Assignment #3 for Managerial Economics, ECO 351M, Fall 2016
Due, Monday October 10.

1. From Ch. 15 of Kreps's *Micro for Managers*, Problem 15.2.
2. From Ch. 15 of Kreps's *Micro for Managers*, Problem 15.3.
3. From Ch. 15 of Kreps's *Micro for Managers*, Problem 15.5.
4. From Ch. 16 of Kreps's *Micro for Managers*,
   a. Problem 16.1.
   b. Problem 16.2.
5. From Ch. 17 of Kreps's *Micro for Managers*,
   a. Problem 17.1.
   b. Problem 17.2, (a)-(c).
6. From Ch. 17 of Kreps's *Micro for Managers*,
   a. Problem 17.3.
   b. Problem 17.4.
7. From Ch. 17 of Kreps's *Micro for Managers*, Problem 17.6.
8. When one looks at statistics measuring the competence with which firms are run, after adjusting for the industry, one finds a weak effect in favor of firms with female CEO's, and a much stronger effect in favor of larger firms. In this problem, you are going to investigate a different advantage of being large, the decreasing average cost aspect of simple inventory systems. Decreasing average costs sometimes go by the name of economies of scale, and economies of scale are a crucial determinant of the horizontal boundary of a firm. In this problem, you will find a power law relating size to costs.

   Your firm needs Y units of, say, high grade cutting oil per year. Each time you order, you order an amount Q at an ordering cost of $F + pQ$, where F is the fixed cost of making an order (e.g. you wouldn't want just anybody to be able to write checks on the corporate account and such sytems are costly to implement), and p is the per unit cost of the cutting oil. This means that your yearly cost of ordering is $\frac{Y}{Q} \cdot (F + pQ)$ because $\frac{Y}{Q}$ is the number of orders per year of size Q that you make to fill a need of size Y.

   Storing anything is expensive, and the costs include insurance, the opportunity costs of the space it takes up, the costs of keeping track of what you actually have, and so on. We suppose that these stockage costs are s per unit stored. Computerized records and practices like bar-coding have substantially reduced s over the last decades. Thus, when you order Q and draw it down at a rate of Y per year, over the course of the cycle that lasts Q/Y of a year, until you must re-order, you store, on average $Q/2$ units. This incurs a per year cost of $s \cdot \frac{Q}{2}$. Putting this together, the yearly cost of running an inventory system to keep you in cutting oil is

$$C(Y) = \min_{Q} \left[\frac{Y}{Q} \cdot (F + pQ) + s \cdot \frac{Q}{2} \right], \tag{1}$$

   and the solution is $Q^*(Y, F, p, s)$.

   a. Without actually solving the problem in equation (1), find out whether Q^* depends positively or negatively on the following variables, and explain, in each case, why your answers makes sense: Y; F; p; and s.

b. Now explicitly find the optimal tradeoff between fixed costs and storage costs to solve for $Q^*(Y, F, p, s)$ and $C(Y)$.
c. Find the marginal cost of an increase in Y. Verify that the average cost, $AC(Y)$, is decreasing and explain how your result about the marginal cost implies that this must be true.
d. With the advent and then lowering expenses of computerized inventory and accounting systems, the costs F and s have both been decreasing. Does this increase or decrease the advantage of being large?

9. When one looks at historical statistics about R&D rates, one finds that it is concentrated in the larger firms. Such figures do not include a recent phenomenom, the growth in the number of firms that specialize in doing contract R&D, often for the government, but increasingly in the recent past, for the large pharmaceutical firms who have been "outsourcing their brains." In this problem, you are going to investigate a simple case of how being large can give a decreasing risk-adjusted average cost of doing R&D. Behind the results you will find here is the notion of portfolio diversification.

We are going to suppose that research projects cost C, that C is "large," and that research projects succeed with probability p, that p is "small," and that if the project does not succeed, then it fails and returns 0. Thus, the distribution of returns for a project are $(R-C)$ with probability p and $0-C$ with probability $1-p$. Since R, if it happens, will be off in the future and the costs, C, must be borne up front, we are supposing that R measures the net present value of the eventual success if it happens.

The expected or average return on a research project is $p(R-C)+(1-p)(-C)$ which is equal to $pR - C$, expected returns minus expected costs. We assume that $pR > C$, that is, that expected returns are larger than expected costs. We are also going to assume that success on different projects are independent of each other. Specifically, if you take on two projects, then the probability that both succeed is p^2, the probability that both fail is $(1-p)^2$, and the probability that exactly one of them succeeds is $[1 - p^2 - (1-p)^2]$, that is, $2p(1-p)$.

A heavily used measure of the risk of a random return is its standard deviation, which is the square root of the average squared distance of the random return from its average. We let $\mu = pR-C$ be the average or expected return of a single project, the standard deviation is then $p\sqrt{(R-C)-\mu} + (1-p)\sqrt{(-C)-\mu}$, which is denoted σ. Of particular interest is the ratio $\frac{\sigma}{\mu}$, a unitless measure giving the risk/reward ratio for the project. Of interest is the comparison of the risk/reward ratio when you have one project and when you have two. Its inverse, $\frac{\mu}{\sigma}$ is a risk adjusted measure of the average return.

a. If $R = 10^7$ and $C = 100,000$, find the set of p for which the expected value, μ, is positive. For these p, give the associated σ and $\frac{\mu}{\sigma}$. Graph your answers in an informative fashion.
b. Now suppose that your research budget is expanded, and you can afford to undertake two projects. Verify that the expected value is now $2\cdot\mu$. Verify that the new σ for the R&D division is $\sqrt{2}$ times the answer you previously found. What has happened to the risk adjusted measure of the average return?
c. Repeat the previous two problems with $R = 10^8$ and $C = 200,000$.

d. In the inventory problem above, there was a power law giving the advantage of being larger. Give the general form of the power law relating the research budget to the risk adjusted rate of return.