Solutions to Assignment #1 for Managerial Economics
ECO 351M, Fall 2016

1. From Ch. 3 of Kreps's *Micro for Managers*,
   a. Problem 3.1. There are (at least) two ways to proceed: write out profit function, take derivative w.r.t. x, set equal to 0, check that you're at a maximum rather than a minimun; set Marginal Revenue equal to Marginal Cost (an implication of the derivative of profits being equal to 0) and solve, then check that the solution is actually the maximum. [See Problem 3.6 for a graphical example where you can check the second order derivatives and think through the logic of having MR = MC while being at a minimum profit point.]
   Profits are
$$\pi(x) = \underbrace{x \cdot (100 - \frac{x}{100})}_{Rev} - \underbrace{(200 + 20x + \frac{x^2}{300})}_{TCost}.$$
   As this is a quadratic in x opening downwards, the unique maximum happens at the point $\pi'(x^*) = 0$ (stating that counts as checking that we've found a maximum rather than a minimum).
$$\pi'(x) = 100 - \tfrac{x}{50} - 20 - \tfrac{x}{150},$$
   setting equal to 0 yields $80 = \frac{4x}{150}$, solving yields $x^* = \frac{80 \cdot 150}{4} = 3,000$.
   b. Problem 3.2. Proceed as in the previous problem, $\pi'(x) = 16x - \frac{x}{500}$ so $x^* = 8,000$.
   c. Problem 3.8. The profit function is
$$\pi(x_p, x_q) = [100x_p - \tfrac{x_p^2}{100} - \tfrac{x_p x_q}{400}] + [80x_q - \tfrac{x_q^2}{50} - \tfrac{x_p x_q}{200}] - 300 - 20x_p - 10x_q - \tfrac{9x_p^2 + 3x_p x_q + x_q^2}{1200}.$$
   Gathering terms yields
$$\pi = [80x_p + 60x_q] - \tfrac{7x_p^2}{400} - \tfrac{x_p x_q}{100} - \tfrac{61x_q^2}{1200} - 300.$$
   Doing the (matrix) algebra to show that this is also a quadratic that opens downwards is tricky if you don't know the material, easy if you do. In any case, set $\partial\pi/\partial x_p = 0$ and $\partial\pi/\partial x_q = 0$. This yields two linear equations in two unknowns, solve them, approximately, for $x_p = 1,888$, $x_q = 1,114$.

2. From Ch. 3 of Kreps's *Micro for Managers*,
   a. Problem 3.9. There are, again, (at least) two ways to proceed: write out the profit as a function of x_r and x_b and simultaneously solve the equations $\partial\pi/\partial x_r = 0$ and $\partial\pi/\partial x_r = 0$, noting that it is the sum of two quadratics opening downwards, so simultaneous solution of these two gives the optimum; find $MR_r(x_r)$, $MR_b(x_b)$, and $MC(x_r + x_b)$, and set them simultaneously equal, check that this gives a maximum rather than a minimum.
$$\pi(x_r, x_b) = 20x_r - \tfrac{x_r^2}{1000} + 17x_b - \tfrac{x_b^2}{2000} - 4(x_r + x_b) - \tfrac{x_r^2 + x_r x_b + x_b^2}{4000},$$
$$\tfrac{\partial\pi}{\partial x_r} = 20 - \tfrac{x_r}{500} - 4 - \tfrac{x_r}{2000} - \tfrac{x_b}{4000}$$
$$\tfrac{\partial\pi}{\partial x_b} = 17 - \tfrac{x_b}{1000} - 4 - \tfrac{x_b}{2000} - \tfrac{x_r}{4000}$$
   Solve these two equations.

b. Problem 3.11. One way to proceed, solve for $x_b(x_r)$ such that the marginal revenues are equal, then solve $x_b + x_b(x_r) = 2000$. Another way to proceed, set $x_b = 2000 - x_r$, write out profits as $\pi(x_r, 2000 - x_r)$ and solve the one-dimensional problem. Because the costs are constant at $4 \cdot 2000 + 2000^2/4000$, you will again have equality of marginal revenues. Either way, $x_r^* = 1,666\frac{2}{3}$ and $x_b^* = 333\frac{1}{3}$.

