The problem is how large a cost to incur while trading off losses and monitoring costs.

1. Give three examples of what the textbook calls "self-dealing."
2. How do the legal structures of duty of loyalty/fiduciary obligations treat self-dealing?
3. Solve the monitoring problem, that is, solve
$$V(\gamma, L) = \min_{c \geq 0} \left(L \cdot \gamma \cdot e^{-c} + c\right)$$
and give optimal probability of loss as a function of L and γ.
4. If $V(\gamma, L)$ above is too large, it may not be profitable to run the firm using a manager who might self-deal. What kind(s) of solutions does the textbook propose? How do the problems (above) on risk and leverage enter into their feasibility?
5. Suppose that the background probability of loss decreases from γ to $\gamma' = 0.1$. What happens to the optimal c? To the optimal probability of loss? Give both a mathematical and an economic argument for your answer.
6. Suppose that the size of loss increases for $L = 100,000$ to $L' = 150,000$. What happens to the optimal c? To the optimal probability of loss? Give both a mathematical and an economic argument for your answer.
7. Returning to first problem (with $\gamma = 0.2$ and $L = 100,000$), suppose that the company has its employees bonded, that is, suppose the company buys complete insurance, at a price p, against their self-dealing. What does this insurance purchase do to the solution to the problem in part N3? [This is an example of what is called "moral hazard."]
8. Suppose now that the insurance contract specifies that the losses will not be made good if the company exerts a care level less than c°. Give the set of prices p and care levels c° that make both the insurance company and the firm happy with the contract.

O. An employee causes a car accident while on company business.
   1. Who is liable? The employee or the company? What is the name of the legal doctrine behind your answer?
   2. What market insurance purchase decisions does this affect?
   3. How might moral hazard enter into your answer?
   4. What kinds of legal structures might the insurance company use to manage this moral hazard?

P. When one looks at historical statistics about R&D rates, one finds that it is concentrated in the larger firms. Such figures do not include a recent phenomenom, the growth in the number of firms that specialize in doing contract R&D, often for the government, but increasingly in the recent past, for the large pharmaceutical firms who have been "outsourcing their brains." In this problem, you are going to investigate a simple case of how being large can give a decreasing risk-adjusted average cost of doing R&D. Behind the results you will find here is the notion of portfolio diversification.

   We are going to suppose that research projects cost C, that C is "large," and that research projects succeed with probability p, that p is "small," and that if the project does not succeed, then it fails and returns 0. Thus, the distribution of returns for a project are $(R - C)$ with probability p and $0 - C$ with probability $1 - p$. Since R, if it happens, will be off in the future and the costs, C, must be borne up front, we are supposing that R measures the net present value of the eventual success if it happens.

   The expected or average return on a research project is $p(R - C) + (1 - p)(-C)$ which is equal to $pR - C$, expected returns minus expected costs. We assume that $pR > C$, that is, that expected returns are larger than expected costs. We are also going to assume that success on different projects are independent of each other. Specifically, if you take on two projects, then the probability that both succeed is p^2, the probability that both fail is $(1 - p)^2$, and the probability that exactly one of them succeeds is $[1 - p^2 - (1 - p)^2]$, that is, $2p(1 - p)$.

   A heavily used measure of the risk of a random return is its variance or its standard deviation. We let $\mu = pR - C$ be the average or expected return of a single project, the standard deviation is then $p\sqrt{(R - C) - \mu} + (1 - p)\sqrt{(-C) - \mu}$, which is denoted σ. Of particular interest are two unitless measures: $\frac{\sigma}{\mu}$, the risk/reward ratio for the project; and its inverse, $\frac{\mu}{\sigma}$, the risk adjusted measure of the average return.

   1. If $R = 10^7$ and $C = 100,000$, find the set of p for which the expected value, μ, is positive. For these p, give the associated σ and $\frac{\mu}{\sigma}$. Graph your answers in an informative fashion.
   2. Now suppose that your research budget is expanded, and you can afford to undertake two projects. Verify that the expected value is now $2 \cdot \mu$. Verify that the new σ for the R&D division is $\sqrt{2}$ times the answer you previously found. What has happened to the risk adjusted measure of the average return?
   3. Repeat the previous two problems with $R = 10^8$ and $C = 200,000$.
   4. In the inventory problem from a previous assignment, there was a power law giving the advantage of being larger. Give the general form of the power law relating the research budget to the risk adjusted rate of return.