Solutions for Assignment #2, Managerial Economics, ECO 351M, Fall 2016
Due, Monday Sept 26.

1. From Ch. 8 of Kreps's *Micro for Managers*,
   a. Problem 8.1. (a), Figure 8.4, fixed, 50, plus rising, $x/8000$, marginal cost. (b) $AC(x) = 10,000,000/x + 50 + x/16,000$. The AC curve opens upwards, comes to a unique minimum at the point where $AC'(x) = 0$, which is the solution to $10^7/x^2 = 1/16 \cdot 10^3$, or $x^\circ = 4 \cdot 10^5$. The point $(x^\circ, AC(x^\circ))$ is where the MC and the AC curves cross. (c) x° is the efficient scale, and $AC(x^\circ) = 100$.
   b. Problem 8.2, any 2 of the 5 parts. Straightforward, except that (c) has no efficient scale because $AC(x) > MC(x)$ for all $x > 0$.

2. From Ch. 8 of Kreps's *Micro for Managers*,
   a. Problem 8.10. For panel a, the intersection of the MC and MR curves must be down and below the minimum of the AC curve because the MC curve increases until it meets the AC curve at its minimum. For panel b, the MC curve crosses the AC curve from below at the minimum of the AC curve. As the MC curve must keep increasing after that for the AC curve to be increasing, the intersection with the MR curve is above and to the right of the minimum of the AC curve. For panel c, we have the trifecta, the MR curve cross the AC curve at the minimum of the AC curve, hence exactly where the MC curve also crosses.
   b. Problem 8.11. There are a couple of ways to proceed, and we'll cover two of them. First, to compare the problems $\max_{x>0} \log(\pi(x))$ and $\max_{x>0} \log(\pi(x)/x) = \log(\pi(x)) - \log(x)$, let $f(x, \theta) = \log(\pi(x)) - \theta \log(x)$, note that $f(\cdot, \cdot)$ is submodular, hence $x^*(1) < x^*(0)$. Second, suppose that maxima exist and that the derivatives are equal to 0 at those points. The FOCs for $\max_{x>0} \pi(x)$ are $\pi'(x^*) = 0$, the FOCs for $\max_{x>0} \frac{\pi(x)}{x}$ are $\frac{x_m \pi'(x_m) - \pi(x_m)}{x_m^2} = 0$. Since $x_m^2 > 0$, we can only satisfy the FOCs if $x_m \pi'(x_m) - \pi(x_m) = 0$, that is only if $\pi'(x_m) = \pi(x_m)/x_m$ and $\pi(x_m)/x_m > 0$. If there is only one local maximum for the problem derivatives are equal to 0 at those points. The FOCs for $\max_{x>0} \pi(x)$, then the only places where $\pi'(x) > 0$ are to the left of (smaller than) x^*.

3. From Ch. 9 of Kreps's *Micro for Managers*,
   a. Problem 9.1. For this problem, I'll take the x-axis as the lathe hours, the y-axis as the labor hours. (a) The isoquant for 1 chair consists of three segments: the line joining $(1, 5)$ to $(4, 2)$; the horizontal line starting at $(4, 2)$ and going to the right; and the vertical line starting at $(1, 5)$ and going upwards. Scaling all of these by 4 or 6 gives the isoquants for 4 chairs or 6 chairs respectively. (b) Along the first line segment, the hours are perfect substitutes, so the firm should use as much labor and as little lathe as possible, i.e. $(6, 30)$ for a total cost of $6 \cdot 15 + 30 \cdot 10 = 390$ for 6 chairs. (c) The MC is constant at $390/6 = 65$, there are no fixed costs, graph as in Ch. 8.
   b. Problem 9.3. (a) No-substitutions. (b) Fixed coefficients. (c) Constant MRS.

4. From Ch. 9 of Kreps's *Micro for Managers*,
   a. Problem 9.6.

Ans. For any $x > 0$, the cost function is

$$c(x; p_m, p_\ell) = 300 + \min\ (p_m \cdot m + p_\ell \cdot \ell) \text{ subject to } m^{1/3}\ell^{1/6} \geq x.$$

Since $(p_m \cdot m + p_\ell \cdot \ell)$ is strictly monotonic, the solution will happen at a point where $m^{1/3}\ell^{1/6} = x$. Solving this for ℓ in terms of m yields $\ell(m) = x^6/m^2$. Substituting this into $(p_m \cdot m + p_\ell \cdot \ell)$ yields $(p_m \cdot m + p_\ell \cdot (x^6/m^2))$, minimizing yields the FOCs

$$p_m = \frac{2p_\ell x^6}{m^3}, \text{ or } m = \left[(2\frac{p_\ell}{p_m})^{1/3}x^2\right].$$

Substituting this into $\ell(m)$ yields

$$\ell = x^2(\frac{p_m}{2p_\ell})^{2/3}).$$

Finally, this yields the cost function

$$c(x; p_m, p_\ell) = 300 + x^2\left[p_m(2\frac{p_\ell}{p_m})^{1/3}) + p_\ell(\frac{p_m}{2p_\ell})^{2/3})\right].$$