3. From Ch. 4 of Kreps's *Micro for Managers*,

a. Problem 4.5. Here the price elasticity is greater than 1 in absolute value, so demand is sufficiently responsive to price cuts that it is optimal to decrease price.

(a) A decrease in the price of \$0.10 is a $100 \cdot \frac{10}{800}$ percentage change, that is, a decrease of %1.25. 3 times that is an increase of %3.75 in quantity, so they will sell $1.0375 \cdot 10,000$ at a price of \$7.90, for revenues of \$81,962.50.

(b) This is a 1.5% decrease in quantity, corresponding to a 0.5% increase in price, yielding revenues $9850 \cdot 0.995 \cdot 8 = 78,406$.

b. Problem 4.6. The price elasticity of demand is $\nu(x) = 1/(\frac{d\log p(x)}{d\log(x)}) = \frac{p(x)}{xp'(x)}$. The one-good profit maximization problem is

$$\max_{x \geq 0} xp(x) - c(x), \text{ with FOCs } p(x) + xp'(x) = c'(x).$$

Rearranging the left-hand side of the FOCs yields

$$p(x)\left[1 + \frac{xp'(x)}{p(x)}\right] = c'(x), \text{ or } p(x)\left[1 + \frac{1}{\nu(x)}\right] = c'(x).$$

With $\nu(x) = -4$, we have $c'(x) = \frac{3}{4}p(x)$, so the marginal cost is (approximately) \$15,000.

c. Problem 4.8. From just above, we have $c'(x) = p(x)\left[1 + \frac{1}{\nu(x)}\right]$. In this problem, we have $c'(x) = c$ and $p(x) = 1.2c$. Combining, $\left[1 + \frac{1}{\nu(x)}\right] \cdot 1.2 = 1$, that is $\nu(x)$ is (approximately) -6.

4. From Ch. 4 of Kreps's *Micro for Managers*,

a. Problem 4.9. The inverse demand functions are $x_y = 10,000 - 1,000 \cdot p_y$, $x_m = 30,000 - 2,000 \cdot p_m$, and $x_s = 10,000 - 800 \cdot p_x$. Adding gives the aggregate inverse demand function, $x = 50,000 - 3,800p$, for a demand function of $p(x) \simeq 13.6 - \frac{x}{3,800}$.

b. Problem 4.10. The demand function is

$$p(x) = \begin{cases} 20 - \frac{x}{5,000} & \text{if } 0 \leq x \leq 30,000 \\ 16 - \frac{x}{15,000} & \text{if } 30,000 \leq x \leq 240,000 \end{cases}$$

This means that marginal revenue is

$$R'(x) = \begin{cases} 20 - \frac{x}{2,500} & \text{if } 0 \leq x \leq 30,000 \\ 16 - \frac{x}{7,500} & \text{if } 30,000 \leq x \leq 240,000 \end{cases}$$

Notice that the marginal revenue is not continuous, it jumps upwards at $x = 30,000$, jumping from $8 = 20 - \frac{30,000}{2,500}$ to $12 = 16 - \frac{30,000}{7,500}$. There are two solutions to $R'(x) = 10$, one at $x = 25,000$ and one at $x = 45,000$. The larger one maximizes profits (as you should check), and corresponds to a price of $16 - \frac{45,000}{15,000} = 13$.

c. Problem 4.13.

The first part of the problem asks about the perfectly discriminating monopolist with demand functions $p_T = 5 - \frac{x_T}{120}$ and $p_L = 3 - \frac{x_L}{180}$. The marginal revenue for tourists is $5 - \frac{x_T}{60}$, setting this equal to the marginal cost, 1.2, yields $x_T^* = 228$ and $p^*T = 3.1$. For locals, the parallel calculations yield $MR_L = 3 - \frac{x_L}{90}$, and this is equal to MC at $x_L = 162$. The corresponding profits are $\pi_T = (5 - \frac{228}{120}) \cdot 228 - 1.2 \cdot 228 = 433.20$ and $\pi_L = (3 - \frac{162}{180}) \cdot 162 - 1.2 \cdot 162 = 145.80$. Total profits are then 579.

One can also do the first part of the problem using prices as the bakery's decision variable: $x_T = 120(5 - p_T) = 600 - 120p_T$ so $\pi_T(p_T) = (p_T - 1.2) \cdot (600 - 120p_T)$ which achieves its maximum at $p_T^* = 3.1$ as above. For the locals, the calculation is $\pi_L(p_L) = (p_L - 1.2) \cdot (540 - 180p_T)$ which achieves its maximum at $p_L^* = 2.1$ as above.

The second part asks you to horizontally sum the inverse demand functions $x = 600 - 120p_T$ for $0 \leq p \leq 5$ and $x = 540 - 180p$ for $0 \leq p \leq 3$. This yields

$$x(p) = \begin{cases} 1140 - 300p & \text{if } 0 \leq p \leq 3 \\ 600 - 120p & \text{if } 3 \leq p \leq 5 \end{cases}$$

Now, profit is $\pi(p) = (p - 1.2) \cdot x(p)$, maximizing yields $p^* = 2.5$, for profits of $1.3 \cdot (1140 - 300 \cdot 2.5) = 507$. As we knew must happen, profits are lower, here 507 vs. 579, when the monopolist cannot discriminate between the two populations.

The third part asks you to compare the welfare of the locals and the Town Council. In the case that you allow the bakery to perfectly discriminate as a monopolist, go back to the first case and calculate the consumer surplus for the local demand function. Consumer surplus decreases as a function of the price they are charged, when they are on their own, they are charged 2.1, when they are tossed in with the less price sensitive tourists, they are charged the higher price 2.5. (The consumer surpluses are 72.9 and 22.5 respectively.) Here, the council's interest in getting money from the baker's contributions and the council's interest in the welfare of their own consumers go hand in hand. However, the surplus of the tourists goes down, they are charged the higher price 3.1 rather than 2.5 if they are separately identifiable by the bakery. If this ends up meaning less tourist revenue, not only because they get a less good deal that they can tell their friends about, but because they now miss the 'local color,' it could be bad.

5. From Ch. 5 of Kreps's *Micro for Managers*,

   a. Problem 5.3. The marginal utilities are $\frac{6}{b}$, $\frac{2}{c}$, $\frac{1}{s}$, and 1. Equalizing the bang-for-the-buck requires $\frac{6}{b} = 1.20$, $\frac{2}{c} = 3.00$, $\frac{1}{s} = 4$. The utility maximizing choices are: $b^* = 5$, $c^* = 2/3$, $s^* = 4$, and $m^* = 148$.

   b. Problem 5.4.

   (a) The marginal utilities multiplied by a constant to be determined, call it κ, must equal the prices of all goods that are consumed at positive levels. Guessing that all are consumed, this yields

$$\kappa\frac{8}{b+2} = 1, \kappa\frac{6}{c+1} = 2, \kappa frac42s + 1 = 4.$$

   These need to be combined with $1b + 2c + 4s = 18$, which yields $b^* = 10$, $c^* = 7.5$, and $s^* = 0.25$.

(b) Attempting the previous with 6.50 to spend ends up with negative consumption of good s, so $s^* = 0$. The remaining demands are $b^* = 4$ and $c^* = 1.25$.
(c) Now we know that the marginal utilities of all goods consumed must be equal to 1. Working through, as long as income is above 16, we will have $b^* = 6$, $c^* = 2$, $s^* = 1.5$, corresponding to expenditures of $1 \cdot 6 + 2 \cdot 2 + 4 \cdot 1.5 = 16$. If $w = 50$, then $m^* = 50 - 16$, if $w = 500$, the $m^* = 500 - 16$, if $w = 18$, then $m^* = 18 - 16$. If $w = 6.50$, then one will spend the money as in problem (b).