The marginal cost is $\partial c(x; p_m, p_\ell)/\partial x$, and

$$\frac{\partial c(x;p_m,p_\ell)}{\partial x} = 2x\left[p_m(2\frac{p_\ell}{p_m})^{1/3}) + p_\ell(\frac{p_m}{2p_\ell})^{2/3})\right].$$

Marginal revenue is $160 - 4x$. Put in the prices $p_m = 1$ and $p_\ell = 4$ and solve $MR = MC$.

b. Problem 9.8. (a) The X is at the point $(28, 20)$. (b) The big dot is at the point $(18, 18)$. Assuming, in addition to decreasing returns to scale, that the production technology is homothetic, draw the line through the dot to the origin, it crosses the 10 isoquant at $(12, 12)$. $18/12 = 1.5$, so the most that a decreasing returns to scale technology could produce at the dot is just under $10 \cdot 1.5 = 15$. Thus, 12 and 14 are possible, 16 and 18 are not.

5. From Ch. 9 of Kreps's *Micro for Managers*,

a. Problem 9.16. This problem extends our previous analyses of transfer pricing between divisions of a firm.

(a) $\max_{x_i \geq 0}\ \pi_i(x_i; q) = qx_i - TC(x_i)$ has $x_i^* = 0$ if $q \leq TC'(0)$ because $MC(x_i) = TC'(x_i)$ is an increasing function — if one loses money on the first unit, one will only lose more money by producing more at an even higher cost. If $q > MC(0)$, then, because MC is an increasing function, $\pi_i(\cdot; q)$ is strictly concave and achieves its unique maximum where $MC(x_i) = q$. (b) If $q < q_i^*$, then producing anything loses money while producing 0 makes and loses 0. For $q > q_i^*$, it is possible to make money by producing, hence it should happen at $q = MC(x_i)$. For $q = q_i^*$, producing at the minimal AC makes 0, as does producing 0, and that is the best that one can do. (c) This is the textbook's one mention of monotone comparative statics, what we have called super/submodularity. Observe that the profit function is supermodular in q and x, hence the conclusion follows. (d) This is the observation discussed in class. (e) Here we are summing supply functions.

6. From Ch. 10 of Kreps's *Micro for Managers*,

a. Problem 10.2.

Ans. $SRTC(x) = 684 + x^3/8$, $LRTC(x) = 300 + 3.4344x^2$. At $x = 16$, these are $LRTC(16) = 1179.21$ and $SRTC(16) = 1196.00$. The status quo production plan is 64 units of ℓ and 256 units of m. This is not the cost minimizing long-run plan at the new input prices. In the short run, the fir is stuck with $\ell = 64$, but in the long run, because ℓ has become more expensive, increase m and decrease ℓ.

7. Four problems on discrete discounting.
   a. What is the maximum amount you would pay for an asset that generates an income of \$250,000 at the end of five years if the opportunity cost of using funds is %8?
   Ans. $250,000/(1.08)^5 \simeq 170,146$.
   Ans. For $0 < \rho < 1$, $\sum_{t=1}^{\infty} \rho^t = \frac{\rho}{1-\rho}$. Setting $\rho = 1/(1-r)$ where $r = 0.05$, $\rho/(1-\rho) = 1/r = 20$. So the answer is $20 \cdot 125 = 2,500$. If the company begins to look riskier, the dividends may be interrupted at some random time T so the expected net present value is $E \sum_{t=1}^{T} 125\rho^t$. Replacing "$\infty$" with a smaller T lowers the value, hence one should use a higher interest rate/lower discount factor to evaluate the value. Note that the assumption here is that we are one year from the first dividend. If you assumed that the dividend was coming tomorrow, then the sum would be $\sum_{t=0}^{\infty} \rho^t = \frac{\rho}{1-\rho}$, which increases the value a little bit.
   b. What is the value of a preferred stock that promises to pay a perpetual dividend of \$125 at the end of each year when the interest rate is %5? In which direction would you move the interest rate you use to evaluate this net present value if the company's prospects begin to look riskier? Why?
   Ans. For $0 < \rho < 1$, $\sum_{t=1}^{\infty} \rho^t = \frac{\rho}{1-\rho}$. Setting $\rho = 1/(1-r)$ where $r = 0.05$, $\rho/(1-\rho) = 1/r = 20$. So the answer is $20 \cdot 125 = 2,500$. If the company begins to look riskier, the dividends may be interrupted at some random time T so the expected net present value is $E \sum_{t=1}^{T} 125\rho^t$. Replacing "$\infty$" with a smaller T lowers the value, hence one should use a higher interest rate/lower discount factor to evaluate the value. Note that the assumption here is that we are one year from the first dividend. If you assumed that the dividend was coming tomorrow, then the sum would be $\sum_{t=0}^{\infty} \rho^t = \frac{\rho}{1-\rho}$, which increases the value a little bit.
   c. An owner can lease her building for \$120,000 per year for three years. The explicit cost of maintaining the building is \$40,000, and the implicit cost is \$55,000. All revenues are received, and costs borne, at the end of each year. If the interest rate is %5, determine the present value of the stream of accounting profits, and the present value of the stream of economic profits.
   Ans. The net present value of the accounting profits are $\sum_{t=1}^{\infty} \frac{120{,}000-40{,}000}{1.05^t}$, for economic profits, $\sum_{t=1}^{\infty} \frac{120{,}000-95{,}000}{1.05^t}$. One could (reasonably) sum from $t = 0$ as well depending on how you interpret the problem. Faced with a real problem, you would look at when the payments/expenses come due and use those times, and if the payments happened at small intervals, continuous discounting would give a more accurate answer.
   d. You are in the market for a new frig and you have narrowed the search to two models. The energy-efficient model sells for \$700 and will save you \$45 per year. For the purposes of use, the standard model is indistinguishable from the energy-efficient model except that it costs \$500. Assuming that your opportunity cost of funds is %6, which frig should you purchase?
   Ans. The net present value of the extra expense is 200. The net present value of the savings is, using discrete discounting and assuming that bills are