c. Problem 5.5.
(a) The marginal utilities multiplied by a constant to be determined, call it κ, must equal the prices of all goods that are consumed at positive levels. Guessing that all are consumed at $w = 83$ yields

$$b = 5\kappa, \ c = \tfrac{1}{5}\kappa - 1, \ s = \tfrac{1}{20}\kappa - 4.$$

Combining with the budget equation yields $\kappa = \frac{76}{23}$. One should plug these in to find the demands, checking that they are positive, and then, just to be sure, check that the expenditures are equal to 83.
(b) Similar.

6. From Ch. 5 of Kreps's *Micro for Managers*,
   a. Problem 5.11.
   b. Problem 5.12.

7. A biotech firm . . . .
   a. Let $f(x, \theta) = (B_1(x) + \theta B_2(x)) - x$. When $\theta = 0$, we have the firm's profit maximization problem, when $\theta = 1$, we have society's total welfare maximization problem. Since $B_2(\cdot)$ is increasing, for $x' > x$ and $\theta' = 1 > \theta = 0$, we have

$$L = f(x', 1) - f(x, 1) = [(B_1(x') - x') - (B_1(x) - x)] + B_2(x') - B_2(x)$$
$$R = f(x', 0) - f(x, 0) = [(B_1(x') - x') - (B_1(x) - x)]$$

   Since $L - R = B_2(x') - B_2(x) > 0$, the function is supermodular, hence society's optimum, $x^*(1)$, should be greater than the firm's optimum, $x^*(0)$.
   b. Let $g(x, \theta) = (B_1(x) + \theta B_2(x)) - x$ for $0 \leq \theta \leq 1$ being the portion of social benefits captured by the firm. By the previous analysis, $x^*(\cdot)$ is increasing in θ.
   One of several possible ways to do the last part of the analysis: let $h(x, \theta) = B_1(x) - (1 - \theta)x$ where θ is the portion of costs subsidized. This is supermodular in x and θ, hence $x^*(\cdot)$ is again increasing in θ.

8. An oil company . . . .
   Let $H(f_i, \theta) = \Pi_i(f_i) - C_i(f_i) - \theta C_j(f_i)$. When $\theta = 0$, the problem $\max_{f_i} H(f_i, \theta)$ is firm i's profit maximization problem, when $\theta = 1$, they take account of the negative externality they impose on firm j. The function $f_i^*(\theta)$ is therefore decreasing in θ.
9. One part . . . .
   Let x be the amount of training, let $f(x, \theta) = B_C(x) + \theta B_i(x) - C(x)$ where $B_C(\cdot)$ is the benefit to the consulting firm and $B_i(\cdot)$ is the benefit to the individual. When $\theta = 0$, the problem $\max_x f(x, \theta)$ is the firms problem, when

$\theta = 1$, the problem takes into account some of the benefits not captured by the consulting firm, and $f(\cdot, \cdot)$ is supermodular.

10. For $x, t \in [1200, 1900]$, let $f(x, t) = xt$.

    The problem $\max_{x \in [1200,1900]} f(x, t)$ has $x^*(t) \equiv \{1900\}$ (not $x^*(t) = \{100\}$ as in the problem). Let $g(x, t) = \log(f(x, t))$, $\partial g/\partial x = \frac{1}{x}$, $\partial^2 g/\partial x \partial t = 0$. Let $h(x, t) = \log(g(x, t))$, $\partial h/\partial x = (\frac{1}{x})/(\log(x) + \log(t))$ and $\partial^2 h/\partial x \partial t < 0$ because the denominator, $(\log(x) + \log(t))$ is a positive and strictly increasing function of t. Thus, $h(\cdot, \cdot)$ is strictly submodular while its monotonic transformation $f(\cdot, \cdot)$ is strictly supermodular.

11. Suppose that an organization $\ldots$.

    This is an application of the textbook's horizontal summing of demand curves to the problem we did in class.