paid yearly at the end of the year, $\sum_{t=1}^{\infty} \frac{45}{1.06^t}$. The expensive, energy efficient frig wins.

8. [A problem on continuous discounting] A project accumulates costs at a rate C for the interval $[0, T]$, measured in years, then accumulates benefits, B, in perpetuity, money is discounted continuously at rate r where $r = 0.12$ corresponds to an interest rate of 12% per annum. Fill in the 8 (eight) blank entries in the following table where "$npv(r)$" stands for the net present value at interest rate r.

C	B	T	r	$npv(r)$
10	15	3	0.12	
10	15	3	0.18	
10	15	3	0.24	
10	15	3		0
20	75	8	0.12	
20	75	8	0.18	
20	75	8	0.24	
20	75	8		0

These problems are applications of the formula

$$npv(r) = \int_0^T (-C)e^{-rt}\,dt + \int_T^{\infty} Be^{-rt}\,dt.$$

From calculus class, $\int_0^T (-C)e^{-rt}\,dt = (-C)\frac{1}{r}(1 - e^{-rT})$ and $\int_T^{\infty} Be^{-rt}\,dt = B\frac{1}{r}e^{-rT}$, this gives

$$npv(r) = \tfrac{1}{r}\left[(-C)e^{-rT} + B(1 - e^{-rT})\right].$$

To find the r such that $npv(r) = 0$ involves solving $\left[(-C)e^{-rT} + B(1 - e^{-rT})\right] = 0$, which yields $e^{-rT} = \frac{B}{B+C}$, or $r = -\frac{1}{T}\log(B/(B+C))$. Start your calculators.

9. You take out a loan for L agreeing to payback at a rate x per year over the course of T years. Interest is continuously compounded at rate r so that $L = \int_0^T xe^{-rt}\,dt$.

   a. Find the payback rate, x, as a function of L, T, and r. Explain the intuitions for why x should depend in the fashion that it does on these three variables.
   Ans. $\int_0^T xe^{-rt}\,dt = x \cdot \frac{1}{r}(1 - e^{-rT})$. Solving $L = x \cdot \frac{1}{r}(1 - e^{-rT})$ for x yields $x = Lr/(1 - e^{-rT})$. If L goes up, you are borrowing more hence have to pay back more per period. If r goes up, you are borrowing at a larger rate of interest, hence have to pay more back. If T goes up, you have longer to pay the loan back, hence have to pay back at a lower per year rate.

   b. Find the necessary payback time T, as a function of x, L, and r. Explain the intuitions for why T should depend in the fashion that it does on these three variables, paying special attention to the case that there is no T solving the problem.
   Ans. First note that paying x back in perpetuity gives a value of $x\frac{1}{r}(1-0) = \frac{x}{r}$, so if $L > \frac{x}{r}$, then you cannot ever pay back the amount L at this x and r. Doing the more detailed algebra, solving $L = x \cdot \frac{1}{r}(1 - e^{-rT})$ for T yields $T = -\frac{1}{r}\log(1 - \frac{x}{r}L)$ which only makes any sense if $(1 - \frac{x}{r}L) > 0$, i.e. $L < \frac{x}{r}$. Provided it makes sense, $[r \Uparrow] \Rightarrow [T \Uparrow]$ because your future payments are

worth less, $[L \uparrow] \Rightarrow [T \uparrow]$ because it takes longer to pay back a larger amount, $[x \uparrow] \Rightarrow [T \downarrow]$ because paying more back per year means you're done faster.

c. Now suppose that bank that is lending you the money believes that your business will fail with probability λdt in any given small interval of time $[t, t + dt)$. Let τ be the random time until you fail, i.e. $P(\tau \leq t) = 1 - e^{-\lambda t}$. If the bank wants to set x such that the expected valued of your repayments until you fail is L, i.e. $E \int_0^\tau x^{-rt}\, dt = L$, find the expected payback rate, x, as a function of L, T, r and λ. [This is one version of what are called risk premia, that is, the extra that someone in a riskier situation must pay.]

Ans. As in class, replace r by $(r+\lambda)$, then repeat the analysis of the previous problem.